AdS Black Branes: A Sound Bound? ①

work in progress with Aron Jansen (Barcelona), Umut Gürsoy (Utrecht), Ronnie Rodgers (Southampton)

0905.0900 Hohler + Stephanov
0905.0903 Cherman, Cohen, Nellore — The proposal
0905.2969 Cherman + Nellore

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hep-th/0508220 Friess, Gubser, Mitra
0904.1716, 1010.5748 Buchel + Pagnutti — Holographic counter-examples
1603.07724 Gürsoy et al.
1603.05950 Janik et al.

1609.03480 Hoyos, Vuorinen et al. — chemical potential $\mu \neq 0$

1611.05808 Mateos et al. — D7 branes / Landau pole

Rhoades + Ruffini PRL 32 324 (1974) — Neutron star mass
Bedaque + Steiner PRL 114 031103 (2015) 1408.5116

I. Review: Sound Modes in Fluids

II. A Bound on the Speed of Sound from Holography?

III. Counterexamples from Holography?

I. Review: Sound Modes in Fluids

Consider (3+1)-dimensional Minkowski space

(although nothing that follows is unique to (3+1) dim.)

metric $[\eta_{\mu\nu}] = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ (t, x^i) $i = 1,2,3$

Consider a relativistic fluid

with conserved stress-energy tensor

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0 \qquad \langle T^{\mu\nu} \rangle = \langle T^{\nu\mu} \rangle$$

in ~~with~~ an isotropic, homogeneous thermal equilibrium state

$$\langle T^{tt} \rangle \equiv \mathcal{E} \qquad \text{energy density}$$

$$\langle T^{ij} \rangle \equiv P \delta^{ij} \qquad \text{pressure}$$

and equation of state $\mathcal{E}(P)$ or $P(\mathcal{E})$

Consider a small fluctuation of stress-energy

$$\langle T^{tt} \rangle = \mathcal{E} + \langle \delta T^{tt} \rangle$$

$$\langle T^{ij} \rangle = P \delta^{ij} + \langle \delta T^{ij} \rangle$$

$$\langle T^{ti} \rangle = \langle \delta T^{ti} \rangle$$

with $\langle \delta T^{tt} \rangle, \langle \delta T^{ij} \rangle, \langle \delta T^{ti} \rangle \ll \mathcal{E}, P$

The fluctuations depend on time and space

Fourier transform

$$\langle \delta T^{\mu\nu} \rangle \equiv \int \frac{d^4 q}{(2\pi)^4} \, e^{-i q \cdot x} \langle \delta \tilde{T}^{\mu\nu} \rangle$$

$$q \cdot x = -\omega t + \vec{q} \cdot \vec{x}$$

From the conservation equation

$$-i\omega \langle \delta \tilde{T}^{tt} \rangle + i q_i \langle \delta \tilde{T}^{ti} \rangle = 0$$

$$-i\omega \langle \delta \tilde{T}^{ti} \rangle + i q_j \langle \delta \tilde{T}^{ji} \rangle = 0$$

We want the <u>eigenmodes</u> of $\langle \delta \tilde{T}^{\mu\nu} \rangle$

Among other things, these will include <u>sound</u>

Simplest example: <u>ideal fluid</u>

Landau + Lifshitz vol. 6 <u>Fluid Mechanics</u>

Chapter <u>VIII</u> "Sound"

"... a sound wave in an ideal fluid is, like any motion in an ideal fluid, adiabatic. Hence the small change [...] in the pressure is related to the small change in [energy] density by"

$$\langle \delta \tilde{T}^{ij} \rangle = \frac{\partial P}{\partial \varepsilon} \langle \delta \tilde{T}^{tt} \rangle \delta^{ij}$$

Plugging into the conservation equations

$$-i\omega \langle \delta \tilde{T}^{tt} \rangle + i q_i \langle \delta \tilde{T}^{ti} \rangle = 0$$

$$-i\omega \langle \delta \tilde{T}^{ti} \rangle + i q_j \delta^{ij} \frac{\partial P}{\partial \varepsilon} \langle \delta \tilde{T}^{tt} \rangle = 0$$

Solving the second equation for $\langle \delta \tilde{T}^{ti} \rangle$ and plugging the result into the first equation

$$\left(\omega^2 - \frac{\partial P}{\partial \varepsilon} \vec{q}^{\,2} \right) \langle \delta \tilde{T}^{tt} \rangle = 0$$

$\Rightarrow$ eigenmodes with $\boxed{\omega = \pm \sqrt{\frac{\partial P}{\partial \varepsilon}}\, |\vec{q}|}$

These are fluctuations of the energy density that propagate with speed V_s given by

$$\boxed{V_s^2 = \frac{\partial P}{\partial \varepsilon}}$$

These are the <u>sound waves</u>

(the <u>only</u> non-trivial excitation of an <u>ideal</u> fluid)

V_s determined <u>entirely</u> by <u>equilibrium thermodynamics</u>!

<u>i.e.</u> the equation of state $P(\varepsilon)$

For a <u>non-ideal fluid</u> in a <u>hydrodynamic</u> regime

to leading order in spatial gradients

$$\langle \delta T^{ij} \rangle = - \frac{1}{\varepsilon + p} \Big[\eta \, (\partial_i \langle \delta T^{ti} \rangle + \partial_j \langle \delta T^{ti} \rangle - \tfrac{2}{3} \delta^{ij} \partial_k \langle \delta T^{tk} \rangle$$
$$+ \zeta \, \delta^{ij} \partial_k \langle \delta T^{tk} \rangle \Big]$$

$\eta \equiv$ shear viscosity

$\zeta \equiv$ bulk viscosity

the term $\propto \eta$ is <u>traceless</u>

the term $\propto \zeta$ has trace $\propto \zeta$

Solving for the eigenmodes again:

① Transverse fluctuations of $\langle \delta T^{ti} \rangle$

shear mode

$$\omega = -i \frac{\eta}{\varepsilon + p} |\vec{q}|^2 + \dots$$

Purely imaginary $\Rightarrow$ non-propagating dissipative

② Sound Modes

$$\omega = \pm v_s |\vec{q}| - \frac{i}{2} \frac{1}{\varepsilon + p} \left(\zeta + \frac{4}{3} \eta \right) |\vec{q}|^2 + \dots$$

Dissipation $\Rightarrow$ non-zero dispersion

<u>i.e.</u> the sound wave now <u>decays</u>

Other ways to write $v_s^2 = \frac{\partial P}{\partial \varepsilon}$

just using thermodynamic identities

WHEN ALL CHEMICAL POTENTIALS VANISH

$$\boxed{\mu = 0}$$

$F = -P$ = free energy density

$s \equiv -\frac{\partial F}{\partial T} = \frac{\partial P}{\partial T}$ = entropy density

$c_V \equiv \frac{\partial \varepsilon}{\partial T}$ heat capacity density

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$$v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\partial P/\partial T}{\partial \varepsilon/\partial T} = \frac{s}{c_V} = \frac{s}{T}\left(\frac{\partial s}{\partial T}\right)^{-1} = \frac{\partial \ln T}{\partial \ln s}$$

$w \equiv \varepsilon + P$ enthalpy

$\langle T^\mu{}_\mu \rangle = \Theta = \varepsilon - 3P$

$\varepsilon = (3w + \Theta)/4 \qquad P = (w - \Theta)/4$

$$v_s^2 = \frac{1 - d\Theta/dw}{3 + d\Theta/dw}$$

Special values of $v_s^2 = \frac{\partial P}{\partial \varepsilon}$

(1) $v_s^2 \geq 0$ because if $v_s^2 < 0$

then v_s must be complex-valued, and

hence for the sound mode $\mathrm{Im}\,\omega \simeq \mathrm{Re}\,\omega$

$\Rightarrow$ No longer a well-defined quasi-particle

Also, when $\mu = 0$ thermodynamic stability $\Rightarrow v_s^2 = \frac{s}{c_v} \geq 0$

(2) Causality requires $v_s^2 \leq 1$

(3) For a theory with conformal invariance

$T^\mu{}_\mu = 0$ as an <u>operator</u>

true in all states

thermal equilibrium $\langle T^\mu{}_\nu \rangle = \begin{pmatrix} \varepsilon & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$

$\langle T^\mu{}_\mu \rangle = 0 \Rightarrow -\varepsilon + 3P = 0 \Rightarrow \varepsilon = 3P$

$\Rightarrow v_s^2 = \frac{1}{3}$ (in d-dim. $v_s^2 = \frac{1}{d-1}$)

First-order hydrodynamics $\langle T^\mu{}_\mu \rangle = 0 \Rightarrow \zeta = 0$

Both v_s^2 and ζ measure <u>deviation</u> from conformal invariance

$g_{\mu\nu}$ = non-dynamical background metric

$Z[g_{\mu\nu}, \ldots]$ = generating functional of connected correlation functions

in Euclidean signature

$$T_{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \ln Z[g_{\mu\nu}, \ldots]$$

conformal transformation

$$x_\mu \longrightarrow x_\mu'(x_\nu)$$

such that $g_{\mu\nu} \longrightarrow e^{2\Omega(x)} g_{\mu\nu}$

$$\Rightarrow \quad T^\mu{}_\mu = \frac{1}{\sqrt{g}} \frac{\delta}{\delta\Omega} \ln Z$$

conformal invariance $\Rightarrow$ $\frac{\delta}{\delta\Omega} \ln Z = 0$

$$\Rightarrow \quad T^\mu{}_\mu = 0$$

II. A Bound on the Speed of Sound from Holography?

0905.0900 Hohler + Stephanov

0905.0903 Cherman, Cohen, Nellore

Consider classical Einstein gravity on asymptotically AdS_5 manifold M minimally coupled to scalar ϕ with almost any potential $V(\phi)$

$$g_{\mu\nu} \longleftrightarrow T_{\mu\nu}$$

$$\phi \longleftrightarrow \mathcal{O}$$

$\mathcal{O}$ = relevant scalar operator responsible for breaking conformality (i.e. introduce dimensionful source for $\mathcal{O}$)

~~Proof~~ ~~Theorem~~ Theorem: For such systems,

as $T \to \infty$, $v_s^2 \to \frac{1}{3}$ from below

In other words, if we start at $T=\infty$ and then reduce T,

the correction to $v_s^2 = \frac{1}{3}$ is

ALWAYS NEGATIVE

PROOF OF THE THEOREM

$$S_{grav} = \frac{1}{2\kappa^2} \int_M d^5x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] + S_{\partial M}$$

κ^2 = gravitational constant $\propto 1/N_c^2$ typically

g = determinant of $g_{\mu\nu}$

R = Ricci scalar

$S_{\partial M}$ = boundary terms
(Gibbons - Hawking + counterterms)

Want solutions describing non-zero T

Phase structure depends on $V(\phi)$

for sufficiently high T, we expect a black brane solution

The most general metric that is static, isotropic and homogeneous in field theory directions, asymptotically approaches AdS_5 and has a regular, non-extremal horizon takes the form

$$ds^2 = A(z)^2 e^{2B(z)} \frac{dz^2}{f(z)} + A(z)^2 \left(-f(z)\,dt^2 + d\vec{x}^2\right)$$

with $z \in (0, z_H]$

Boundary conditions: asymptotically AdS_5

$$\lim_{z\to 0} f(z) = 1 \qquad \lim_{z\to 0} e^{2B(z)} = 1$$

as $z \to 0$ $\quad A(z) \to \frac{L}{z}$

$\lim_{z\to 0} \phi(z) = 0$ (normalizable)

Boundary condition: non-extremal horizon

$$f(z_H) = 0 \qquad \text{(simple zero)}$$

Demand that ~~all metric functions~~ $A(z), B(z)$ and $\phi(z)$ are regular at z_H (except g_{zz})

Hawking Temperature

$$T = \frac{\left| A(z_H)^2 f'(z_H)\, e^{-B(z_H)} \right|}{4\pi}$$

Bekenstein - Hawking entropy

$$S = \frac{2\pi}{\kappa^2} \left| A(z_H) \right|^3$$

where $S \propto \frac{1}{\kappa^2} \propto N_c^{\,2}$

suggests a <u>deconfined</u> phase

The scalar ϕ depends only on z

and vanishes as $z \to 0$

<u>Assumption</u>: $\lim_{z \to 0} V(\phi) = -\frac{12}{L^2} + \frac{1}{2} m^2 \phi^2 + \mathcal{O}(\phi^4)$

with $m^2 = \Delta(\Delta - 4)/L^2$

$\Rightarrow \quad \Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$

Δ_+ is the dimension of the dual $\mathcal{O}$

Breitenlohner - Freedman stability bound

$$m^2 l^2 \geq -4$$

Stay above that : $m^2 l^2 > -4$

which $\Rightarrow$ $\Delta_+ > 2$

Restrict to relevant operators

which $\Rightarrow$ $\Delta_+ < 4$

Asymptotically : $\phi(z) = c_- z^{\Delta_-} + c_+ z^{\Delta_+} + \ldots$

c_- = source for dual $\mathcal{O}$

c_+ = VEV for dual $\mathcal{O}$

with choice $2 < \Delta_+ < 4$

these are both dimensionful

$[c_-] = \Delta_-$ $\quad\quad$ $[c_+] = \Delta_+$

$$\langle \mathcal{O} \rangle = - \frac{\delta S_{grav}}{\delta c_-}\Big|_{on\text{-}shell} = -(\Delta_+ - \Delta_-)\, c_+$$

Here I will follow 0905.0900 Kohler + Stephanov

Strategy: Compute $\langle T^{tt} \rangle$ and $\langle T^{xx} \rangle$

compute $\frac{\partial P}{\partial \mathcal{E}}$

Asymptotically $g_{\mu\nu}(z) = \frac{L^2}{z^2} \, \overset{\circ}{g}_{\mu\nu} + \dots$

$$\langle T^{\mu\nu} \rangle = 2 \left. \frac{\delta S_{grav}}{\delta g^{0}_{\mu\nu}} \right|_{on\text{-}shell}$$

~~They~~ "Bare" correlator diverges

could "Renormalize" using counterterms in $S_{\partial M}$

Kohler + Stephanov instead

renormalize by subtracting $T=0$ values

$$\mathcal{E} \equiv \langle T^{tt} \rangle_{T>0} - \langle T^{tt} \rangle_{T=0}$$

$$P \equiv \langle T^{xx} \rangle_{T>0} - \langle T^{xx} \rangle_{T=0}$$

$$w \equiv \mathcal{E} + P \qquad \text{enthalpy}$$

A straightforward calculation gives

$$\varepsilon = \left(3w - c_- c_+ \Delta_- (\Delta_+ - \Delta_-)\right)/4$$

$$P = \left(w + c_- c_+ \Delta_- (\Delta_+ - \Delta_-)\right)/4$$

which also immediately gives

$$\langle T^\mu{}_\mu \rangle = -\varepsilon + 3P = c_- \Delta_- \langle \mathcal{O} \rangle$$

as required by a Ward identity, and

$$v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{1 + c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w}}{3 - c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w}}$$

So far we haven't really used the high-T limit

Do so now, in the form

$$c_- / T^{\Delta_-} \ll 1$$

since c_- is the only dimensionful scale besides T in the limit $c_-/T^{\Delta} \ll 1$

$$V_s^2 = \frac{1}{3} + \frac{4}{9} c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w} + \ldots$$

We can <u>linearize in ϕ</u>

The metric is then approximately AdS_5-Schwarzschild

$$B = 0 \quad \forall z$$

$$f(z) = 1 - \frac{w}{4} z^4 \qquad A(z) = \frac{L}{z}$$

$$\Rightarrow V_s^2 = \frac{1}{3}$$

The most general solution to ϕ's linearized equation is then

$$\phi(z) = c_- z^{\Delta_-} {}_2F_1(\Delta_-/4, \Delta_-/4, \Delta_-/2, wz^4/4)$$

$$+ c_+ z^{\Delta_+} {}_2F_1(\Delta_+/4, \Delta_+/4, \Delta_+/2, wz^4/4)$$

BOTH terms diverge logarithmically at $z = z_H$

regularity of $\phi(z_H)$ fixes c_+ in terms of c_-

$$C_+ = -c_- w^{(\Delta_+ - \Delta_-)/4} D(\Delta_-)$$

$$D(\Delta_-) \equiv \frac{\pi 2^{\Delta_-}}{2-\Delta_-} \cot\left(\frac{\pi\Delta_-}{4}\right) \frac{\Gamma(\Delta_-/2)^2}{\Gamma(\Delta_-/4)^4}$$

$$\phi(z_H) = c_- w^{\Delta_+/4 - 1} 2^{-\Delta_+/2} (2\Delta_+ - 4) \frac{\Gamma(\Delta_+/4)^2}{\Gamma(\Delta_+/2)}$$

with these results, we find

$$V_s^2 = \frac{1}{3} - \frac{1}{9} c_-^2 \Delta_- (\Delta_+ - \Delta_-)^2 w^{-\Delta_-/2} D(\Delta_-) + \dots$$

↑

NEGATIVE FOR ANY $V(\phi)$

In terms of $\phi(z_H)$

$$V_s^2 = \frac{1}{3} - \frac{1}{18\pi} (4-\Delta_+)(4-2\Delta_+) \tan\left(\frac{\pi\Delta_+}{4}\right) \phi(z_H)^2 + \dots$$

0905.0903 Cherman, Cohen, Nellore

compute $\phi(z)$'s leading order back-reaction (keeping fixed $c_- L$ and z_H)

and compute $V_s^2 = \frac{\partial \ln T}{\partial \ln s}$ OBTAIN THE SAME RESULT

GENERALIZATIONS

0912.2100 Yarom

generalize AdS_5 to AdS_{d+1}

$$V_s^2 = \frac{1}{d-1} - (d-2\Delta_+)\tan\left(\frac{\pi\Delta_+}{d}\right)\frac{c_-^2}{T^{2(d-\Delta_+)}} D(d,\Delta_+)$$

$$D(d,\Delta_+) \equiv \frac{16^{1-\Delta_+/d}\,(d-\Delta_+)\,d^{2(d-\Delta_+-1)}}{2(4\pi)^{2(d-\Delta_+)}\,(d-1)^2}\left[\frac{\Gamma\left(\frac{\Delta_+}{d}\right)}{\Gamma\left(\frac{\Delta_+}{d}+\frac{1}{2}\right)}\right]^2$$

0905.2969 Cherman + Nellore

generalize from one to multiple scalars

Find the "least relevant" ~~…~~ $\mathcal{O}$

Plug ~~…~~ into the above

that $\mathcal{O}$'s Δ_+

(always assuming ~~…~~ ~~…~~ $\mathcal{O}$ is relevant)

That was an explicit calculation,

but Cherman, Cohen, and Nellore (0905, 0903)

go farther, and ask

> Is $v_s^2 = \frac{1}{d-1}$ an <u>upper bound</u>
>
> on the speed of sound?

For all fluids?

Or just some subset?

relativistic fluids? fluids of (strongly) interacting particles?

gauge theories? superfluids?

Difficult to survey ALL KNOWN FLUIDS!

Restrict to cases similar to their

holographic systems

- relativistic
- uncharged (chemical potential $\mu = 0$)
- translationally invariant (in holography: black brane)
- no symmetry breaking (not superfluid)
- approaches a CFT in the UV
  (in holography: asymptotically AdS)

Bedaque + Steiner 1408.5116

- Non-relativistic systems have $v_s^2 \ll 1$
- Relativistic free <u>massless</u> particles have $v_s^2 = \frac{1}{3}$
- Relativistic free <u>massive</u> particles have $v_s^2 < \frac{1}{3}$
- Weak coupling among the particles $\Rightarrow v_s^2 < \frac{1}{3}$

<u>Ex</u> $\lambda \phi^4$ with $\lambda \ll 1$ and mass m

when $T \gtrsim m/\sqrt{\lambda}$

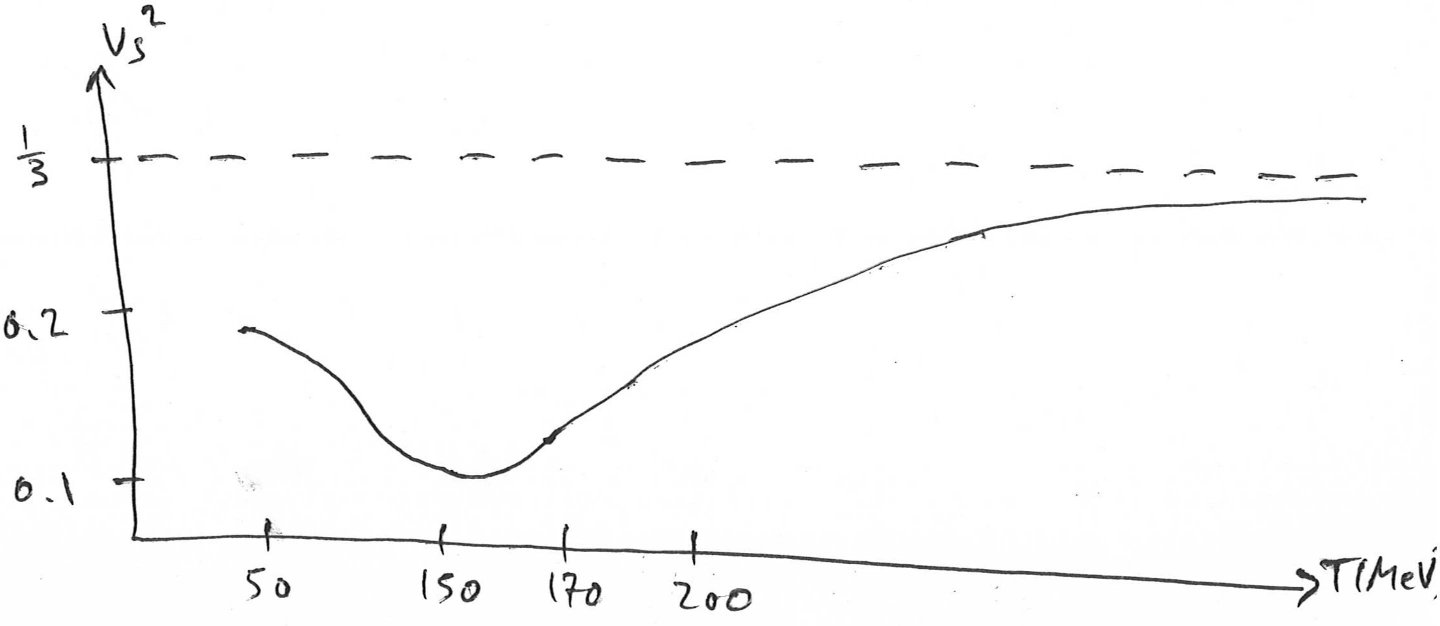
hep-ph/9409250 Jeon, hep-ph/9512263 Jeon + Yaffe

$$v_s^2 = \frac{1}{3} - \frac{5}{12\pi^2} \frac{m^2}{T^2} + O(\lambda^{3/2})$$

- <u>Quantum Chromodynamics</u> has $v_s^2 < \frac{1}{3}$

Lattice QCD results for v_s^2 (with $\mu = 0$)

hep-lat/0601013 Karsch, 1007.2580 Borsányi et al.



Upper Bound on Neutron Star Masses

Rhoades + Ruffini PRL 32 324 (1974)

Assume P and ρ are bounded
(non-zero but $< \infty$)

$$v_s^2 = \frac{\partial P}{\partial \rho} \geq 0 \quad \text{and} \quad \leq 1$$

$$M = \int_0^R dr\, 4\pi r^2 \rho(r)$$

EXTREMIZE M

"... the maximum mass is obtained for that equation of state which maximizes at <u>each</u> density the velocity of sound ..."

$\rho < 4.6 \cdot 10^{14} \frac{g}{cm^3}$ free degenerate neutron EOS maximizes v_s^2

$\rho > 4.6 \cdot 10^{14} \frac{g}{cm^3}$ they assume EOS with $v_s^2 = 1$

$$\Rightarrow \boxed{M \leq 3.2 M_\odot}$$

Bedaque and Steiner PRL 114 031103 (2015)
1408.5116

n = nucleon density

n_0 = nuclear saturation density $\simeq 0.16/\text{fm}^3$

n_p = proton density

$$x \equiv \frac{n_p}{n} = \text{proton fraction}$$

$$T_0 = \left(\frac{3\pi^2 n_0}{2}\right)^{2/3} \frac{1}{2M_N}$$

M_N = neutron mass

M_p = proton mass

"Skyrme-like parametrization" of EOS:

$$\begin{aligned}\mathcal{E}(n,x) = {} & (1-x)M_N + x M_p \\ & + \frac{3}{5}\left[x^{5/3} + (1-x)^{5/3}\right]\left(\frac{2n}{n_0}\right)^{2/3} \\ & - \left[(2\alpha - \alpha_L)x(1-x) + \alpha_L\right]\frac{n}{n_0} \\ & + \left[(2\eta - 4\eta_L)x(1-x) + \eta_L\right]\left(\frac{n}{n_0}\right)^{\gamma}\end{aligned}$$

five parameters: $\alpha, \alpha_L, \eta, \eta_L, \gamma$

Determined by empirical knowledge of

$$-B = \mathcal{E}(n_0, \tfrac{1}{2}) + \tfrac{1}{2}(M_N + M_p)$$

$$P = n^2 \frac{\partial \mathcal{E}/n}{\partial n}\Big|_{\substack{n=n_0 \\ x=1/2}} = 0$$

$$K = 9 n_0^2 \frac{\partial^2 \mathcal{E}}{\partial n^2}\Big|_{\substack{n=n_0 \\ x=1/2}}$$ nuclear incompressibility

$$S = \frac{n_0}{8} \frac{\partial^2 \mathcal{E}}{\partial x^2}\Big|_{\substack{n=n_0 \\ x=1/2}}$$ symmetry energy

$$L = \frac{3n_0}{8} \frac{\partial^3 \mathcal{E}}{\partial n\, \partial x^2}\Big|_{\substack{n=n_0 \\ x=1/2}}$$

various experimental / observational results give

$B = 16 \pm 0.1$ MeV

$K = 235 \pm 25$ MeV

$S = 32 \pm 2$ MeV

$L = 50 \pm 15$ MeV

$B, n_0, K \Rightarrow \alpha, \eta, \gamma$

then $S, L \Rightarrow \alpha_L, \eta_L$

finally, minimize $\mathcal{E}(n, x)$ in x with fixed n

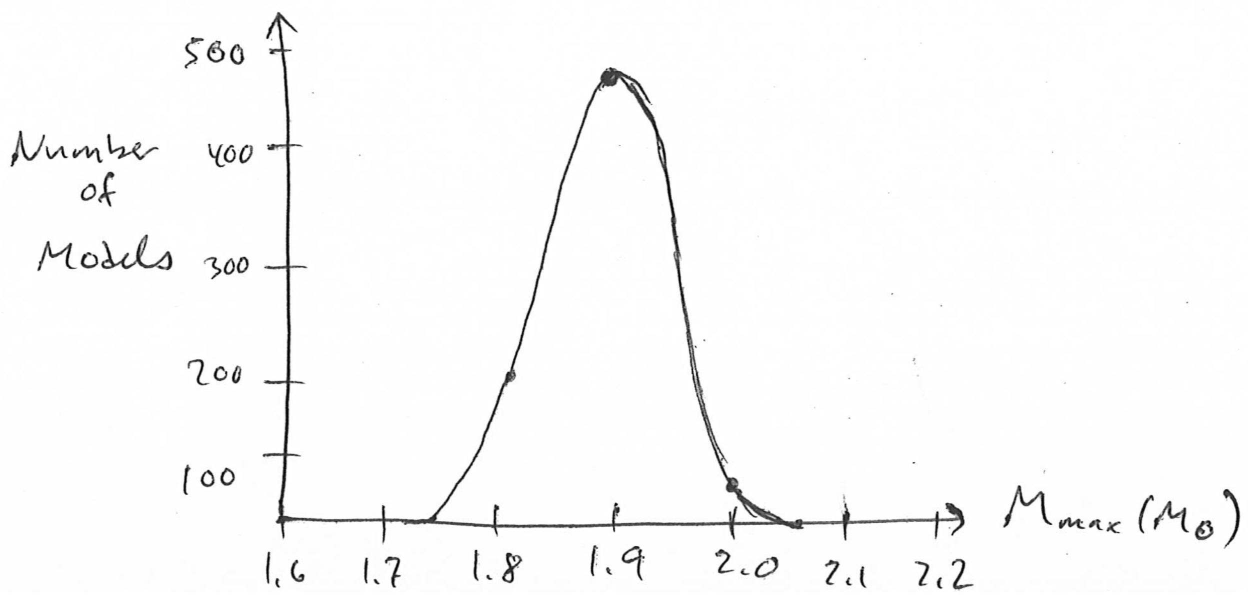
following Rhoades + Ruffini, write EOS that maximizes v_s^2 at each ρ now imposing upper bound $v_s^2 \le \frac{1}{3}$

$$\varepsilon = \begin{cases} \min_x \ \varepsilon(n(\rho), x) & n < 2n_0 \\ \min_x \ \varepsilon(2n_0, x) + 3p & n > 2n_0 \end{cases}$$

(unrealistic: ε continuous but v_s^2 jumps at $2n_0$ only meant to bound NS mass)

select K, S, L from Gaussian distribution centred on observed values above standard deviation given by ± in values

each EOS → TOV equations → maximum mass



Bedaque + Steiner:

"... the main point of this paper is the abrupt disappearance of viable models at masses larger than about $2M_\odot$."

"... the $v_s^2 < \frac{1}{3}$ bound is in strong tension with known empirical facts."

Similar analyses, with various models for the EOS

1608.00344	Moustakidis et. al	disfavor $v_s^2 < \frac{1}{3}$
1709.07889	Alsing, Silva, Berti	
1801.01923	Tews, Carlson, Gandolfi, Reddy	
1811.07071	Ma + Rho	favor $v_s^2 < \frac{1}{3}$

CAN WE <u>PROVE</u> $v_s^2 \leq \frac{1}{3}$

FOR ANY CLASS OF SYSTEMS?

NON-TRIVIAL, INTERACTING

Can we prove $v_s^2 \leq \frac{1}{d-1}$

for fluids with holographic duals?

I checked $\gtrsim$ 200 papers

HOLOGRAPHIC COUNTER-EXAMPLES:

~~FOUR EXCEPTIONS:~~

① Probe brane zero sound (many papers)

② 1007.3431 Albrecht + Erlich

Hard-wall AdS/QCD with isospin $\mu_I \neq 0$

$\Rightarrow$ pion condensate

$$v_s^2 = \frac{\mu_I^4 - m_\pi^4}{\mu_I^4 + 3m_\pi^4}$$

Compare to hep-ph/0011365 Son + Stephanov

chiral perturbation theory at $T=0$

$m_\pi \approx 140$ MeV $\ll \mu_I \ll m_\rho \approx 775$ MeV $\approx 5.66\, m_\pi$

$\Rightarrow$ pion condensate

$$v_s^2 = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3m_\pi^2}$$

COUNTER-EXAMPLE

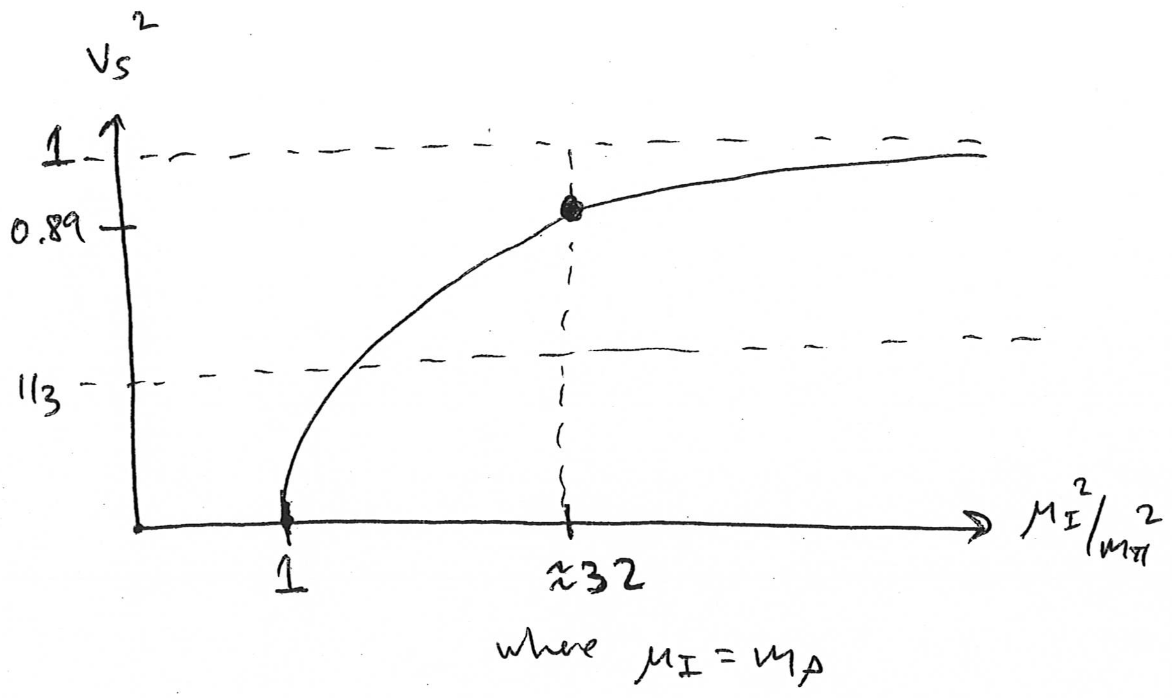
hep-ph/0011365 Son + Stephanov

QCD with $T = 0$ and non-zero isospin chemical potential μ_I

in regime $m_\pi \approx 140\ \text{MeV} \ll \mu_I \ll m_\rho \approx 775\ \text{MeV}$
$\approx 5.66\, m_\pi$

Chiral Perturbation Theory reveals the ground state is a Bose-Einstein Condensate of Pions

$$v_s^2 = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3m_\pi^2}$$



(3) 1611.05808 Faedo, Mateos, Pantelidou, Tarrio

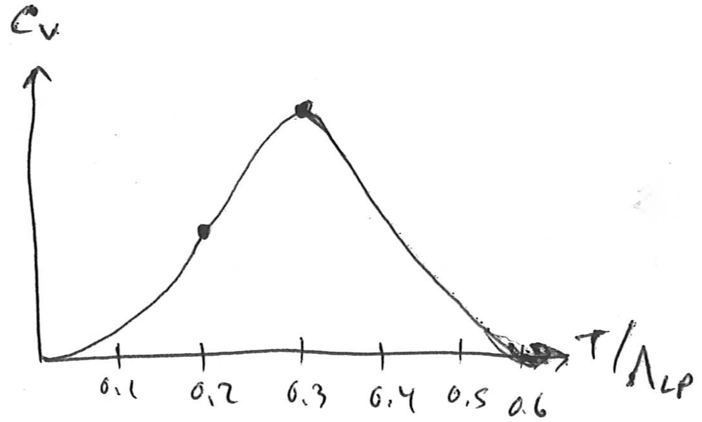
D3 - Branes + smeared D7-branes

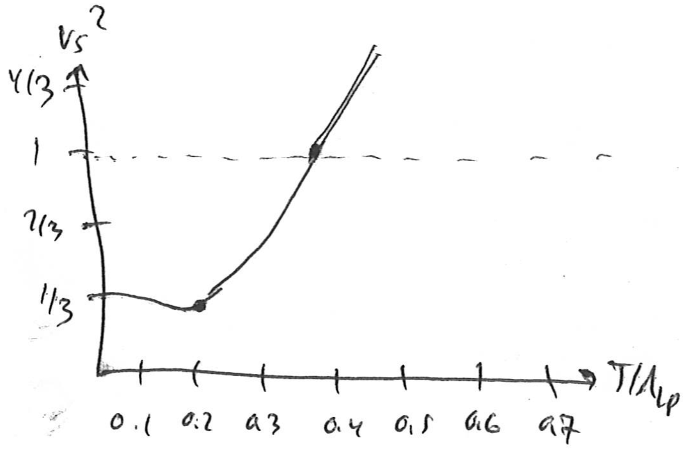
NOT asymptotically AdS_5

Landau pole in the UV

rather, hyperscaling violating fixed point

with $z = 1$ $\theta = 7/2$





(4) 1609.03480 Hoyos, Jokela, Fernandez, Vuorinen

holographic model of NS EOS

$\mu \neq 0$

they find solutions with $v_s^2 > \frac{1}{3}$

... that appear to have an

unstable QNM

(5) hep-th/0508220 Gubser, Mitra

0904.1716 } Buchel + Pagnutti
1010.5748

1603.07724 Gürsoy, Jansen, van der Schee

1603.05950 Janik, Jankowski, Soltanpanahi

Einstein - Hilbert + Negative cosmological constant

+ 1 or 2 scalars + potential
(no gauge field ⇒ $\mu = 0$)

Meet Hohler + Stephanov's assumptions

with some exceptions

Ex) some cases have <u>irrelevant</u> scalar: $\Delta > d$

Each case has complicated phase structure:
many branches of black brane solutions

<u>All</u> have (at least) one branch with $v_s^2 > \frac{1}{d-1}$

What do those solutions have in common?

- in CFT a source is non-zero : $c_- \neq 0$
  to break conformal symmetry
- Locally thermodynamically stable

$$c_V \geq 0$$

(obvious from $v_s^2 = \frac{s}{c_V} > \frac{1}{d-1} > 0$)

- NOT globally thermodynamically stable

another branch solutions always exists

with lower free energy / higher entropy

<u>i.e.</u> these are "small black holes (branes)"

- OUR MAIN RESULT (so far)

ALL have an unstable quasi-normal mode (QNM)

Consider linearized fluctuations of

the metric and scalar(s)

CRUCIALLY : ~~set frequency $\omega = 0$~~

momentum $\vec{q} = 0$

$\Rightarrow$ scalar fluctuation decouples from all others

EX) 1603.05950 Janik Jankowski, Soltanapanahi

one scalar ϕ in AdS_5

"Improved Holographic QCD" potential

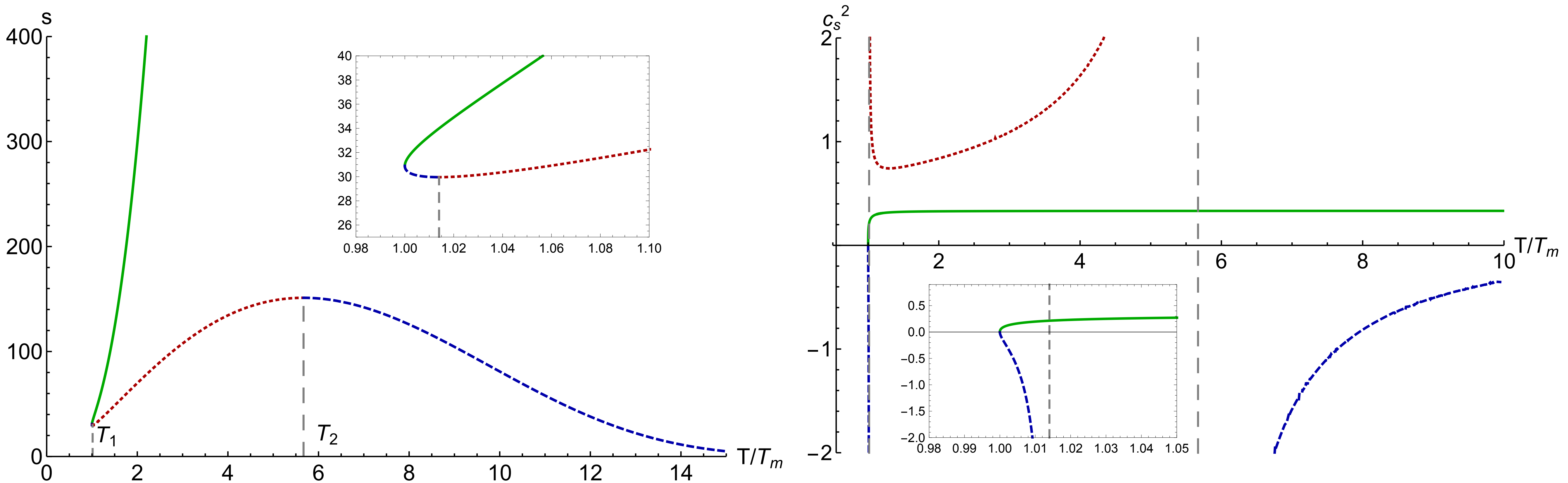
$$V(\phi) = -12(1+\phi^2)^{1/4}\cosh(\gamma\phi) + \frac{25}{4}\phi^2$$

$$\gamma = \sqrt{2/3}$$

field ϕ $\longleftrightarrow$ operator $\mathcal{O}$ with $\Delta \approx 3.58$

black brane solutions only exist for

$T \geq$ minimum temperature T_m



The last point to discuss is the spectrum of modes for temperatures, $T_1 < T < T_2$, in the small black hole branch, which shows anomalously large speed of sound. In fact, $c_s^2 > 1/3$, and for some temperatures it is even superluminal, leading to causality violation. In this range of temperatures the system does not exhibit any instability in thermodynamic quantities. However, there appears to be a novel *dynamical* instability, signaled by the positive imaginary part of the first non-hydrodynamical mode [4]. The difference with respect to the usual spinoidal region is that for $k = 0$ the mode stays positive on the imaginary axis.

[4]The nomenclature is chosen because at high temperatures this modes continuously transforms into first non-hydrodynamic mode.

Ex 1603.07724 Gürsoy, Jansen, van der Schee

one scalar ϕ in AdS_5

potential from Gubser + Nellore 0804.0434 0804.1950

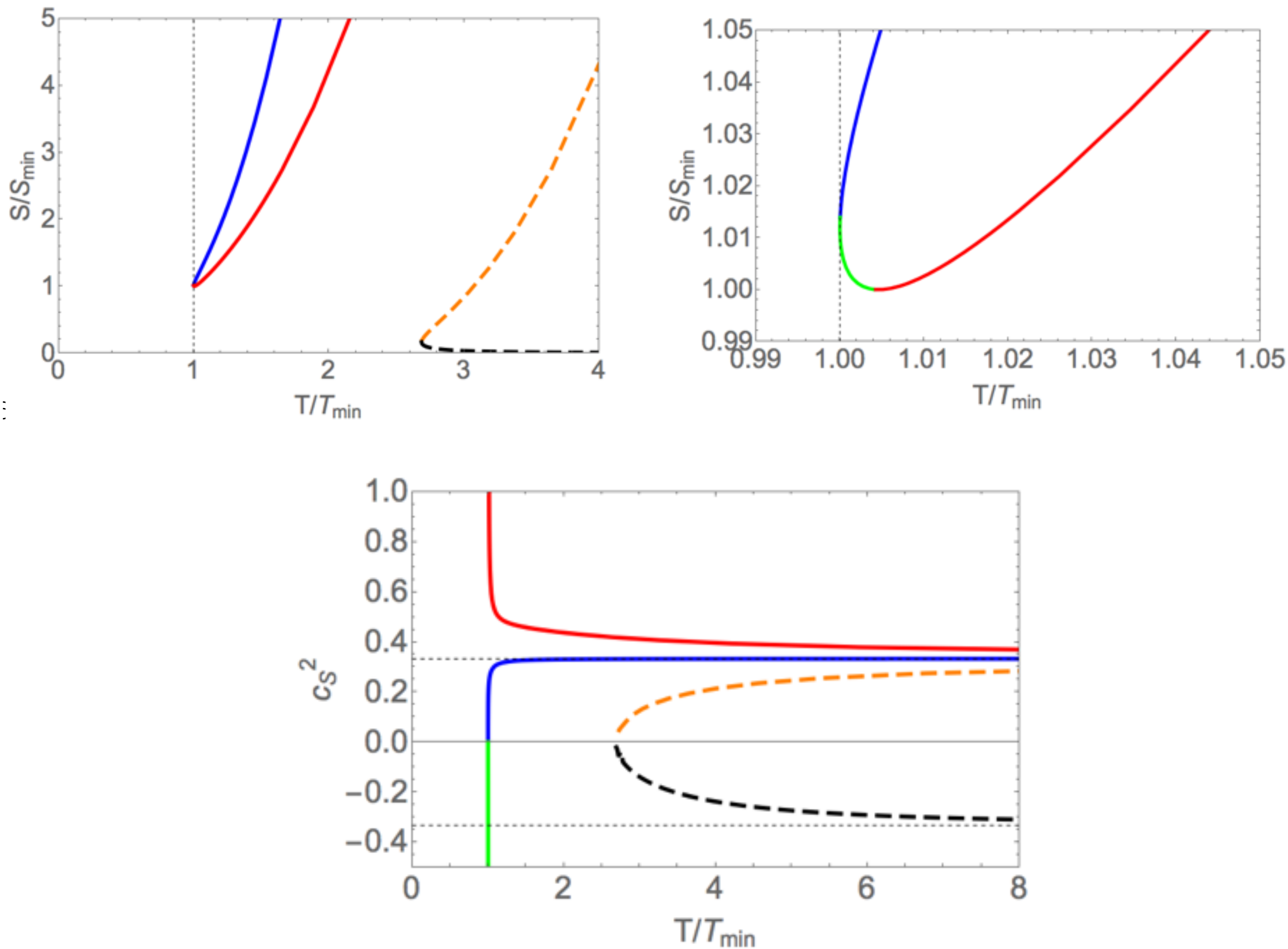
$$V(\phi) = -12 \cosh(\gamma \phi) + \frac{19}{9} \phi^2$$

$$\gamma = \frac{4}{3}\sqrt{2}$$

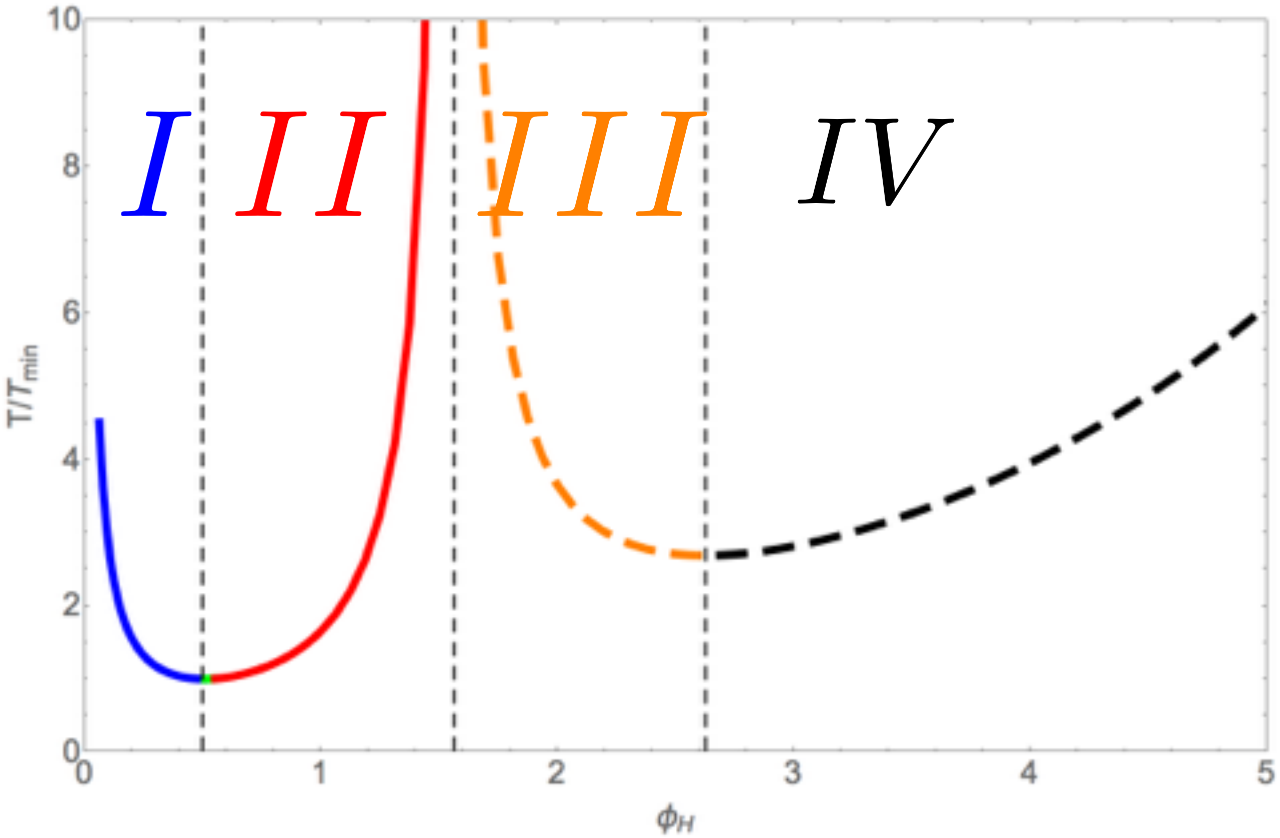
field ϕ $\leftrightarrow$ operator $\mathcal{O}$ with $\Delta = 3$

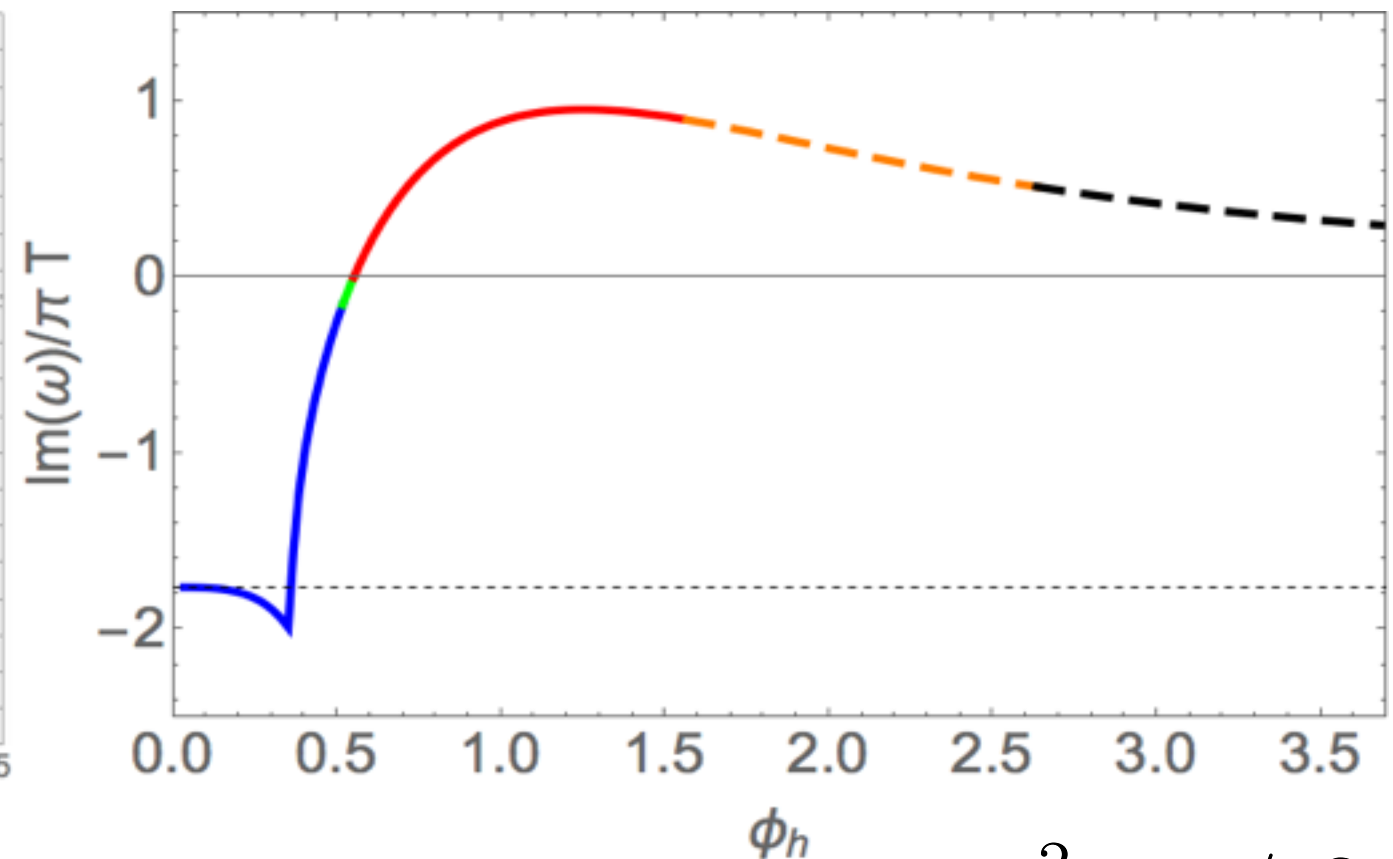
black brane solutions only exist for

$T \geq$ minimum temperature T_m



S/S_min
T/T_min
S/S_min
T/T_min
c_S^2
T/T_min





	Thermodynamic Stability ($C_V > 0$)	Dynamic Stability	$c_s^2 \leq 1/3$?
I	✓	✓	✓
IIa	✗	✓	$c_s^2 < 0$
IIb	✓	✗	✗
III	✓	✗	✓
IV	✗	✗	$c_s^2 < 0$

Compare to other known instabilities

Gregory - Laflamme hep-th/9301052

has $\vec{q} \neq 0$

Gubser - Mitra "correlated Stability Conjecture"

hep-th/0009126, 0011127

"For a black brane solution to be free of dynamical instabilities, it is <u>necessary</u> and <u>sufficient</u> for it to be locally thermodynamically stable"

(not just asymptotically AdS_{d+1})

~~The~~ All of the above are <u>counter-examples</u> of Gubser - Mitra conjecture

Q For asymptotically AdS_{d+1} uncharged black brane solutions

can we prove in full generality

that <u>if</u> $v_s^2 > \frac{1}{d-1}$ <u>then</u> $\exists$ an unstable QNM at $\vec{k} = 0$?

<u>Note</u>: converse is not always true

unstable QNM with $k = 0$ $\not\Rightarrow$ $v_s^2 > \frac{1}{d-1}$

as known in many examples