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## The Optimal Marketing Mix of Posted Prices, Discounts and Bargaining

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# The Optimal Marketing Mix of Posted Prices, Discounts and Bargaining 

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#### Abstract

We analyze the optimal marketing mix of pricing and bargaining when price takers buy at posted prices but bargainers attempt to negotiate discounts. The optimal bargaining strategy involves the firms offering bargainers randomly-sized discounts. Competing firms keep posted prices high to weaken the bargainers' outside option, and posted prices increase in the proportion of bargainers. We show that consumer surplus is concave and profits are convex in the proportion of bargainers: competition authorities desire a balance between bargainers and price takers while firms would rather all consumers bargained. Finally, we study the firms' strategic decision about how much bargaining discretion sales staff should be allowed. Both firms allowing full bargaining flexibility is always an equilibrium - but not always the most profitable one. If there are enough bargainers, both firms committing to only matching the rival's posted price is also an equilibrium as price matching moderates competition.


Keywords: Posted prices; list prices; bargaining; negotiation; haggling; discounts; outside option; price takers; competition policy; price matching.

JEL Classification: C78; D43; L13.

[^0]
## 1 Introduction

Retailers often set posted prices which are potentially negotiable. In this paper we want to discover the optimal marketing mix of posted prices and discounts off them in markets in which some consumers attempt to bargain while others dislike the notion of bargaining and instead buy at the posted prices. In particular, we would like to answer the following types of question:

1. How does the presence of the bargainers affect the posted prices that the firms choose to set and the profits that the firms earn?
2. What level of discounts should a firm offer bargainers as part of its marketing mix?
3. When should firms and competition authorities encourage more consumers to bargain and how might they attempt to do so?
4. How are the optimal pricing strategies affected by the bargainers' level of skill?
5. If the firms are able commit to limit the bargaining discretion of their sales staff, will they choose to restrict the availability of discounts?

In the United Kingdom (UK) discounts on big ticket items such as cars and package holidays have been available for some time. The Competition Commission (2000) report on the UK car market found that discounts off the public list price were common, large on average and highly variable (see in particular Appendices 7.1 and 7.2). For example, private buyer discounts off Ford models in 1997/98 averaged 11\%, discounts off Vauxhauls averaged $12 \%$ and discounts off Fiats averaged $10 \%$, with some discounts in the $30-40 \%$ range in each case. Yet many consumers received no reduction at all (e.g., $13 \%$ in the case of Ford). A similar pattern holds for the automobile market in the United States: according to Goldberg (1996, p. 641), "the data reveal substantial variation in dealer discounts, a large fraction of which cannot be explained just by financing, or model- and time-specific variables". The Office of Fair Trading (2004) report on the UK estate agency (realtor) market found that almost $50 \%$ of house sellers using an estate agent had tried to negotiate fees, with $80 \%$ of those receiving a reduction (Section 4.48).

With the advent of the credit crunch, the range of products over which negotiation is possible appears to be expanding, raising the salience of our research question. In a story entitled "Shops cave in on hagglers in 'souk Britain' ", The Sunday Times (2008) reported that: "Haggling has hit the high street. Jewellers, shoe shops, travel agents and electrical retailers are offering price cuts of up to $60 \%$ to shoppers prepared to negotiate the prices quoted on labels... A spokesman for Which? magazine said: It would seem that where people have the confidence to ask for a discount, they will get it. The classic British reserve stops us from haggling but you've got nothing to lose." The Daily Telegraph (2009) story "Haggle your way to a bargain" reported that: "Items such as washing machines and sofas often have high mark-ups, leaving room for
reduction. Independent retailers have always been open to negotiation, but now chain stores are getting in on the act, too. Carol Ratcliffe of the retail sector analysts Verdict Research says: The big chains are being very responsive to consumers looking to bag a bargain... In America, we see a lot of haggling - we Brits are more shy because we don't want to cause offence, but we are getting used to it. It does no harm to ask!"

In our price-setting model two competing firms simultaneously set posted prices, which become common knowledge. 'Price takers' dislike bargaining, or don't appreciate that discounts are possible, and so buy at the posted prices. 'Bargainers', on the other hand, approach at least one of the firms to ask for a better price than the one posted. To model bargaining, we adapt Burdett and Judd (1983)'s non-sequential search framework. In Burdett and Judd there are no posted prices and consumers are offered quotes from a probability distribution. Similarly to Burdett and Judd, we find that the firms offer bargainers quotes of a random size; the twist in our analysis is that the upper bound of this distribution is determined endogenously by the posted prices that are set for the price takers. As part of its optimal marketing mix, each firm offers bargainers discounts of an unpredictable size. The firms trade off the profits when pricing high from bargainers who don't approach the rival firm to ask for a better price against the greater probability when pricing low of selling to bargainers who also approach the rival.

The presence of the bargainers in the market affects the optimal marketing strategy in interesting and important ways. Of course, bargainers succeed in negotiating discounts off the posted prices as each firm is worried that a bargainer will be tempted by a lower offer from the rival firm. However, bargaining also impacts on the firms' incentives when they set their posted prices. The firms understand that the higher the posted prices, the worse the outside option for the bargainers, and so the higher the average price that the bargainers pay. If the posted prices are too high, though, each firm has too big an incentive to undercut the rival's posted price to win price taker market share at a high price. The presence of the bargainers thus allows posted prices to rise; in fact, we find that they permit a range of equilibrium posted prices to exist. Our results thus provide a possible justification for the findings of Cason et al. (2003), whose experiments show that when consumers can haggle below a posted price, prices tend to be higher and efficiency lower (Davis and Holt, 1994, present similar experimental findings).

When the proportion of bargainers in the market increases, the highest possible equilibrium posted price rises as each firm's incentive to undercut the rival's posted price to win price takers is weakened. Assuming the firms succeed in raising their posted prices towards the new highest possible equilibrium, those consumers who remain price takers pay higher posted prices and those who were already bargaining also pay higher prices on average as their outside option in the bargaining process becomes worse. However, those consumers who switch to bargaining get better prices as they now receive discounts off the posted price.

We show that if, through marketing, a firm can make almost all consumers bargainers, then its profits are maximized. With barely any price takers, the competitive pressure on posted prices is removed, allowing these to rise toward the full consumer willingness to pay. The firms thus increase their bargaining profits as the outside option for bargainers becomes so poor. We also show that profits are not monotonic in the proportion of bargainers. In the paper we determine when exactly firms and competition authorities might attempt to alter the proportion of bargainers incrementally. ${ }^{1}$

Finally, we extend our analysis to consider the case where firms are able to commit strategically to limit the bargaining discretion of their sales staff. In an initial stage the firms choose simultaneously the level of discretion to allow, either permitting full discretion to bargain below the posted price (as in the benchmark model analyzed in the rest of the paper) or committing their sales staff to only matching the rival's posted price when asked for a discount (price matching). We find that both firms allowing full discretion to bargain is always an equilibrium. By deviating to price matching, competition for the bargainers becomes less intense, but the deviator loses market share among the bargainers. If there are enough bargainers, both firms committing in advance to price matching can also be sustained as an equilibrium: when the firms play such an equilibrium, they succeed in moderating competition for both bargainers and price takers, and so raise their profits.

## Related Literature

Our work adds to the limited amount of literature which looks at the impact of bargaining when some consumers take posted prices as given. Like us, Korn (2007), Raskovich (2007) and Gill and Thanassoulis (2009) find that an increase in the proportion of bargainers can raise posted prices. In Korn (2007)'s monopoly setting, the bargaining process is not modeled: bargainers are assumed to negotiate a fixed proportionate discount off the price-cost margin. In the absence of bargaining the monopolist can set the monopoly price, which means that, in contrast to our results, the presence of bargainers always reduces profits. In Raskovich (2007) a big enough proportion of bargainers causes posted prices to jump from marginal cost to the monopoly price. The mechanism which raises the posted prices is different from that in our model: firms setting higher posted prices are assumed to be weaker bargainers and so are more attractive to bargaining consumers. In earlier work Gill and Thanassoulis (2009) present a quantity-setting model where a Cournot auctioneer sets a binding public list price off which bargainers negotiate discounts. Gill and Thanassoulis (2009) consider the distributional implications of bargaining markets. They show that if a consumer starts to bargain then although she raises her own surplus, the surplus of those who were already bargaining, and of those who remain price takers, is reduced. Here, instead, we study the optimal pricing and marketing strategy of a competitive

[^1]firm facing some bargaining consumers. We show how many bargainers such a firm would like to see in the market, how much discretion to bargain should be left to sales staff, and what form posted prices and final bargained discounts should optimally take.

Like us, Desai and Purohit (2004) consider whether firms will commit to restricting bargaining when some consumers take posted prices as given but the haggling type of consumer bargains. This important contribution to the literature focuses on whether the firms will choose to commit to never bargain with the hagglers. Unlike in our set-up, when both firms choose to haggle the posted prices are unaffected by the presence of the hagglers as they are never effective outside options for the haggling type. Thus the firms can use a haggling policy to separate the markets for the hagglers and non-hagglers and price discriminate effectively.

Much of the rest of the literature exploring bargaining in consumer markets examines the choice between committing to a fixed price and allowing consumers to bargain in the absence of a posted price (see e.g., Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, Camera and Delacroix, 2004 and Myatt and Rasmusen, 2009). Zeng et al. (2007) are notable in considering the marketing strategy of a monopolist who can use both a bargaining channel and a fixed price channel. They discover that if consumers differ in their bargaining abilities then using both channels can be optimal. Our analysis considers competing firms and so allows the optimal sales person discretion and marketing strategy to be determined in this richer context. There is also a small literature on bargaining below a posted price when all consumers bargain (e.g., Chen and Rosenthal, 1996a, 1996b and Camera and Selcuk, 2009).

Finally, as we find that both firms committing to only match the rival's posted price moderates competition and raises profits, we add to the literature which shows that price-matching guarantees can reduce the competitive pressures on firms (see for instance Moorthy and Winter, 2006, and the references therein).

## Plan of the paper

The paper is organized as follows: Section 2 sets out the model; Section 3 analyzes the firms' optimal bargaining strategy given the posted prices; Section 4 derives the equilibrium choice of posted prices; Section 5 conducts comparative statics on profits and consumer surplus; Section 6 considers how and when firms and competition authorities might influence the proportion of bargainers; Section 7 studies how much bargaining discretion the firms will allow their sales staff; Section 8 concludes; and the Appendix collects the proofs.

## 2 The Model

Two competing firms are located at the ends of a Hotelling (1929) line of length 1 with a uniform density of consumers along it. Firm $a$ is located at 0 , the left end of the line, and firm $b$ at 1 , the right end. The firms have the same constant marginal cost of production, which we normalize
to 0 , and have no fixed costs. As in Hotelling (1929) every consumer purchases exactly one unit and the market is always covered. The firms simultaneously choose a publicly posted price $l_{i} \geq 0, i \in\{a, b\}$. The posted prices are assumed to be binding: a firm can't refuse to sell at its chosen posted price. We define $l$ to be the lower of the two posted prices, i.e., $l \equiv \min \left\{l_{a}, l_{b}\right\}$.

A proportion $\mu \in(0,1)$ of the consumers are 'price takers', i.e., consumers who do not seek to bargain, instead taking the posted prices as given. The price takers have a standard linear Hotelling 'transport cost' with parameter $t>0$, so a price taker located at $x$ on the line incurs a total cost of $t x+l_{a}$ if she buys from firm $a$ and $t(1-x)+l_{b}$ if she buys from firm $b$. The price takers choose which firm to buy from to minimize this cost (randomizing in the event of a tie). We restrict $\mu>0$ to ensure that there are always some price takers for whom the firms set posted prices.

The other $1-\mu \in(0,1)$ consumers are 'bargainers'. ${ }^{2}$ We assume that the bargainers have no transport costs, i.e., they view the products as homogeneous, or at least of equal value, so they are entirely price-focused. After the firms have set their posted prices, these consumers ask for second quotes. A proportion $q \in(0,1)$ of the bargainers randomly select just one of the two firms for a second quote. One could interpret these as 'unskilled' bargainers. The remainder ask both firms for a second quote: we can think of these as 'skilled' bargainers. Note that the bargainers are doing more than searching; they are actively inviting sellers to beat their publicly posted prices.

The assumption that the bargainers see the goods as perfectly substitutable is technically convenient. It allows us tractably to determine the optimal distribution of second quotes which sellers should offer, even given non-identical posted prices. The assumption also bears a clear economic interpretation. We can think of the price takers as consumers who suffer significant bargaining and switching costs, which could be entirely psychological or due to a lack of information. ${ }^{3}$ Given the consumers always purchase one unit, Hotelling transport costs are equivalent to a switching cost of $2 t\left|\frac{1}{2}-x\right|$. The bargainers, by contrast, are entirely price-focused, and to get the best price they are willing to bargain with at least one seller. Desai and Purohit (2004) and Lal and Rao (1997) similarly assume that, respectively, hagglers (who behave like our bargainers) and cherry-pickers (who search for supermarket price promotions) have lower transport costs than other consumers.

If a firm is asked to offer a better price it is ignorant of whether the particular bargainer is also approaching the rival firm for a second quote. Given the posted prices are binding, the firm

[^2]can make a bargained offer of any price $p \geq 0$ weakly lower than its posted price. The bargained price the firm quotes is assumed to be a final take-it-or-leave-it offer to this consumer, so the firm cannot lower price further. ${ }^{4}$ A skilled bargainer who asks both firms for a second quote buys at the lower of the two quotes, randomizing if the quoted prices are the same. An unskilled bargainer who asks just firm $i$ for a second quote buys from firm $i$ if the quoted price is lower than or matches firm $j^{\prime} s$ posted price, and otherwise buys from firm $j$ at its posted price $l_{j}$.

Each firm therefore has two strategic marketing decisions to make. The first is to decide what posted price to set, which will determine demand from the price takers and also set an upper bound on the prices that can be quoted to bargainers (as the bargainers can always buy at the posted price). The second strategic marketing choice faced by the firms is how to bargain: should a firm be invited to offer a better price by a bargainer, it must decide what price to offer. The firms seek to maximize their expected profits. The model's parameters $\{t, \mu, q\}$ and payoff functions are all common knowledge, as are the posted prices once they have been set. Where possible we restrict attention to pure strategies.

## 3 Strategic Bargaining

To determine the optimal marketing approach we must analyze optimal bargaining and strategic pricing together as they will have competitive feedback effects on each other. This section determines the optimal bargaining strategy as a function of the posted prices which have been set. In other words given the market parameters (proportion of bargainers, propensity to approach multiple firms for second quotes) what reductions in price, if any, should a firm consider offering to bargainers as part of its marketing mix?

Suppose that the firms have set positive posted prices $l_{a}$ and $l_{b}$ with $l \equiv \min \left\{l_{a}, l_{b}\right\}$. The price $l$ acts as an upper bound on the prices that are quoted to bargainers by either firm. If a seller with $l_{i}>l_{j}$ should refuse to lower its price to at least the rival's price $l_{j}=l$ when requested to beat its own posted price then it will fail to sell to any of the bargainers. If instead it were to offer a price of $l$ then it would at least make sales to those bargainers who approach it for a second quote but don't approach its rival. Thus optimal bargaining will require the firms to at least match the lowest posted price.

Furthermore refusing to offer prices below $l$ to the bargainers cannot be part of the optimal bargaining strategy. If a firm adopted this strategy then her rival would bargain by just undercutting her when a second quote was requested. In fact, the bargaining stage cannot have a pure strategy equilibrium in which a firm always offers bargainers the same price. Such a price is either too low and should be raised (which increases profits from those bargainers who do

[^3]not approach the rival firm) or would be undercut by the rival firm to win the business of those bargainers who seek two second quotes. Unless a firm can commit in advance to deny her sales force the discretion to lower prices (analyzed in Section 7), active bargaining in which prices are lowered with some probability must be part of the optimal bargaining mix.

## Proposition 1 [Optimal Bargaining Strategy]

There is a unique symmetric equilibrium bargaining strategy. ${ }^{5}$ Each firm stands ready to lower prices to bargainers, but to an unpredictable extent. Under the optimal strategy:

1. Both firms choose to offer bargainers second quoted prices $p$ lying in the range $\left[\frac{l q}{2-q}, l\right]$. When $l>0$, the price offers are drawn from the distribution $F(p)=1-\frac{(l-p) q}{2 p(1-q)}$.
2. Average bargained prices are given by $l q$ and each firm makes profits of $\pi_{i}=\frac{(1-\mu) l q}{2}$ from the bargainers.
3. Average bargained prices and profits from the bargainers grow if the lowest posted price, $l$, rises or if bargainers become less skilled, approaching both firms for a second quote less frequently ( $q$ rises).

In equilibrium the firms must trade off the incentive to offer high prices to maximize profit from the bargainers who don't approach the rival against the incentive to price low to win the business of bargainers who also approach the rival firm for a second quote. Part 1 of Proposition 1 demonstrates that this trade-off yields price choices drawn from a distribution $F(p)$. Differentiating this entity allows us to characterize the probability density function of second quoted prices as $f(p)=l q / 2 p^{2}(1-q)$. Recall that the firms' marginal costs are normalized to zero, so $f(p)$ should be interpreted as the probability of lowering the margin above cost from $l$ to $p$ when pressed to improve on the posted prices by a consumer. This density function can be readily calculated for any given parameter values. For example the probability of various ranges of percentage discounts offered relative to $l$ is given by Figure 1 when half of the bargainers are skilled and half unskilled ( $q=1 / 2$ ).

The optimal bargaining strategy has, perhaps surprisingly, larger reductions in margin being offered more frequently than smaller ones. Equilibrium, and so optimality, requires that a bargaining firm must be indifferent between all the prices which it will offer with some probability to bargainers demanding a second quote. Therefore the firm is indifferent to lowering the bargained price by some small amount. When the initial price is near $l$, the initial probability of a sale is low, while the price reduction will generate a lot of extra profit from any extra sale to bargainers. When instead the initial price is low, the initial sale probability starts higher, while the price reduction would generate much more modest extra profits from any extra sale. As the

[^4]firm must be indifferent to either price change, the increase in the probability of a sale must be greater when lowering price from a low level as compared to a high level. This is equivalent to saying that high prices must be quoted by the firm's rival less frequently, and so generates the shape of margin reductions given in Figure 1.


Figure 1: Percentage discounts when $q=\frac{1}{2}$
Notes: When interpreting Figure 1, recall that we have normalized marginal cost to zero: the reductions should thus be thought of as percentage discounts offered off the margin a firm makes above marginal cost when selling at $l$. The final bin ( $65 \%$ to $70 \%$ reduction in margin) has a lower probability than the previous bin because the support of the pricing distribution ends somewhere within this bin.

Next consider Parts 2 and 3 of Proposition 1. When the lower of the posted prices takes a higher value ( $l$ is higher) the bargainers' outside option becomes less valuable, allowing the firms to bargain to higher prices on average as the posted prices impose less of a constraint on pricing. The posted prices will therefore be set strategically: we turn to this analysis in the next section. When the bargainers seek a single quote with higher probability, so $q$ is higher and bargainers are less skilled, there is a greater incentive to price high to profit from bargainers who have not approached the rival firm, pushing up the offered prices. ${ }^{6}$ A higher q also raises average bargained prices because unskilled bargainers pay higher prices in expectation for a given pricing distribution: they only receive one draw from $F(p)$, while the skilled bargainers

[^5]pay the lower of two draws. As the market is always covered in our Hotelling framework with all consumers buying one unit, higher bargained prices translate directly into higher profits for the firms from the bargainers.

## 4 Competitive Posted Prices

We now turn to the marketing decision of what posted price to set. The firms will decide this strategically as different posted prices have strategic implications for price takers and alter the bargaining profits which can be made. Thus to determine optimal pricing we must build on the bargaining analysis of Section 3.

If the firms considered only the price takers, the equilibrium posted price would be $l^{*}=t$ as prices would be set just as in the standard Hotelling model. The firms, however, are of course aware of the impact of the posted prices on the expected profits they make from the bargainers. Strategic considerations then yield a continuum of possible outcomes.

## Proposition 2 [Competitive Posted Prices]

Posted prices in excess of the Hotelling competitive level of $t$ are optimal. In particular:

1. The competing firms set the same posted price: all pure strategy Nash equilibria of the posted price setting stage are symmetric.
2. There is a continuum of possible posted price equilibria $l^{*}$ given by

$$
\begin{equation*}
l^{*} \in\left[t,\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t\right] . \tag{1}
\end{equation*}
$$

3. The top of the pricing range grows as the proportion of bargainers increases ( $\mu$ falls) and as the bargainers become less skilled (q rises).

Proposition 2 demonstrates that in markets in which some consumers bargain, the optimal pricing strategy is to raise posted prices above the level that standard Hotelling competition for the price takers would deliver. At these high prices a firm could increase its profits from price takers by undercutting the rival: the increase in market share would outweigh the loss in inframarginal revenues. However lowering the posted price would allow bargainers a better outside option - thus the bargained prices would fall and so profits would be lost on bargainers (Proposition 1). This concern allows the firms to forgo the price war for the price takers and maintain equilibrium posted prices above $t$. Profits from the bargainers increase when the bargainers become less skilled or the proportion of bargainers rises (Proposition 1), thus reducing the incentive to undercut the rival's posted price and allowing the range of equilibria to expand. As the proportion of bargainers tends to zero, so $\mu \rightarrow 1$, the top of the pricing range falls to the standard Hotelling price $t$.

We have yet to explain however why a range of posted prices can be supported. Why are high prices not guaranteed?' The reason here is that firms have a 'kinked profit curve'. Suppose that a rival sets a posted price above $t$ but below the maximum that can be supported as an equilibrium. Undercutting this price is not optimal by the reasoning above. However raising posted prices above the level of a rival is never optimal. If a firm were to raise its posted price higher than the rival's, profits would be lost on the price takers. In addition no profits would be gained from the bargainers as the lowest posted price, $l$, would not change. Bargainers see the products as undifferentiated as they are price-focused, and so can use the rival's posted price as a credible outside option. Thus the lowest posted price is the relevant variable off which bargained prices are negotiated, and this outside option cannot be raised by one firm acting alone.

## 5 Prices, Profits and Consumer Surplus

We have established that in a market in which firms set posted prices and yet some consumers bargain, the optimal marketing mix requires competing firms to set identical posted prices lying within a given range (Proposition 2) and then to offer reductions of a non-predictable size to bargainers (Proposition 1). We are therefore now in a position to analyze how equilibrium profits from this market will vary with the market parameters.

To conduct this comparative statics exercise we introduce a parameter $\alpha \in(0,1]$ which measures the degree of coordination the firms manage to achieve. ${ }^{8}$ Specifically, we assume that the firms set an equilibrium posted price of

$$
\begin{equation*}
l^{*}(\alpha)=\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t \tag{2}
\end{equation*}
$$

so the firms achieve a proportion $\alpha$ of the maximum possible increase above the standard Hotelling price $t$ which Proposition 2 allows. From Part 2 of Proposition 1, each firm makes profits of $(1-\mu) q l^{*} / 2$ from the bargainers. As the market is always covered, and firms set the same posted price in equilibrium, the firms share the proportion $\mu$ of price takers at price $l^{*}$. Thus, using (2), the expected overall profits of each firm will be:

$$
\Pi_{i}(\alpha, q, \mu, t)=\underbrace{\frac{\mu}{2}\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t}_{\begin{array}{c}
\text { Profit from }  \tag{3}\\
\text { price takers }
\end{array}} t+\underbrace{(1-\mu) \frac{q}{2}\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t}_{\begin{array}{c}
\text { Profit from } \\
\text { bargainers }
\end{array}} .
$$

Equation (3) is an important step as it allows us to capture equilibrium profitability in a

[^6]market in which firms compete with the optimal marketing mix of posted prices and subsequent bargaining strategy.

## Proposition 3 [Firm Profits and Consumer Surplus]

In equilibrium, each firm's profits are given by (3) and are:

1. Increasing in the degree of coordination $\alpha$ achieved by the firms.
2. Decreasing in bargainers' skill. That is profits rise in the propensity with which bargainers ask only one firm for a second quote, $q$.

Furthermore:
3. Consumer surplus moves in the opposite direction to firm profits and to average prices.

The competing firms in our market are striving to maximize profits gained from two separate groups of consumers: the price takers and the bargainers. Increased coordination allows greater profits to be made on both consumer types. If the firms succeed in coordinating on the same high posted price (high $\alpha$ ) within the equilibrium range found in Proposition 2, neither firm loses price taker market share to the other and each firm serves half the price taker market at a high price. At the same time bargainers secure reductions off high posted prices and so profits from these consumers are also high (Proposition 1). Bigger profits immediately imply that consumer surplus must fall given the market is covered.

When the bargainers become less skilled, profits from both types of consumer also rise. For a given degree of coordination, (2) shows that the equilibrium posted price increases in the proportion of bargainers who request a second quote from a single firm, $q$. This is because a higher $q$ raises the profits from the bargainers (Proposition 1), which in turn reduces the incentive to undercut a rival's posted price as the profit gain from the price takers becomes smaller relative to the foregone profits from the bargainers. The increase in the equilibrium posted price directly increases profits from the price takers and also leads to higher bargained prices.

## 6 Encouraging Consumer Bargaining and Marketing Policy

We now move beyond the question of the optimal marketing mix of posted price setting and bargaining; instead we turn to the wider question of whether encouraging consumers to bargain, or the opposite, should form part of the firms' broader marketing strategy. The same question could be put to national competition authorities and consumer champions, though their objectives are (presumably) the reverse of that of the firms. So would consumers or firms gain from an increase in the proportion of bargainers?

On the one hand bargainers pay lower prices than price takers, so one might imagine that increasing the number of bargainers would improve consumer outcomes. This has certainly been
the view of the UK competition authority. In 2004 the UK's Office of Fair Trading launched an information campaign to advertise the benefits of bargaining in the UK estate agency (realtor) market:
"We will therefore undertake an information campaign to raise consumer awareness of the benefits of shopping around before choosing an estate agent and of negotiating fee rates." (Section 1.12, Office of Fair Trading, 2004)

The rationale for this campaign was that:
"Greater shopping around and negotiation by consumers will increase competitive pressures on estate agents and result in better value for money in terms of both lower prices and higher service quality." (Section 1.12, Office of Fair Trading, 2004)

However such a conclusion is not clear. As the proportion of bargainers increases we will show that posted prices rise. Hence bargainers receive a reduction, but from a higher level. Thus it is quite possible that average prices could rise, and consumer surplus fall, as the proportion of bargainers increases. In this light it is interesting to note that policymakers sometimes do try to limit bargaining: until 2001 the Rabattgesetz (statute on discounts) and the Zugabeverordnung (regulation governing free gifts with sales) severely restricted the ability of German retailers to offer discounts off posted prices (Finger and Schmieder, 2005, Korn, 2007).

### 6.1 The Comparative Statics of Changes in the Bargaining Population

In this section we analyze how a change in the proportion of bargainers affects prices for both bargainers and price takers. Our result splits the consumers into three camps: those who remain price takers; those who were already bargaining; and those who swap from taking prices as given to asking for discounts.

## Proposition 4 [Prices and the Proportion of Bargainers]

As the proportion of bargainers increases ( $\mu$ goes down):

1. The equilibrium posted price increases, so the remaining price takers pay more.
2. The expected bargained price increases, so the existing bargainers pay more.
3. If the proportion of consumers who switch to bargaining is sufficiently small, the new bargainers pay less on average.

As we increase the proportion of bargainers, the incentive to keep the posted prices high to make the bargainers' outside option less attractive goes up as the price takers impose less of a constraint on pricing. The higher equilibrium posted price hurts the price takers directly: this is captured by the first result above. In addition, however, the higher posted price forces up the
average price offered to bargainers from the distribution of second quotes. Thus the consumers who remain price takers as well as those who were already bargaining become worse off. Of course, bargainers expect to receive a discount off the posted price, so those consumers who switch to bargaining get a better price on average as long as not too many consumers switch so the equilibrium posted price does not rise too much. We consider the overall impact of the proportion of bargainers on profits and consumer surplus in Section 6.3.

### 6.2 How Would a Firm Alter the Proportion of Bargainers?

Before proceeding we first address how a firm (or competition authority) might hope to alter the proportion of bargainers in the population.

Empirical evidence from a number of sources (see the Introduction) shows that in numerous markets some consumers bargain while others do not. Psychological and informational costs may deter consumers from bargaining, so to the extent that firms can reduce the embarrassment of bargaining and make it more widely known that discounts are available, the proportion of bargainers should increase. Let us consider how firms might influence the psychological costs of bargaining. Consumers may be more inclined to bargain if firms employ a greater proportion of more senior or better trained sales staff who are perceived as more likely to have the authority to offer a discount. Consumers may also feel that bargaining is more acceptable when the number of retail staff is higher, so the negotiation does not inconvenience other shoppers as much. The propensity of consumers to bargain will also be affected by social norms: in certain industries bargaining below a posted price is a well-established and acceptable norm of behavior. ${ }^{9}$ Over time, firms may be able to induce a culture of bargaining in their industry; indeed, there may be a snowball effect whereby initial increases in the proportion of bargainers encourage more reticent consumers to join in, thus creating a new norm.

### 6.3 Who Wants Bargainers?

We now turn to the question of when it is in the firms' interest to try to increase the proportion of bargainers. To abstract from the complications involved in analyzing potential competition between firms to attract more bargainers, we consider only how a generalized increase in the proportion of bargainers affects the firms jointly. To simplify, we also abstract from any costs directly incurred in attempting to influence this proportion.

Lemma 1 Firm profits are convex in the proportion of bargainers. The profits are maximized when everyone bargains and no one is a price taker, i.e., as $\mu \rightarrow 0$.

[^7]Hence if firms' ability to alter the proportion of bargainers was unrestrained, and further if alterations in the proportion of bargainers didn't alter the bargaining skill of consumers or the degree of coordination between firms ( $q$ and $\alpha$ ), then firms would want all consumers to bargain and none to be price takers. From the perspective of a consumer surplus focused competition authority, this would be the worst possible outcome. If everyone is a bargainer then there are essentially no lost profits from refusing to undercut a rival's posted price. Hence the downward force on prices is removed and firms can set posted prices with a view to weakening the bargainers. This allows posted prices to rise as high as consumers' full willingness to pay for the item - competition imposes no restraint. And profits from bargainers grow as the posted prices grow from Proposition 1.

However the convexity of profits in the proportion of bargainers (Lemma 1) implies that profits do not change monotonically in the proportion of bargainers, $1-\mu$. This is because of the conflicting effects of raising the proportion of bargainers described in Proposition 4: posted prices rise, which hurts existing bargainers and the remaining price takers, while those who switch to bargaining now receive a discount. Further the firms' ability to manipulate the proportion of bargainers is likely to be constrained: consumer behavior can be changed by degrees and not immediately moved to some entirely new practice. Hence when would an incremental increase in the proportion of bargainers be a part of an optimal firm marketing policy?

The convexity of profits established in Lemma 1 implies that there is a profit minimizing, consumer surplus maximizing, proportion of bargainers. This key proportion of price takers is given explicitly by $\hat{\mu}$ in Proposition 5, which shows that if the proportion of price takers is reduced incrementally from a level above $\hat{\mu}$ then profits will decline due to the convexity of profits, while if the proportion of price takers starts below $\hat{\mu}$ profits rise (the consumer surplus results are reversed).

## Proposition 5 [Incremental Bargaining Proportions]

Profits are lowest, and consumer surplus highest, if the proportion of price takers in the population is given by

$$
\begin{equation*}
\hat{\mu}=\min \left\{\sqrt{\frac{\alpha q^{2}}{(1-q)(1-\alpha q)}}, 1\right\} \tag{4}
\end{equation*}
$$

Thus:

1. A small reduction in the proportion of price takers, $\mu$, increases firm profits and reduces consumer surplus if there are fewer price takers than $\hat{\mu}$.
2. A small reduction in the proportion of price takers, $\mu$, reduces firm profits and increases consumer surplus if there are more price takers than $\hat{\mu}$.
3. If bargainers become less skilled (q rises) or the degree of coordination achieved by the firms goes up ( $\alpha$ rises), then $\hat{\mu}$ increases so the profit minimizing, consumer surplus maximizing
point occurs with fewer bargainers.

It follows from Proposition 5 that incrementally increasing the proportion of bargainers is in the firms' interests (and hurts the consumers) if the current proportion of price takers is below $\hat{\mu}$. Thus the critical level $\hat{\mu}$ is key to answering the question "who wants more bargainers?" Part 3 of Proposition 5 addresses how $\hat{\mu}$ varies with the model parameters. Suppose first that bargainers become less effective at negotiating discounts as they tend to ask for fewer second quotes ( $q$ rises). The firms now benefit more strongly from the presence of the bargainers in the market as bargained profits rise (Proposition 1), which acts to push up the posted prices (see (2)). Thus when the proportion of bargainers rises, the discounts secured by the new bargainers more quickly become overwhelmed by the higher prices faced by existing price takers and bargainers. As a result profits rise and consumer surplus falls even at high initial levels of price taking: that is $\hat{\mu}$ rises. Similar reasoning applies if coordination, $\alpha$, increases.

Of course, if the firms or a competition authority were successful in changing the proportion of bargainers they might also indirectly affect the average 'bargaining skill' of consumers, $q$, and the degree of coordination achieved by the firms, $\alpha$. The direction in which these might move is somewhat ambiguous. For example consider the variable $q$. Inducing more bargaining by making it easier for consumers may, on the one hand, stimulate more of those consumers who were already bargaining to source quotes from both firms; on the other hand a high proportion of the price takers who are induced to start bargaining might ask for a single quote. It is clear, however, that an increase in $q$ always raises prices and profits: from (2) the equilibrium posted price rises in $q$, as do profits from bargainers from Proposition 1. Thus the firms might also pursue strategies to increase $q$ directly: for example, if the firms locate their stores further apart, they are likely to increase $q$ as it will be more costly for bargainers to source quotes from both firms. Proposition 5 allows us to gain insight without having to take a stand on this issue by analyzing the benchmark case where $q$ and $\alpha$ are constant.

Proposition 5 makes clear that in the absence of some market analysis competition authorities should be wary of encouraging bargaining in markets as increasing the proportion of bargainers will often have the perverse effect of raising average prices and lowering consumer surplus. The naive policy prescription that increasing bargaining is good because it increases competitive pressure on firms and leads to more discounts can be counterproductive.

## 7 How Much Bargaining Discretion Should Firms Allow?

So far we have studied the optimal marketing policy for competing firms when some consumers bargain while others accept the posted prices, assuming that the firms are not able to commit to restricting the discounts that their sales staff can offer. We now embed our earlier results into an analysis of how much discretion a firm should give its sales staff to offer discounts below
posted prices when some degree of commitment is possible. We consider two possibilities. A firm can: (a) allow its sales staff full discretion to bargain (as modeled above); or (b) restrict its sales staff to only be able to match the rival's posted price if approached by a bargaining consumer. ${ }^{10}$ Both of these approaches are common - yet their overall impact on competition is difficult to capture without a model like ours which endogenizes bargaining strategies and posted price choices.

### 7.1 The Dynamic Bargaining/Price Matching Game

To conduct the analysis, we assume that each firm's sales staff are appropriately incentivized so that they wish to maximize firm profits, and we formulate the following two stage dynamic bargaining/price matching game.

1. The two firms decide simultaneously how much discretion to allow their sales staff. Each firm can decide to either:
(a) Bargain (BA), which means that the firm allows its sales staff full discretion to offer discounts (as per the model analyzed in Sections 3 to 6); or
(b) Price match (PM), which means that if a consumer requests a discount, the firm's sales staff match the rival's posted price by offering the consumer the lower of the two firms' posted prices $l$, but never offer further reductions. ${ }^{11}$

Each firm becomes aware of its rival's sales staff discretion choice. We let (BA,BA), (PM,PM), (BA,PM) and (PM,BA) represent, respectively, both firms bargaining, both firms price matching, firm $i$ bargaining and $j$ price matching, and the reverse.
2. Given the sales staff discretion choices of the firms, they simultaneously set posted prices. The consumers behave as outlined in Section 2, where a proportion $q$ of bargainers approach just one firm for a second quote.

### 7.2 Pricing and Profits Given Bargaining Discretion Choices

Before we can analyze the equilibrium choices of how much bargaining discretion to allow, we must first determine the pricing and profit implications of the different possible constellations of bargaining discretion choices.

[^8]
### 7.2.1 Both Firms Bargain

This case has been solved as the benchmark model. Proposition 2 gives the range of equilibrium posted prices as

$$
\begin{equation*}
l^{*(\mathrm{BA}, \mathrm{BA})} \in\left[t,\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t\right] \tag{5}
\end{equation*}
$$

Assuming a degree of coordination $\alpha$, from (3) firm profits can be simplified to:

$$
\begin{equation*}
\Pi_{i}^{(\mathrm{BA}, \mathrm{BA})}=\frac{1}{2}(1-(1-\mu)(1-q))\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t \tag{6}
\end{equation*}
$$

### 7.2.2 Both Firms Price Match

Lemma 2 If both firms choose to price match then they set identical equilibrium posted prices lying in the range

$$
\begin{equation*}
l^{*(\mathrm{PM}, \mathrm{PM})} \in\left[t,\left(1+\left(\frac{1-\mu}{\mu}\right)\right) t\right] \tag{7}
\end{equation*}
$$

and all bargainers transact at the posted prices, splitting themselves equally between the sellers. Assuming a degree of coordination $\alpha$, each firm's profits equal

$$
\begin{equation*}
\Pi_{i}^{(\mathrm{PM}, \mathrm{PM})}=\frac{1}{2}\left(1+\alpha\left(\frac{1-\mu}{\mu}\right)\right) t \tag{8}
\end{equation*}
$$

so when both firms price match, profits are higher than when both firms bargain.
If both firms strategically decide to price match then when bargainers request a better quote from either seller they will be offered the lower of the two posted prices. Hence the firms split the bargaining population between them and serve them at the lowest posted price $l$. For a given $l$, this implies that profits can be extracted from bargainers more effectively than when both firms bargain as bargaining firms offer discounts of $(1-q) l$ on average (see Part 2 of Proposition 1).

Turning to the posted price choice, each firm must trade off the profit to be gained from price takers when undercutting the rival's posted price against the profits lost from bargainers by lowering the best posted price. From above, bargaining consumers are more profitable when both firms price match than when both firms bargain, so the incentive to undercut the rival's posted price is weakened. It follows that higher posted prices can be sustained when both firms price match: the range of possible equilibrium posted prices (7) grows relative to (5), so price matching moderates competition between the firms. Because price matching allows both higher posted prices and smaller discounts off these prices, price matching firms earn higher profits than bargaining firms.

### 7.2.3 One Firm Bargains and the Other Price Matches

Lemma 3 If one firm bargains and its rival price matches then the firms set identical equilibrium posted prices lying in the range

$$
\begin{equation*}
l^{*(\mathrm{BA}, \mathrm{PM})} \in\left[t,\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t\right] . \tag{9}
\end{equation*}
$$

All bargainers transact at the posted prices. A proportion $q / 2$ buy from the price matching firm while the majority $(1-q / 2)$ buy from the bargaining firm. Assuming a degree of coordination $\alpha$, the firms' profits are given by:

$$
\begin{align*}
\Pi_{i}^{(\mathrm{BA}, \mathrm{PM})} & =\frac{1}{2}(1+(1-\mu)(1-q))\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t  \tag{10}\\
\Pi_{j}^{(\mathrm{BA}, \mathrm{PM})} & =\frac{1}{2}(1-(1-\mu)(1-q))\left(1+\alpha q\left(\frac{1-\mu}{\mu}\right)\right) t \tag{11}
\end{align*}
$$

Thus the bargaining firm $i$ makes more profit than when both firms bargain, while the price matching firm $j$ makes the same profit as when both bargain.

Any bargainer who only approaches the price matching firm (a proportion $q / 2$ ) will be offered the lower of the posted prices, $l$, and will buy at that price. The bargaining firm will win the business of all the other bargainers (a proportion $1-q / 2$ ) by just undercutting $l$ (so these bargainers essentially pay $l$ ).

Now consider the question of which posted price to set. The same trade-off as before persists: each firm could gain price taker market share by undercutting the rival's posted price, but this would entail a loss on the bargainers. For the price matching firm, the profit to be made from the bargainers is given by $(1-\mu) l q / 2$, which is identical to the profits it makes from bargainers when both firms bargain (in that case the firms share the bargainers, but at a discounted average price $l q$ ). The price matching firm is the one whose incentives to undercut are binding on the possible equilibrium (the bargaining firm would like to see yet higher posted prices as it makes more profits from bargainers). Hence the range of possible posted price equilibria is the same as in our benchmark model, explaining why (9) matches (5). As the posted price equilibria are the same as when both firms bargain, and the price matching firm earns the same amount of profits from the bargainers, the price matcher makes the same overall profits as when both firms bargain. The bargaining firm, however, sells to a greater proportion of bargainers and so does better than when both firms bargain.

### 7.3 Equilibrium Bargaining Discretion Choices

Using the profit expressions (6), (8), (10) and (11), we are now in a position to find the equilibria of the bargaining/price matching game. Proposition 6 describes the equilibria, while Figure 2 illustrates how the set of equilibria varies in the parameters.

## Proposition 6 [Equilibrium Sales Staff Discretion Choices]

Assuming a constant degree of coordination $\alpha$ achieved by the firms, the dynamic bargaining/price matching game has the following pure strategy equilibria:

1. Both firms allowing their sales staff to bargain $(B A, B A)$ is always an equilibrium.
2. In addition to ( $B A, B A$ ), both firms price matching ( $P M, P M$ ) is also an equilibrium when

$$
\begin{equation*}
\alpha-\alpha q(1-\mu)-\mu \geq 0, \tag{12}
\end{equation*}
$$

that is, when the degree of coordination, $\alpha$, is sufficiently high, the proportion of price takers, $\mu$, is sufficiently low, or (when $\alpha \geq \mu$ ) the proportion of bargainers that are unskilled, $q$, is sufficiently low.
3. In addition to $(B A, B A)$, the hybrid cases in which one firm bargains and the other price matches $((B A, P M)$ and $(P M, B A))$ are also equilibria when the inequality in (12) is reversed.

Proposition 6 demonstrates that both firms bargaining ( $\mathrm{BA}, \mathrm{BA}$ ) is always an equilibrium of the strategic bargaining/price matching game. This is true even though both committing to only matching the rival's posted price (PM,PM) always returns higher profits, and is an equilibrium when (12) holds. If one firm chooses bargaining, its rival can do no better than to allow its sales staff to bargain also. If a firm deviated from (BA,BA) by choosing to price match, it would lose market share among the bargainers. At the same time, competition for bargainers would become less intense, pushing up bargained prices to the posted prices. These effects exactly cancel and the equilibrium posted prices would be unchanged, so the deviator's profits would remain constant (see the discussion of Lemma 3). Neither firm is able unilaterally to coordinate a move to both price matching (PM,PM), which would moderate competition and allow posted prices and both firms' profits to rise. The $(\mathrm{BA}, \mathrm{BA})$ equilibrium appears weak, but note that we have not considered that firms might care about profit and sales volume comparisons: if a firm deviates from (BA,BA) to price matching its profits are unchanged, but the other firm gains bargainer market share and sees its profits increase.

Part 2 of Proposition 6 shows the conditions under which both firms choosing to price match (PM,PM) is an equilibrium. Consider a firm thinking about deviating to bargaining. On the one hand, the deviator gains a bigger share of the bargainers; on the other hand, the equilibrium posted price falls as the remaining price matching firm has less of an incentive to keep prices high as it loses bargainer share. When the degree of coordination $\alpha$ is higher, the proportion of price takers $\mu$ is lower or the proportion of bargainers that are unskilled $q$ is lower, the fall in the posted price from deviation becomes bigger, making deviation less attractive. Because the fall in the posted price lowers profits from both price takers and bargainers, this effect outweighs any bargainer share considerations.


Figure 2: Equilibrium sales staff discretion choices

Notes: Both firms allowing their sales staff to bargain (BA,BA) is an equilibrium for all parameter values. The curves for different values of $q$ mark the boundary between the region in which both firms price matching (PM,PM) is also an equilibrium and the region in which the hybrid cases $(\mathrm{BA}, \mathrm{PM})$ and ( $\mathrm{PM}, \mathrm{BA}$ ) are also equilibria.

Finally let's consider the hybrid equilibria (BA,PM) and (PM,BA). We saw above that a firm is indifferent between both firms bargaining (BA,BA) and the hybrid situation in which it price matches and the other bargains, so the hybrid equilibria are stable to deviation by the price matcher. In thinking about deviation, the bargainer in the hybrid cases faces exactly the reverse incentives as a firm thinking about deviating from the equilibrium in which both firms price match (PM,PM), so the conditions sustaining the hybrid equilibria are reversed from those sustaining (PM,PM).

In summary, Proposition 6 shows that both firms giving their sales staff full discretion to bargain is always an equilibrium, allowing fully for the impact on posted prices and the subsequent bargaining strategy. This result holds in the benchmark case in which sales staff are incentivized to maximize profits and a change in bargaining regime does not affect the degree of coordination within the equilibrium range of posted prices that the firms are able to achieve. However, Proposition 6 also shows that both firms committing in advance to only matching their rival's posted price can sometimes be sustained as an equilibrium: when the firms play such an
equilibrium, they succeed in moderating competition and raising profits. Hybrid equilibria are also possible. Hence the optimal level of sales staff discretion depends not only on the market parameters, but also on the equilibrium the industry is currently in.

## 8 Conclusion

In this paper we analyze a model of duopoly competition between firms when some consumers bargain while others buy at the posted prices. The implications for the optimal marketing strategy of a firm are profound. Perhaps the key insight is that the posted prices set will have strategic implications for the profits available from consumers who bargain. Understanding this insight allows us to address the main questions raised in the Introduction.

We have demonstrated that the presence of bargainers can work to the firms' advantage. Bargaining consumers will seek reductions off the posted prices. As a result, high posted prices allow more profits to be extracted from the bargainers. Thus the presence of bargainers lowers the pressure to reduce the posted prices in competition for the price takers. The optimal marketing mix for a firm facing both price takers and bargainers therefore involves raising the posted prices from which the bargainers negotiate discounts.

Can any insight be given as to the firms' optimal bargaining strategy? In our model the answer is clear: bargained reductions should be substantial on average and not predictable. The reason is that a predictable bargaining strategy will allow the rival firm to always undercut and steal bargainer business. In numerical simulations reductions of as much as $50 \%$ off margins are feasible, a fact which for instance fits in with the available evidence for the automobile industry (Competition Commission (2000)).

If bargainers can be beneficial to firms does it follow that an optimal marketing mix should strive to increase the proportion of consumers who try to bargain? Here the answer is more nuanced. It does follow, in our model, that if almost all consumers try to bargain then firm profits are maximized. This is because the competitive pressure is taken off the posted prices, allowing these to rise as high as valuations allow so more profits can be extracted from bargainers. However, incrementally increasing the proportion of bargainers need not raise profits. While posted prices and average bargained prices rise, the new bargainers swap a high posted price for a lower bargained price. Nevertheless profits are likely to increase if many bargainers are unskilled or if the coordination achievable on high posted prices is great enough.

An important question faced by all firms in designing their business and marketing strategy is how much discretion to leave their sales force to lower prices to clinch a deal. We show that allowing sales staff to lower prices even below the rival's posted price is indeed equilibrium play. Perhaps paradoxically, if enough of the consumers seek to bargain then both firms refraining from bargaining below the best posted price is also an equilibrium, and a more profitable one at
that. When both firms restrict sales staff to price matching, the heat is taken out of competition for the bargainers and so profits rise compared to the equilibrium in which both firms allow their sales staff bargaining flexibility.

We have assumed a bargaining formulation which is innovative as compared to much of the existing marketing research. However the specific sequence of offers and counter-offers is not the source of our results. Rather the main driver is the link between the posted prices and the bargainers' outside option, together with the fact that some consumers are not willing to bargain, while others are with varying levels of skill. Our bargaining formulation is, we believe, realistic and allows a comparatively rich set of results when set against the extant literature on this topic.

## Appendix

Proof of Proposition 1. Suppose that $l=\min \left\{l_{i}, l_{j}\right\}>0$. We start by showing that any symmetric equilibrium must be mixed with: (i) no mass points in the density function; (ii) $F(p)<1$ for $p<l$; and (iii) $F(l)=1$.
(i) If there were a mass point at price $p>0$, a firm could deviate profitably by lowering its quote price to $p-\varepsilon>0$ just below the mass point whenever it would have proposed $p$. This increases total sales by a discrete amount (when the bargainer also gets a quote of $p$ from the rival firm) in return for a vanishingly small loss and so is a profitable deviation. If there were a mass point at $p=0$, a firm would deviate upward to sell at a strictly positive price to bargainers who approach it but not the rival for a second quote.
(ii) Suppose that the support of $F$ stops at $p<l$. From (i) the probability that the rival firm quotes this highest price is zero. Thus any firm charging the highest price could deviate profitably by raising price towards $l$ as in either case the firm will make the sale if and only if the bargainer doesn't also approach the rival firm for second quote. ${ }^{12}$
(iii) $F(l)=1$ as we argued in the second paragraph of Section 3 that the firms will never price above $l$.

Properties (i) to (iii) allow us to write a firm's expected profits from the proportion $1-\mu$ of bargainers at any quoted price $p$ in the support of $F$ as

$$
\begin{equation*}
\pi_{i}(p)=(1-\mu) p\left(\frac{q}{2}+(1-q)(1-F(p))\right) . \tag{13}
\end{equation*}
$$

This follows because of the $q$ proportion of bargainers who ask for just one quote, half will approach a particular firm and of those all will buy at $p$, while all those who ask for two quotes will buy at $p$ if and only if the rival does not offer them a better quote (from (i) the rival firm

[^9]will never offer the same quote). At the top of the pricing distribution ( $p=l$ ) we have
\[

$$
\begin{equation*}
\pi_{i}(l)=\frac{(1-\mu) l q}{2} \tag{14}
\end{equation*}
$$

\]

For the firms to be willing to randomize, their profits must be constant at all points in the support of $F$, so

$$
\begin{align*}
& \frac{(1-\mu) l q}{2}=(1-\mu) p\left(\frac{q}{2}+(1-q)(1-F(p))\right)  \tag{15}\\
& \Leftrightarrow F(p)=1-\frac{(l-p) q}{2 p(1-q)} \tag{16}
\end{align*}
$$

To find the lower bound of the distribution set $F(\underline{p})=0$ and solve for $\underline{p}$. When $l=0$, at least one firm has $l_{i}=0$ and so is forced to offer $p=l=0$. The rival can do no better than to also offer $p=l=0$, so $F(l)=F(0)=1$ and (14) remains valid. This concludes Part 1 .

For Part 2, let $E(p)$ be the average bargained price across both types of bargainer. As the market is covered, and each firm earns the same expected profit from the bargainers, each firm earns $\pi_{i}=\frac{E(p)}{2}$ per bargainer on average. As the firms' profits are constant at all prices in the support of the pricing distribution, using (14) each firm's profit from the bargainers is given by

$$
\begin{equation*}
\pi_{i}=(1-\mu) \frac{E[p]}{2}=\frac{(1-\mu) l q}{2} \tag{17}
\end{equation*}
$$

and so $E[p]=l q$. Part 3 then follows immediately.
Proof of Proposition 2. Given $l_{a}$ and $l_{b}$ the indifferent price taker on the Hotelling line is at a location $\bar{x}$ such that $\bar{x}=\frac{1}{2}+\frac{l_{b}-l_{a}}{2 t}$. Thus firm $i$ 's demand from the proportion $\mu>0$ of price takers is given by

$$
\begin{equation*}
D_{i}\left(l_{i}, l_{j}\right)=\mu \min \left\{1, \max \left\{\frac{1}{2}+\frac{l_{j}-l_{i}}{2 t}, 0\right\}\right\} \tag{18}
\end{equation*}
$$

and profit from the price takers is $l_{i} D_{i}\left(l_{i}, l_{j}\right)$. From (14), expected profit from the bargainers is given by

$$
\begin{equation*}
\pi_{i}\left(l_{i}, l_{j}\right)=\frac{(1-\mu) \min \left\{l_{i}, l_{j}\right\} q}{2} \tag{19}
\end{equation*}
$$

Given $l_{j}$, firm $i$ will never set $l_{i} \geq l_{j}+t$. By doing so it receives no demand from the price takers, while by setting $l_{i} \in\left(l_{j}, l_{j}+t\right)$ profit from any bargainers is the same as when $l_{i} \geq l_{j}+t$ and profit from the price takers is strictly positive. Furthermore, if feasible firm $i$ will never set $l_{i}<l_{j}-t$ as by doing so profit from any bargainers is strictly lower than setting $l_{i}=l_{j}-t$ and profit from the price takers is also strictly lower as demand stays at $\mu$. Thus we can restrict attention to $l_{i} \in\left[l_{j}-t, l_{j}+t\right)$, and hence we can write firm $i$ 's expected total profits as

$$
\begin{equation*}
\Pi_{i}\left(l_{i}, l_{j}\right)=\frac{(1-\mu) \min \left\{l_{i}, l_{j}\right\} q}{2}+\mu l_{i}\left(\frac{1}{2}+\frac{l_{j}-l_{i}}{2 t}\right) . \tag{20}
\end{equation*}
$$

When $l_{i}<l_{j}$, using (20):

$$
\begin{align*}
\frac{\partial \Pi_{i}\left(l_{i}, l_{j}\right)}{\partial l_{i}} & =\frac{(1-\mu) q}{2}+\frac{\mu\left(t+l_{j}-2 l_{i}\right)}{2 t}  \tag{21}\\
\frac{\partial^{2} \Pi_{i}\left(l_{i}, l_{j}\right)}{\partial l_{i}^{2}} & =-\frac{\mu}{t}<0 \tag{22}
\end{align*}
$$

When $l_{i}>l_{j}$ :

$$
\begin{align*}
\frac{\partial \Pi_{i}\left(l_{i}, l_{j}\right)}{\partial l_{i}} & =\frac{\mu\left(t+l_{j}-2 l_{i}\right)}{2 t}  \tag{23}\\
\frac{\partial^{2} \Pi_{i}\left(l_{i}, l_{j}\right)}{\partial l_{i}^{2}} & =-\frac{\mu}{t}<0 \tag{24}
\end{align*}
$$

Finally, when $l_{i}=l_{j}$ the right-hand side derivative is given by (23), the left-hand side derivative is given by (21), and the right-hand side and left-hand side second derivatives remain equal to $-\frac{\mu}{t}<0$.

Now consider symmetric posted prices with $l_{i}=l_{j}=l$. The right-hand side derivative

$$
\begin{equation*}
\frac{\mu(t-l)}{2 t} \gtreqless 0 \quad \Leftrightarrow \quad l \lesseqgtr t \tag{25}
\end{equation*}
$$

while the left-hand side derivative

$$
\begin{equation*}
\frac{(1-\mu) q}{2}+\frac{\mu(t-l)}{2 t} \gtreqless 0 \Leftrightarrow l \equiv\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t \tag{26}
\end{equation*}
$$

Thus if $l<t$, the right-hand side derivative is strictly positive so the firms have a local incentive to raise price, and hence we cannot have a symmetric equilibrium. If $l>\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t$, the left-hand side derivative is strictly negative so the firms have a local incentive to lower price and again we cannot have a symmetric equilibrium. For

$$
\begin{equation*}
l^{*} \in\left[t,\left(1+q\left(\frac{1-\mu}{\mu}\right)\right) t\right] \tag{27}
\end{equation*}
$$

the right-hand side derivative is weakly negative while the left-hand side derivative is weakly positive, so the firms do not have a local incentive to deviate. Furthermore, firm $i$ 's objective function is strictly concave when $l_{j}=l^{*}$ and (27) holds: from above the second derivatives equal $-\frac{\mu}{t}<0$ everywhere and at the kink where $l_{i}=l_{j}=l^{*}$ the slope of the objective function switches from being positive to negative. Thus we have a global optimum, so $l^{*}$ forms a symmetric equilibrium, giving Part 2.

For Part 1, we must show that no pure strategy asymmetric equilibrium can exist. Suppose to the contrary that posted prices $0 \leq l_{j}<l_{i}$ form an equilibrium. Remembering from above that $l_{i}<l_{j}+t$, so $D_{i}>0$, firm $i^{\prime} s$ first-order condition implies that $(23)=0$, so we must have $l_{i}=$ $\frac{1}{2}\left(t+l_{j}\right)$. Thus $l_{j}<\frac{1}{2}\left(t+l_{j}\right)$, which implies $l_{j}<t$. But then $t+l_{i}-2 l_{j}=\left(t-l_{j}\right)+\left(l_{i}-l_{j}\right)>0$
so using (21) $\frac{\partial \Pi_{j}\left(l_{j}, l_{i}\right)}{\partial l_{j}}>0$ and hence firm $j$ would like to increase its posted price.
Part 3 follows immediately from Part 2.

Proof of Proposition 3. Parts 1 and 2 are immediate from (3), which is a strictly increasing function of $\alpha$ and $q$. Part 3 is immediate from the fact that total welfare is constant in a Hotelling model with all consumers being served with a single unit.

Proof of Proposition 4. For Part 1 note that the price takers pay the equilibrium posted price $l^{*}$ given by (2). This clearly increases as $\mu$ decreases. From Part 2 of Proposition 1, bargainers pay a proportion $q<1$ of $l^{*}$ on average. Thus Part 2 follows from the fact that $l^{*}$ increases as $\mu$ falls. Furthermore, if the proportion of consumers who switch to bargaining is sufficiently small, the increase in $l^{*}$ will be small enough that the new expected bargained price will remain below the initial equilibrium posted price, giving Part 3 .

Proof of Lemma 1. From (3) firm profits can be written as

$$
\begin{equation*}
\Pi_{i}=\frac{1}{2}\left(\mu+\alpha q(1-\mu)+q(1-\mu)+\alpha q^{2} \frac{(1-\mu)^{2}}{\mu}\right) t \tag{28}
\end{equation*}
$$

Differentiating this with respect to $\mu$ yields:

$$
\begin{align*}
\frac{\partial \Pi_{i}}{\partial \mu} & =\frac{1}{2}\left(1-q(\alpha+1)+\alpha q^{2}\left(-\frac{1}{\mu^{2}}+1\right)\right) t  \tag{29}\\
\frac{\partial^{2} \Pi_{i}}{\partial \mu^{2}} & =\frac{\alpha q^{2} t}{\mu^{3}}>0 \tag{30}
\end{align*}
$$

Hence profits are strictly convex in $\mu$, and so are also strictly convex in $1-\mu$. Finally note that $\lim _{\mu \rightarrow 1} \Pi_{i}=\frac{t}{2}$ and $\lim _{\mu \rightarrow 0} \Pi_{i}=\infty$, so $\Pi_{i}$ is maximized as $\mu \rightarrow 0$.

Proof of Proposition 5. Using (29), $\frac{\partial \Pi_{i}}{\partial \mu}=0$ if and only if

$$
\begin{equation*}
1-q(\alpha+1)+\alpha q^{2}=\frac{\alpha q^{2}}{\mu^{2}} \Leftrightarrow \mu=\sqrt{\frac{\alpha q^{2}}{(1-q)(1-\alpha q)}} . \tag{31}
\end{equation*}
$$

Given that profits are strictly convex in the proportion of price takers (see the proof of Lemma 1), the price-minimizing proportion of price takers is thus given by (4). Parts 1 and 2 then follow immediately from the convexity of the profit function, remembering that consumer surplus moves in the opposite direction to profits given all consumers buy a single unit. Part 3 follows immediately from (4).

Proof of Lemma 2. Suppose the firms set posted prices $l_{i}$ and $l_{j}$. As both firms price match the bargainers all receive an offered price of $l=\min \left\{l_{i}, l_{j}\right\}$, and so split themselves equally between the firms and transact at $l$. Thus profit from the bargainers is given by (19) with $q$ set
to 1 . The proof of Proposition 2 then extends to show that the firms set identical posted prices in the range given by (7). As the firms share the price takers at the symmetric equilibrium posted prices, (8) is then immediate, and (8) is clearly greater than (6).

Proof of Lemma 3. Suppose firm $i$ bargains while firm $j$ price matches. Any bargainers who approach firm $j$ will be offered $l=\min \left\{l_{i}, l_{j}\right\}$, the price match price. The optimal bargaining response to this is for firm $i$ to offer bargainers $l-\varepsilon$ for some arbitrarily small $\varepsilon>0$. With this bargaining strategy firm $i$ wins the business of all those bargainers who ask for second quotes from both firms as well as those who only ask it for a second quote. Thus, the bargainers all transact at (essentially) the lower posted price $l$, while the price matching firm $j$ secures only a proportion $q / 2$ of the bargainers and the bargaining firm $i$ secures a proportion $1-q / 2$. Conditional on the posted prices, $i$ 's profits are therefore given by the expressions in the proof of Proposition 2, replacing $q / 2$ with $(1-q / 2)$, while $j$ 's are exactly the same as those in the earlier proof. The proof then extends to show that the firms set identical posted prices in the range given by (9). The only complication is that $i$ 's left-hand side derivative is larger than $j$ 's as $1-q / 2>q / 2$, so it is the price matching firm $j$ whose incentives to deviate downward act as the binding constraint on the upper bound of the equilibrium price range. The profit expressions (10) and (11) then follow given the firms share the price takers at the symmetric equilibrium posted prices. The profit comparisons are immediate from comparing (10) and (11) to (6).

Proof of Proposition 6. From (6) and (11), $\Pi_{j}^{(\mathrm{BA}, \mathrm{BA})}=\Pi_{j}^{(\mathrm{BA}, \mathrm{PM})}$. Hence if firm $i$ is bargaining then $j$ is indifferent between bargaining and not, giving Part 1. For Parts 2 and 3 note that from (8) and (10):

$$
\begin{align*}
\Pi_{i}^{(\mathrm{PM}, \mathrm{PM})} \gtreqless \Pi_{i}^{(\mathrm{BA}, \mathrm{PM})} & \Leftrightarrow \frac{\alpha}{\mu}(1-q(1+(1-\mu)(1-q))) \gtreqless(1-q)  \tag{32}\\
& \Leftrightarrow \alpha-\alpha q(1-\mu)-\mu \gtreqless 0 . \tag{33}
\end{align*}
$$

Thus, (PM,PM) is an equilibrium if and only if (12) holds. Furthermore, the bargainer will not want to deviate from a hybrid equilibrium if and only if the reverse inequality holds, while the price matcher has no incentive to deviate given $\Pi_{j}^{(\mathrm{BA}, \mathrm{BA})}=\Pi_{j}^{(\mathrm{BA}, \mathrm{PM})}$ from above.

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[^1]:    ${ }^{1}$ Section 6 gives an example of an information campaign launched by the Office of Fair Trading to encourage bargaining in the UK estate agency (realtor) market.

[^2]:    ${ }^{2}$ To preserve tractability, we assume that the proportion of bargainers is given exogenously and conduct comparative statics with respect to this proportion. The bargaining behavior we model would be the endogenous outcome of a consumer optimization problem if we introduced explicit costs of bargaining. Such an extension adds much complication, and we conjecture that as in Gill and Thanassoulis (2009) our qualitative results would remain unchanged.
    ${ }^{3}$ The psychological embarrassment of starting a negotiation and lack of information that discounts are available can explain bargaining costs. See Burnham et al. (2003) for a discussion of psychological and informational switching costs.

[^3]:    ${ }^{4}$ Our simple bargaining model is tractable and reflects real-world bargaining in that bargainers actively approach a seller to ask for a better price than the one posted, different bargainers approach different numbers of sellers and (as shown below) bargained prices are random.

[^4]:    ${ }^{5}$ In fact there can be no asymmetric equilibrium. The proof works by showing that the two firms' pricing distributions have to be continuous on the same connected support with a supremum at $l$. The details are laborious and hence omitted, but it is then straightforward to establish symmetry.

[^5]:    ${ }^{6}$ The bargained pricing distribution for a higher $l$ or $q$ first-order stochastically dominates that for a lower $l$ or $q$.

[^6]:    ${ }^{7}$ The range of equilibria we find has parallels in the price-matching guarantee literature (see e.g., Chen, 1995 and Moorthy and Winter, 2006).
    ${ }^{8}$ If $\alpha=0, l^{*}=t$ for any $\mu$, so the posted prices are not affected by the proportion of bargainers, and hence neither are the bargained prices from Proposition 1.

[^7]:    ${ }^{9}$ See for example the discussion of the UK car market in Section 1. Drozdenko and Jensen (2005, p.264) note that discounting is affected by industry convention and historical precedent. Zettelmeyer et al. (2006) consider the impact of online referral services such as Autobytel.com which request quotes from dealers on behalf of automobile buyers.

[^8]:    ${ }^{10}$ We do not consider firms committing to rigid posted prices. As bargainers select only on price, a refusal to match even a rival's posted price will mean the loss of all bargainers to a rival who sets a lower posted price. Further, if both firms chose to stick to rigid posted prices, the incentive to $\varepsilon$-undercut the rival's posted price to gain all the bargainers means that there could be no pure strategy equilibrium at the posted price setting stage.
    ${ }^{11}$ If a firm's sales staff were given discretion to offer any price between their own posted price and a rival's lower posted price, they would always choose to price match to avoid losing all the bargainers.

[^9]:    ${ }^{12}$ Even if the density is zero at the highest price in the support of the distribution, by continuity profit at this price must be the same as for prices in the interior of the mixing distribution.

