

The effect of f - modes on the gravitational waves during a binary inspiral

Tanja Hinderer

(AEI Potsdam)

PRL 116, 181101 (2016), arXiv:1602.00599

and arXiv:1608.01907



A.Taracchini

A.Buonanno

J.Steinhoff

F.Foucart

M.Duez

L.E.Kidder

H.P.Pfeiffer

M.A.Scheel

B.Szilagyi

C.W.Carpenter

K.Hotokezaka

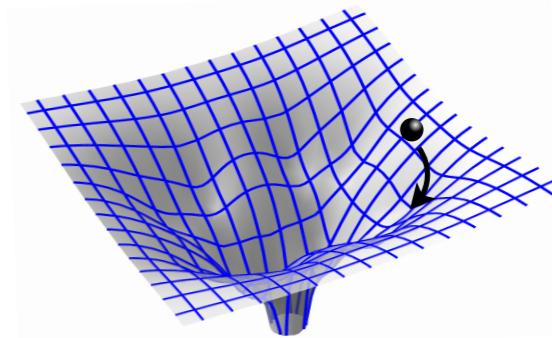
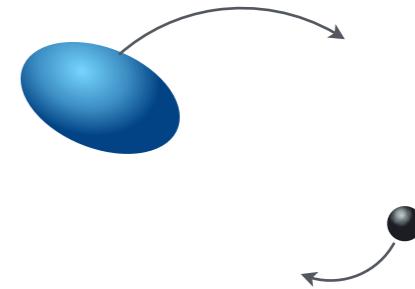
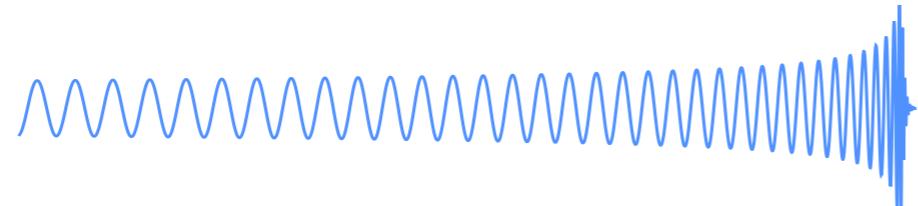
K.Kyutoku

M.Shibata

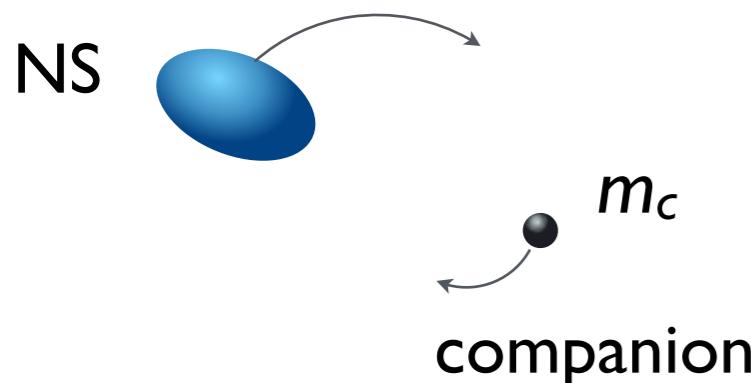


Overview

- Dynamical f -mode tides in General Relativity
- Inclusion in the Effective-One-Body (EOB) model
- Tests against results from Numerical Relativity simulations
- Outlook

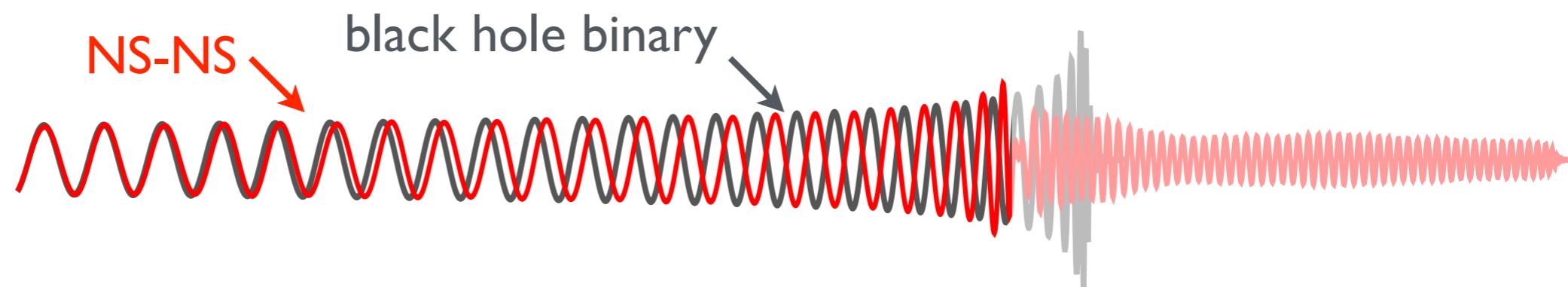


Newtonian tidal excitation of f-modes



- companion's tidal field induces a fluid displacement ξ
- ξ is often decomposed into spherical harmonics
(equivalent alternative: STF tensors)
- oscillator-like Lagrangian for the mode amplitudes

- Effect on GWs:



tidal imprint in the phase evolution

for models: convenient to work directly with the NS's induced multipole moments Q_{ij}, Q_{ijk}, \dots

(differ from mode amplitudes by a normalization factor)

Newtonian tidal excitation of f-modes

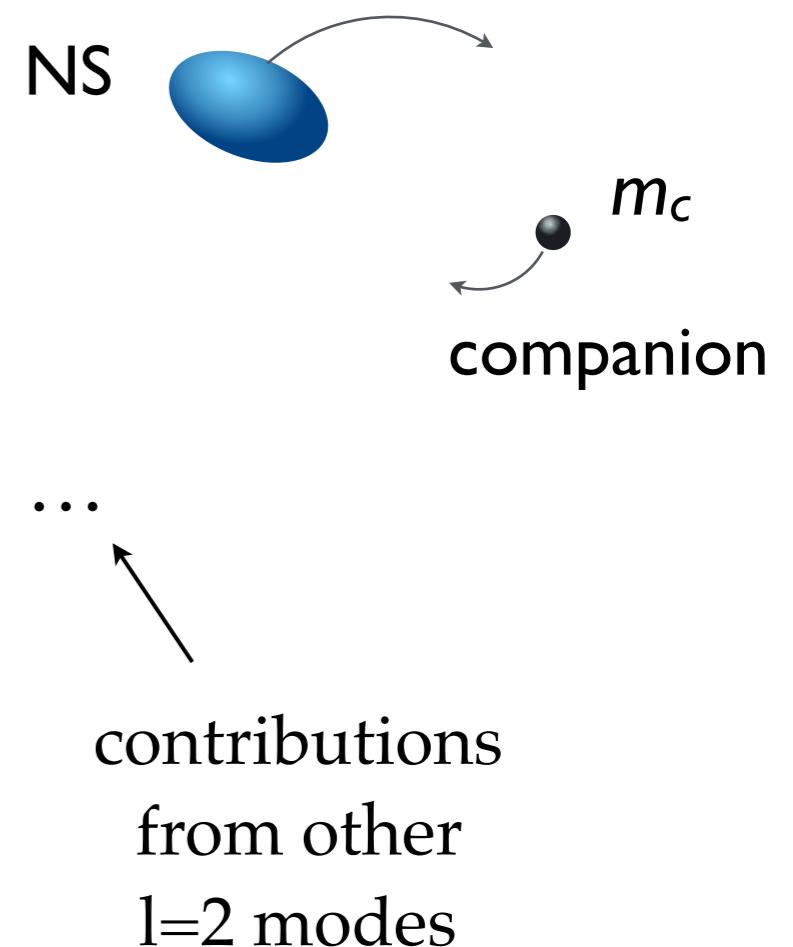
- NS's induced multipole moments Q_{ij}, Q_{ijk}, \dots

- tidal Lagrangian (quadrupole only)

$$L = \frac{1}{4\lambda\omega_f^2} \left[\frac{dQ_{ij}}{dt} \frac{dQ^{ij}}{dt} - \omega_f^2 Q_{ij} Q^{ij} \right] - \frac{1}{2} E_{ij} Q^{ij} + \dots$$

↑ ↑ ↑
 tidal deformability f-mode frequency tidal field
 Love number NS radius

$$\lambda_\ell = \frac{2(\ell-2)}{(2\ell-1)!!} k_\ell R^{2\ell+1}$$



- for two NSs: simply add the same terms with the parameters of the other NS

Adiabatic limit

- f -mode frequency scales as $\omega_f \sim \sqrt{m_{\text{NS}}/R^3}$

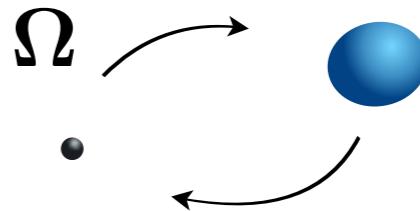
- **tidal forcing** frequency: $2\Omega \sim 2\sqrt{M/r^3}$

- for $2\Omega \ll \omega_f$: adiabatic tides (AT)

$$Q_{ij}^{\text{AT}} = -\lambda E_{ij}$$

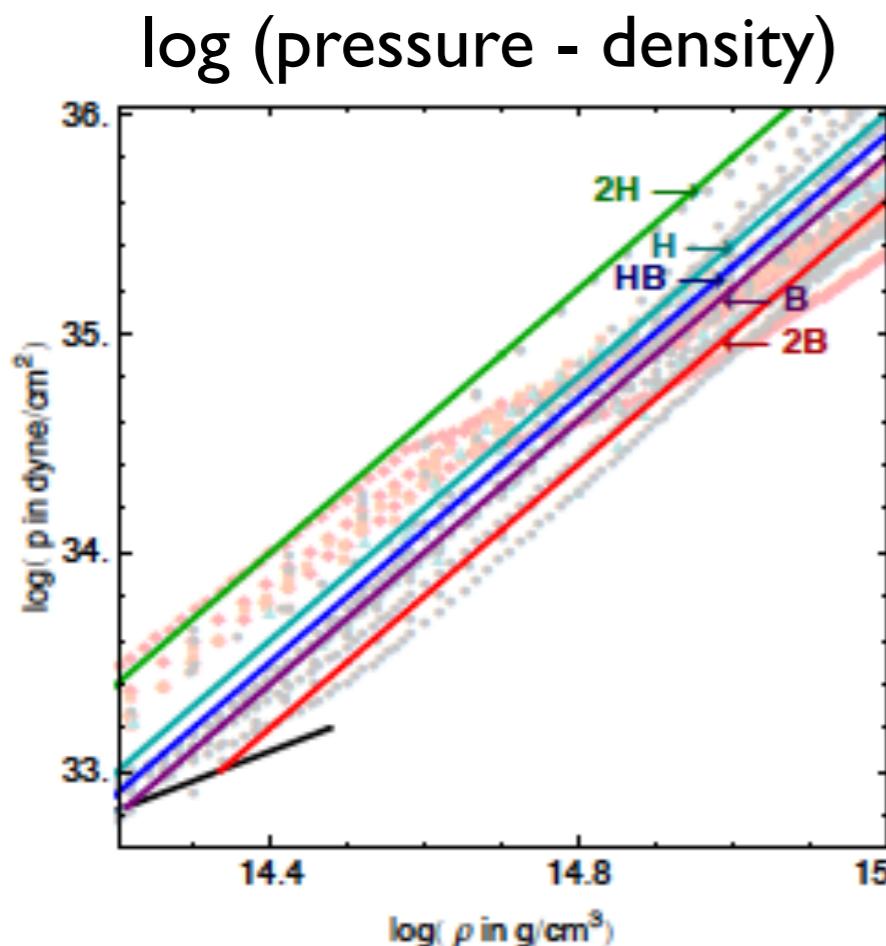
adiabatic tidal Lagrangian: $L_{\text{AT}} = \frac{\lambda}{4} E_{ij} E^{ij}$

involves only orbital variables & tidal deformability coefficient λ

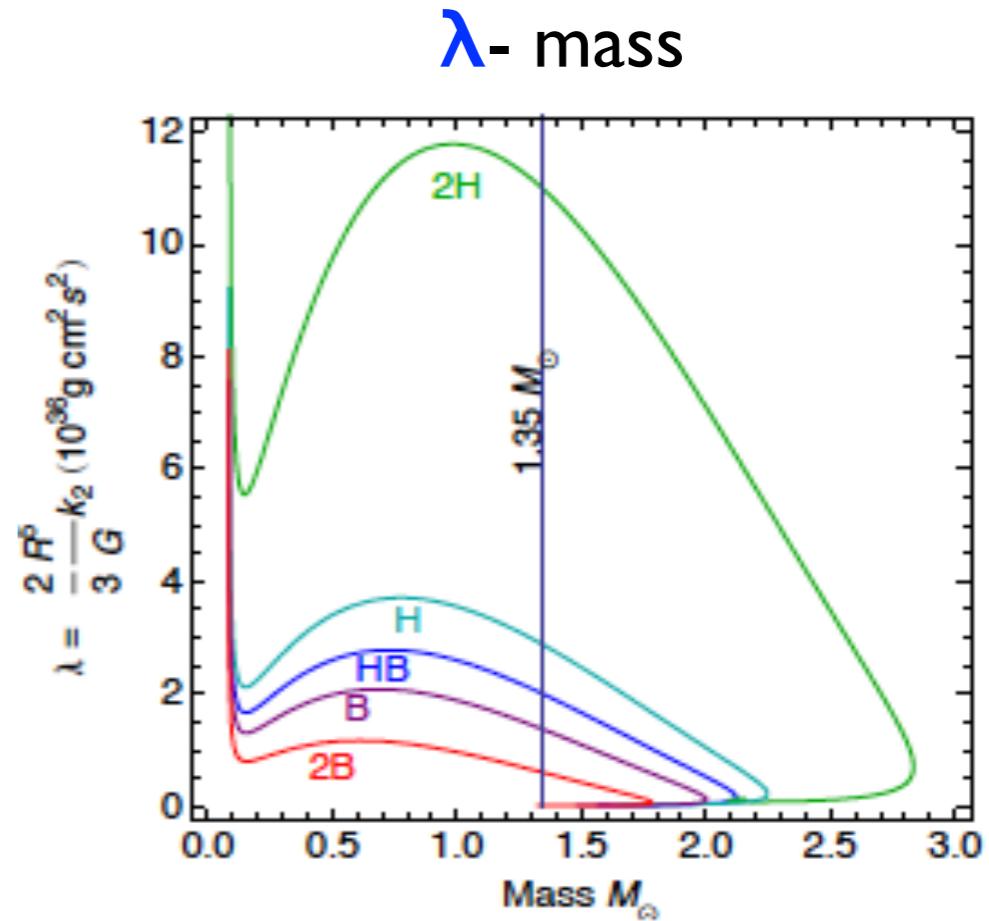


Tidal deformability

- characterizes dominant EoS influence on GWs during inspiral

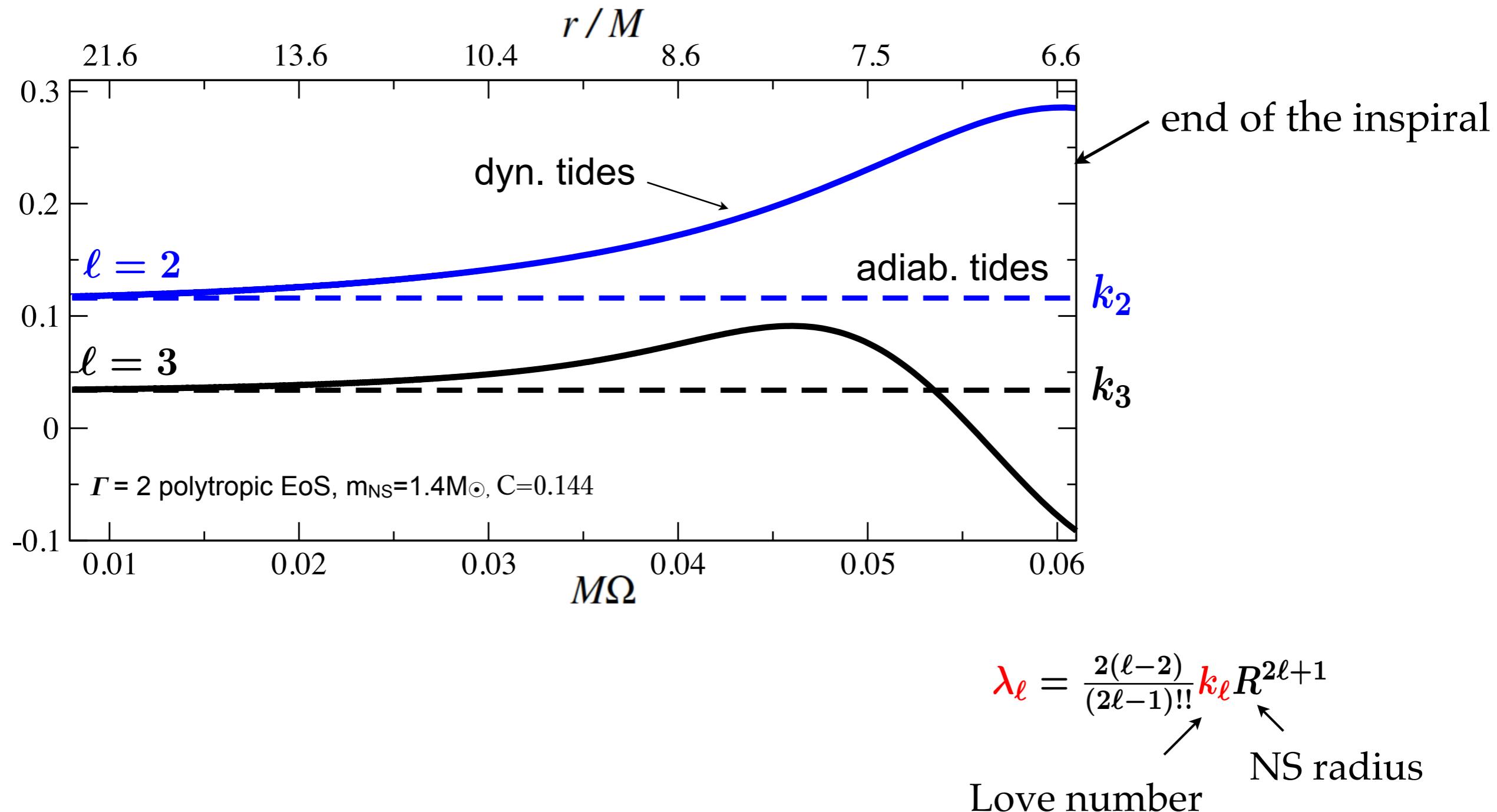


Einstein's Eqs:
linear perturbations
to equilibrium sol.
[One 2nd order ODE]

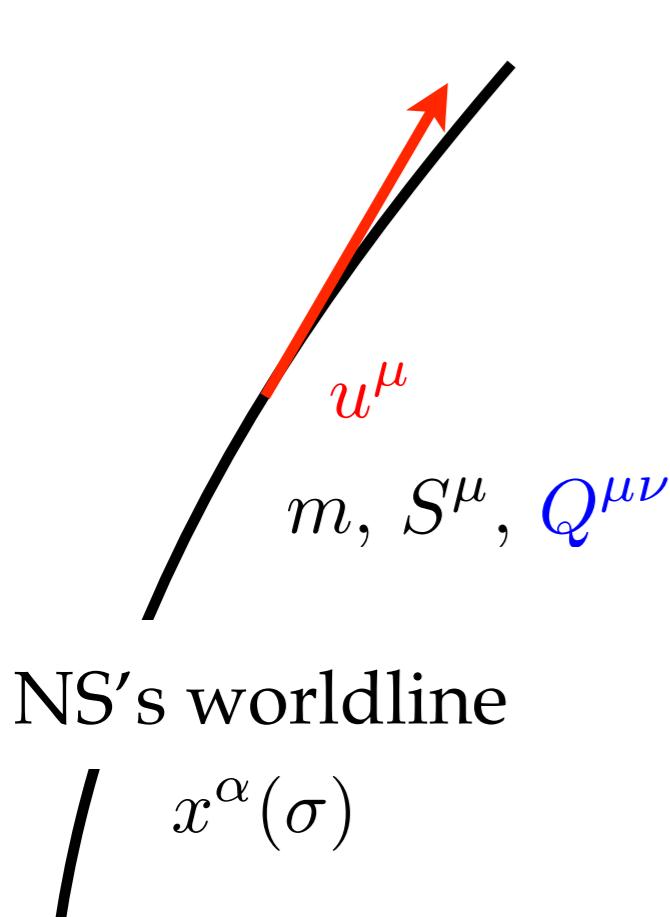


credit: B. Lackey

NS's tidal response during the inspiral



Tidal excitation of f-modes in GR



recall Newtonian tidal Lagrangian:

$$L = \frac{1}{4\lambda\omega_f^2} \left[\frac{dQ_{ij}}{dt} \frac{dQ^{ij}}{dt} - \omega_f^2 Q_{ij} Q^{ij} \right] - \frac{1}{2} E_{ij} Q^{ij}$$

other $I=2$ modes
& other stuff e.g. due to
incompleteness of modes

relativistic version:

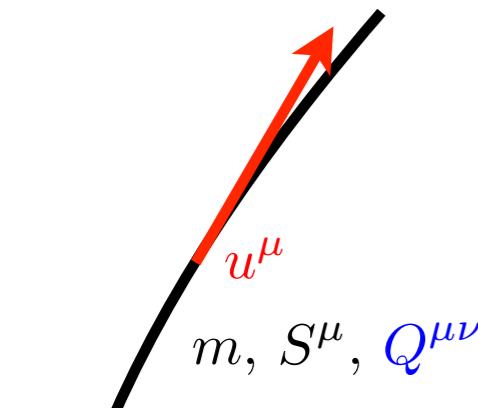
$$L = \frac{z}{4\lambda z^2 \omega_f^2} \left[\frac{DQ_{\mu\nu}}{d\sigma} \frac{DQ^{\mu\nu}}{d\sigma} - z^2 \omega_f^2 Q_{\mu\nu} Q^{\mu\nu} \right] - \frac{z}{2} E_{\mu\nu} Q^{\mu\nu} + \dots$$

redshift

$\frac{D}{d\sigma}$ = covariant derivative along worldline

$$E_{\mu\nu} = C_{\mu\alpha\nu\beta} \frac{u^\alpha u^\beta}{z^2}, \quad C_{\mu\alpha\nu\beta} : \text{Weyl curvature tensor (companion)}$$

Tidal excitation of f-modes in GR



NS's worldline

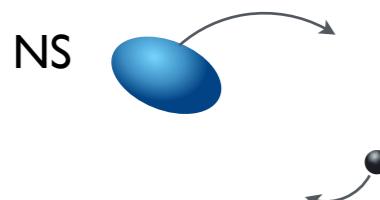
$$x^\alpha(\sigma)$$

relativistic tidal Lagrangian:

$$L = \frac{z}{4\lambda z^2 \omega_f^2} \left[\frac{DQ_{\mu\nu}}{d\sigma} \frac{DQ^{\mu\nu}}{d\sigma} - z^2 \omega_f^2 Q_{\mu\nu} Q^{\mu\nu} \right] - \frac{z}{2} E_{\mu\nu} Q^{\mu\nu}$$

redshift

- 3+1 decomposition to make explicit the physical degrees of freedom:
frame-dragging interaction
between orbital and Q_{ij} 's angular momentum characterized by the
“tidal spin” tensor:



$$S_Q^{ij} = \frac{2}{\lambda \omega_f^2} Q^{k[i} \dot{Q}^{j]k}$$

Explicit expressions in the weak field limit

post-Newtonian (PN) results for tidal effects **without** any new PN **calculations**:

(i) tidal interaction terms vs. **known PN results for spin-induced quadrupoles**

$$-\frac{1}{2} E_{\mu\nu} Q^{\mu\nu}$$

$$\frac{C_{ES^2}}{2m} E_{\mu\nu} S^\mu S^\nu$$

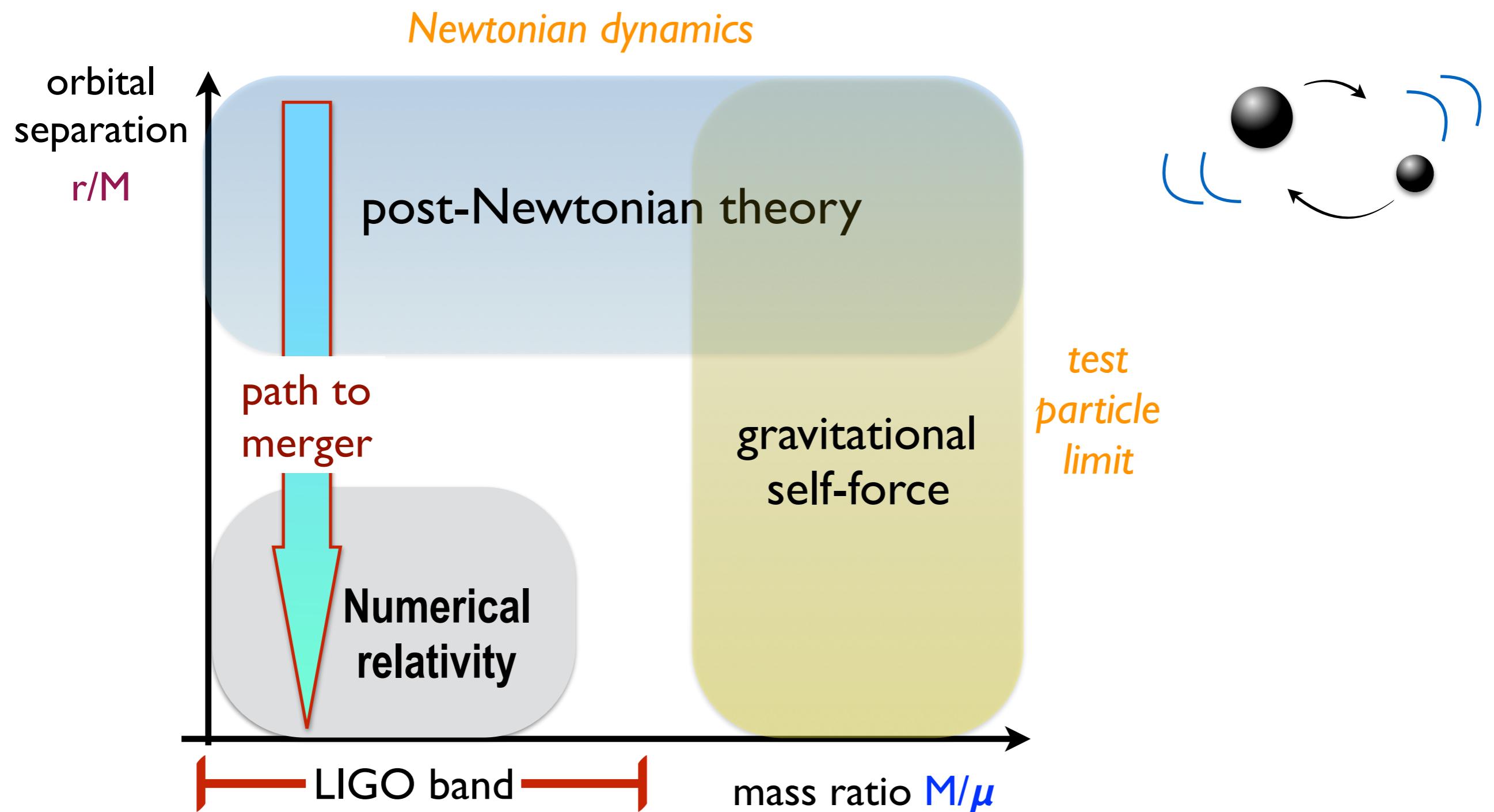
↪ simply **substitute** $C_{ES^2} S^j S^i \rightarrow -m Q^{ij}$

(ii) **frame-dragging** interaction between orbital angular momentum and **tidal spin**

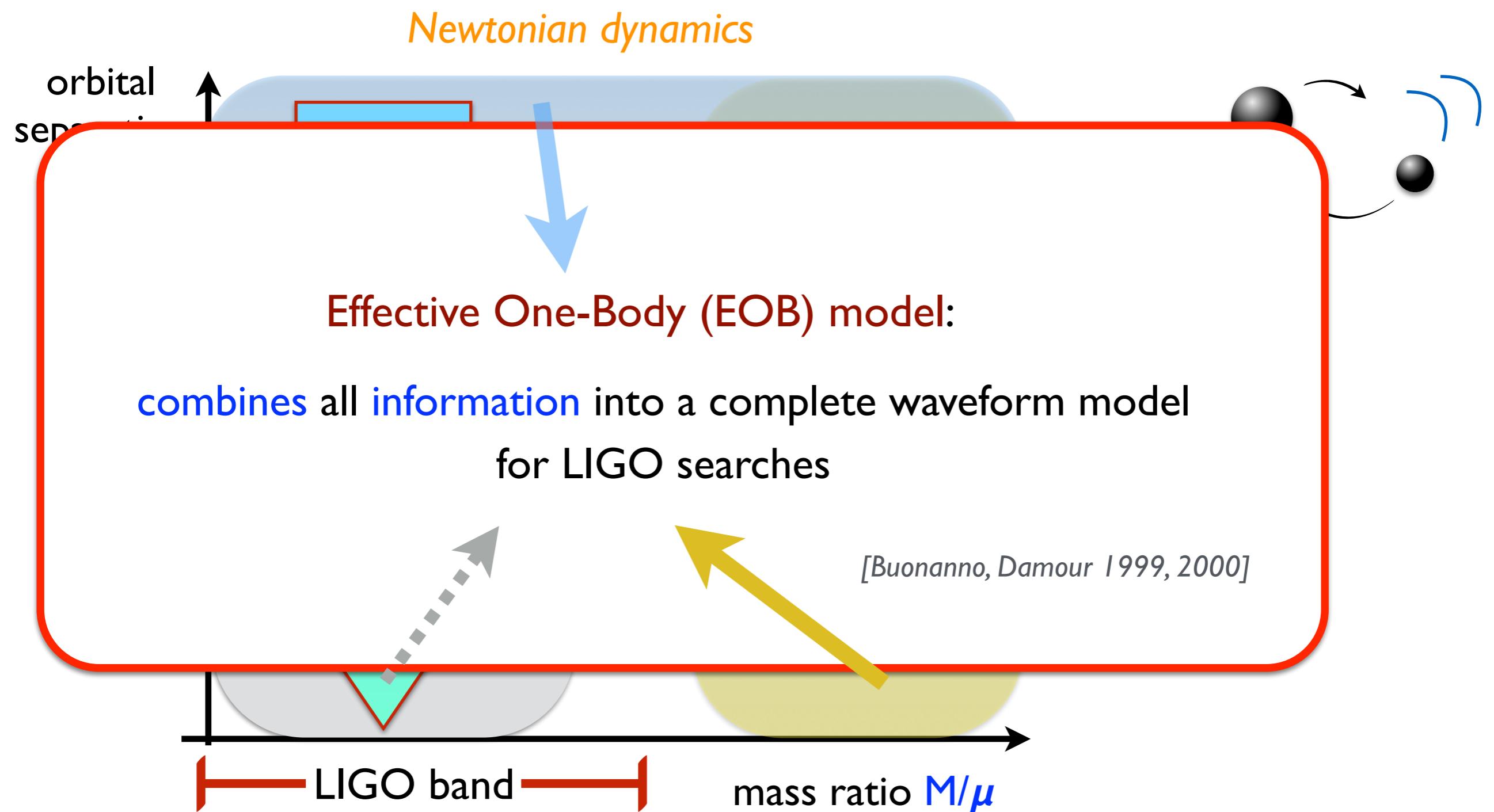
↪ simply **replace** spin by S_Q in known PN frame-dragging potentials

the resulting 1PN accurate tidal Lagrangian agrees with Vines & Flanagan 2013

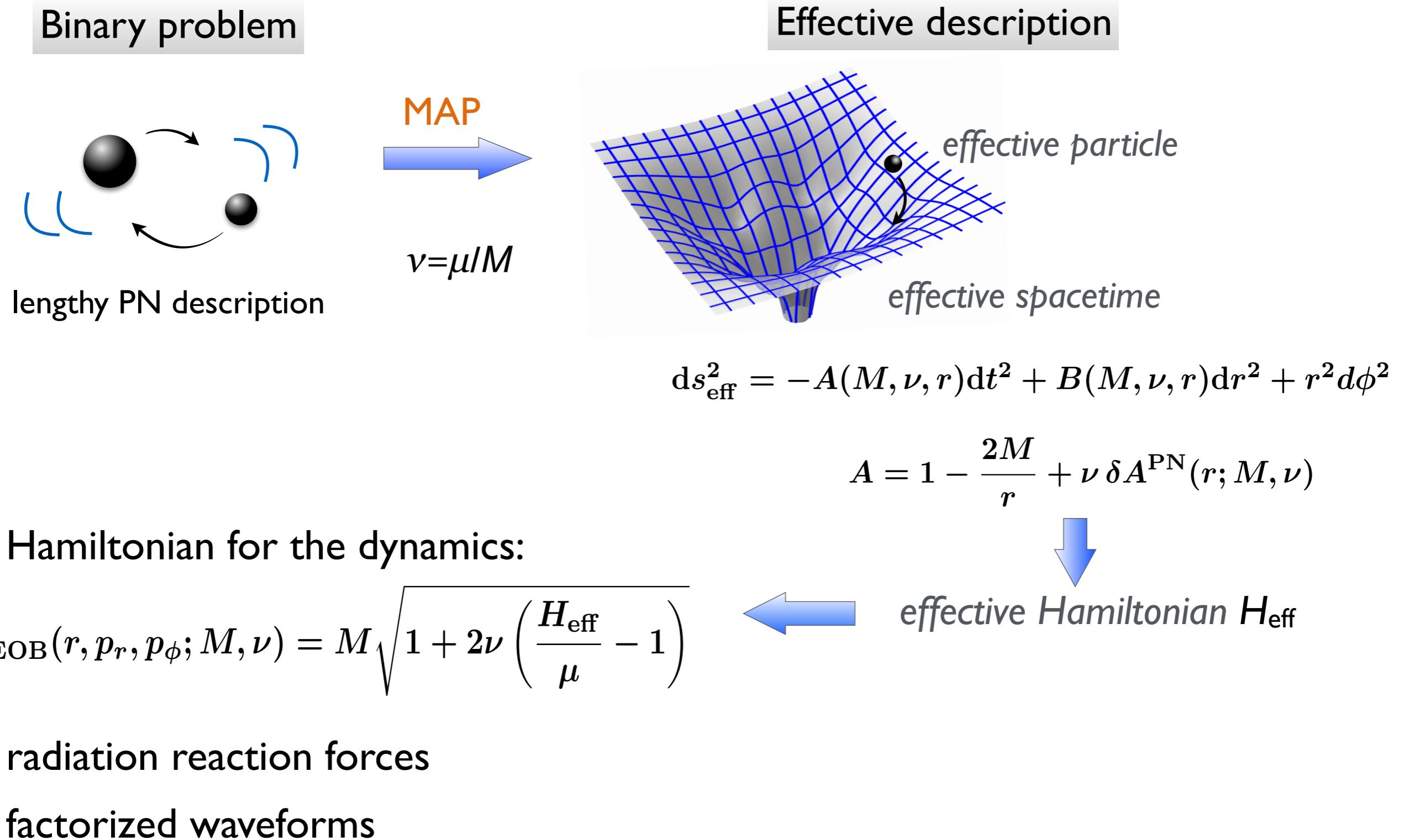
Approaches to computing waveforms



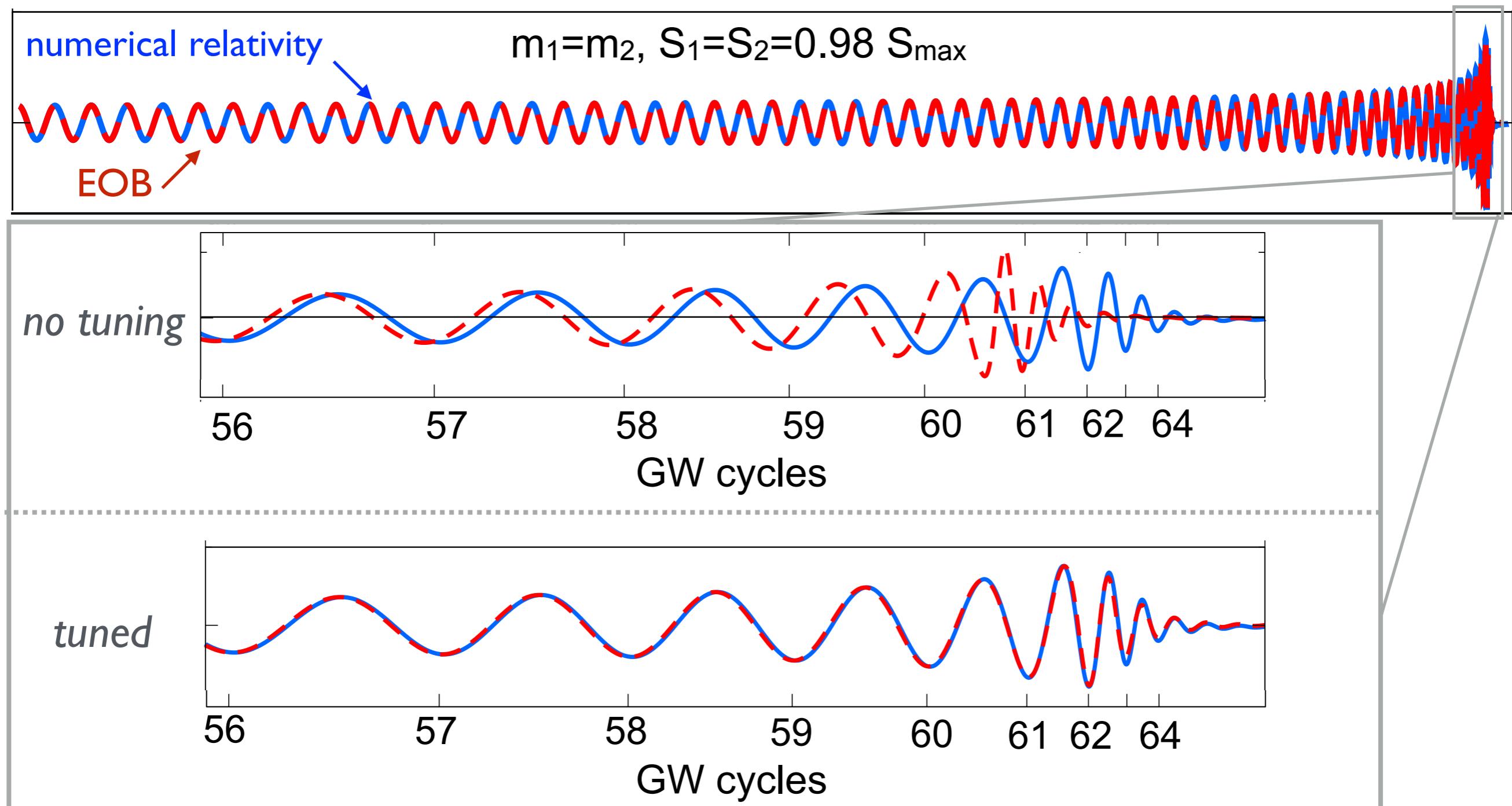
Approaches to computing waveforms



Effective-One-Body (EOB) approach



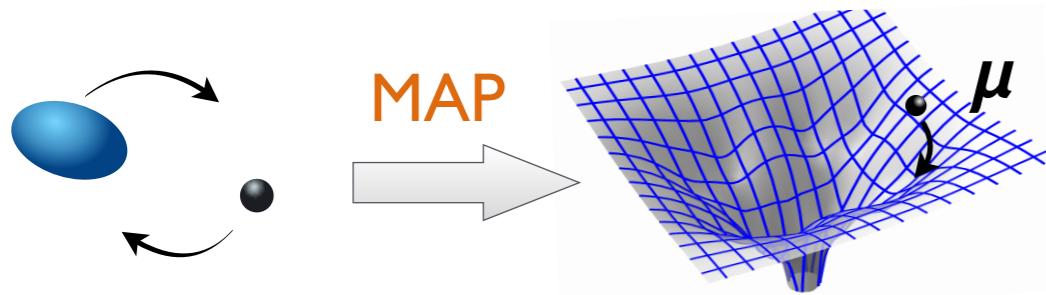
Performance of EOB waveforms



[courtesy A. Taracchini]

- recent extension to precessing spins [Babak, Taracchini+ 2016]

Adiabatic tides in the EOB Hamiltonian



$$ds_{\text{eff}}^2 = -A dt^2 + B dr^2 + r^2 d\phi^2$$

$$A = A^{\text{pp}}(M, \nu, r) - \lambda_\ell A^{\text{AT}}(M, \nu, r)$$

[Damour, Nagar, Bini, Faye, Bernuzzi+2009-2014]

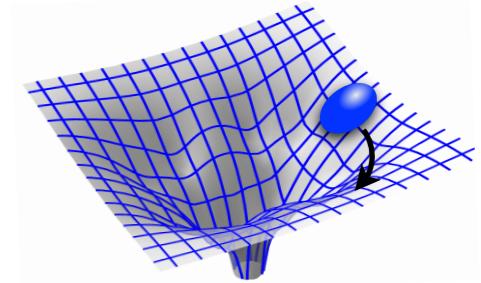
- different possibilities for A^{AT} :
- post-Newtonian: $A_{\text{PN}}^{\text{AT}} = \frac{3q}{r^6} \left[1 + \frac{p_1(\nu)}{r} + \frac{p_2(\nu)}{r^2} + O\left(\frac{1}{r^3}\right) \right]$ $q = m_c/m_{NS}$
- self-force (GSF): $A_{\text{GSF}}^{\text{AT}} = \frac{3q}{r^6} \left[1 + \frac{3}{r^2 \left(1 - \frac{r_{\text{LR}}}{r}\right)} + \frac{g_1(r)}{q \left(1 - \frac{r_{\text{LR}}}{r}\right)^{7/2}} + O\left(\frac{1}{q^2}\right) \right]$ $r_{\text{LR}} = \text{light ring}$
- tuned GSF: $A_{\text{tGSF}}^{\text{AT}} = \frac{3q}{r^6} \left[1 + \frac{3}{r^2 \left(1 - \frac{r_{\text{LR}}}{r}\right)} + \frac{g_1(r)}{q \left(1 - \frac{r_{\text{LR}}}{r}\right)^{7/2}} + \frac{p_2''(\nu)/2}{q^2 \left(1 - \frac{r_{\text{LR}}}{r}\right)^p} \right]$

↑
 adjustable:
 $4 \leq p \leq 6$

EOB Hamiltonian with dynamic tides

- full evolutions: $H_{\text{EOB}}(r, p_r, p_\phi, Q_{lm}, P_{lm}; M, \nu, \lambda_l, \omega_f)$

↑
conjugate momentum to Q_{lm}

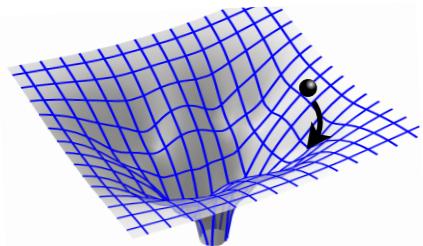


tidal modifications to the effective spacetime, the mass of the effective particle, etc.
derived from PN results for the Lagrangian

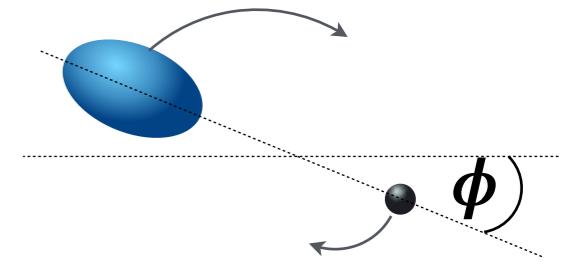
- no additional approximations
- many extra variables (24 if up to hexadecapole tides are included)
- no problems in evolving the dynamics
- but inefficient for data analysis implementations, no direct control of the effects

EOB with approximate dynamic tides

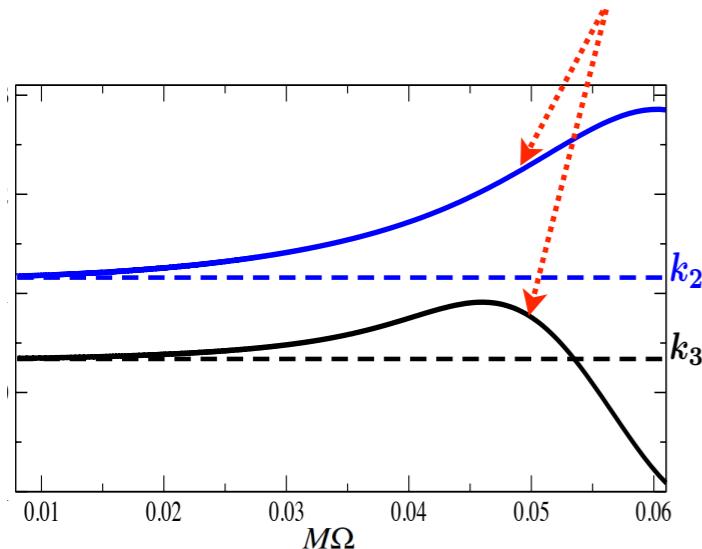
- effective description from an approximate solution for Q_{lm} :



$$A = A^{\text{pp}}(M, \nu, r) - \lambda_\ell^{\text{eff}} A_{\text{PN}}^{\text{AT}}(M, \nu, r)$$



$$\frac{\lambda_\ell^{\text{eff}}}{\lambda_\ell} \sim \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} \& \frac{\omega_f^2}{(\phi - \phi_f)^2} \& \cos [(\phi - \phi_f)^2] \text{FresnelS}(\phi - \phi_f)$$



before
resonance

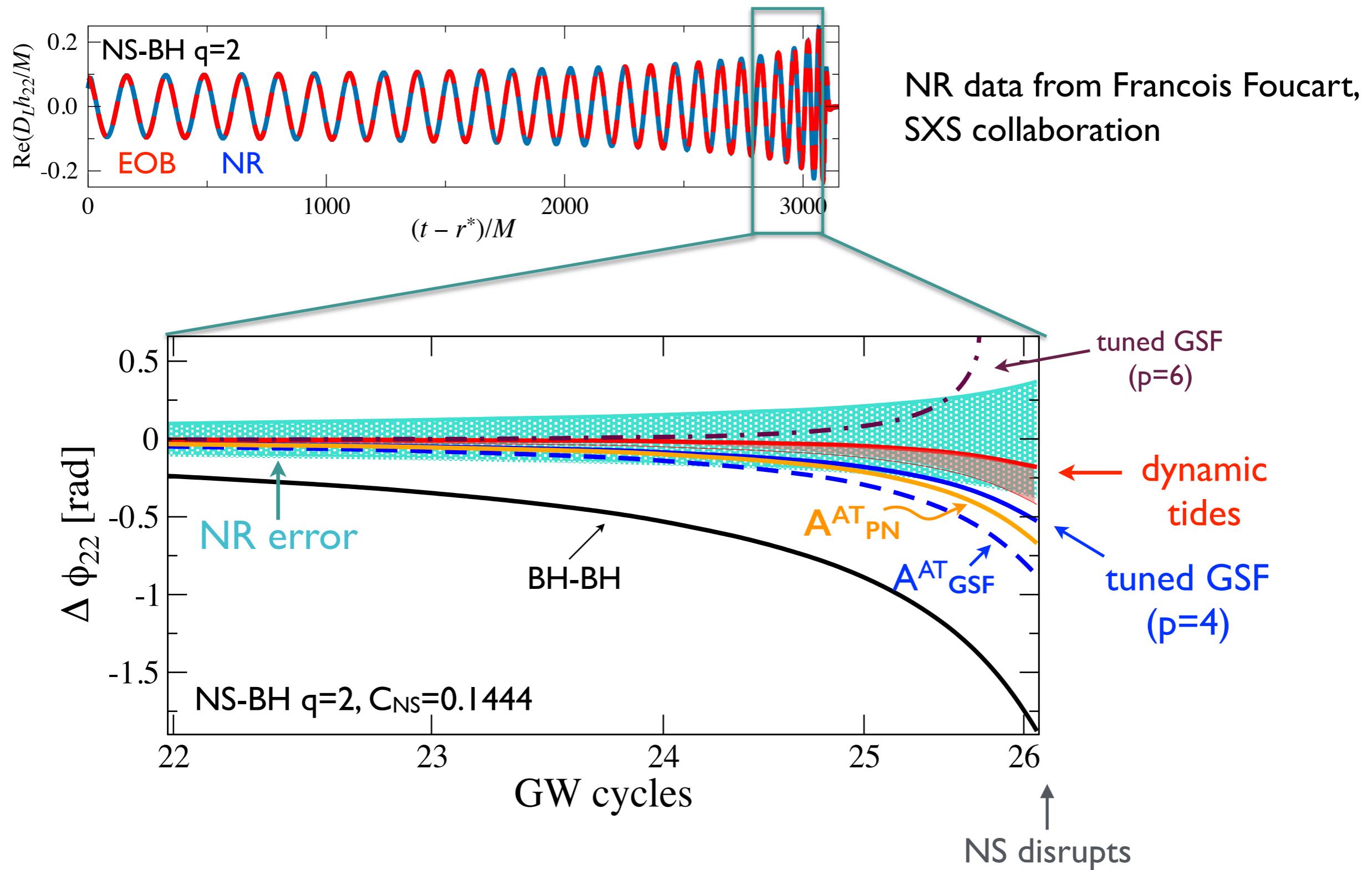
— common
term

near resonance
where $\phi \sim \phi_f$

all fns. of $\{M, \nu, \omega_f, r\}$ using a Newtonian inspiral

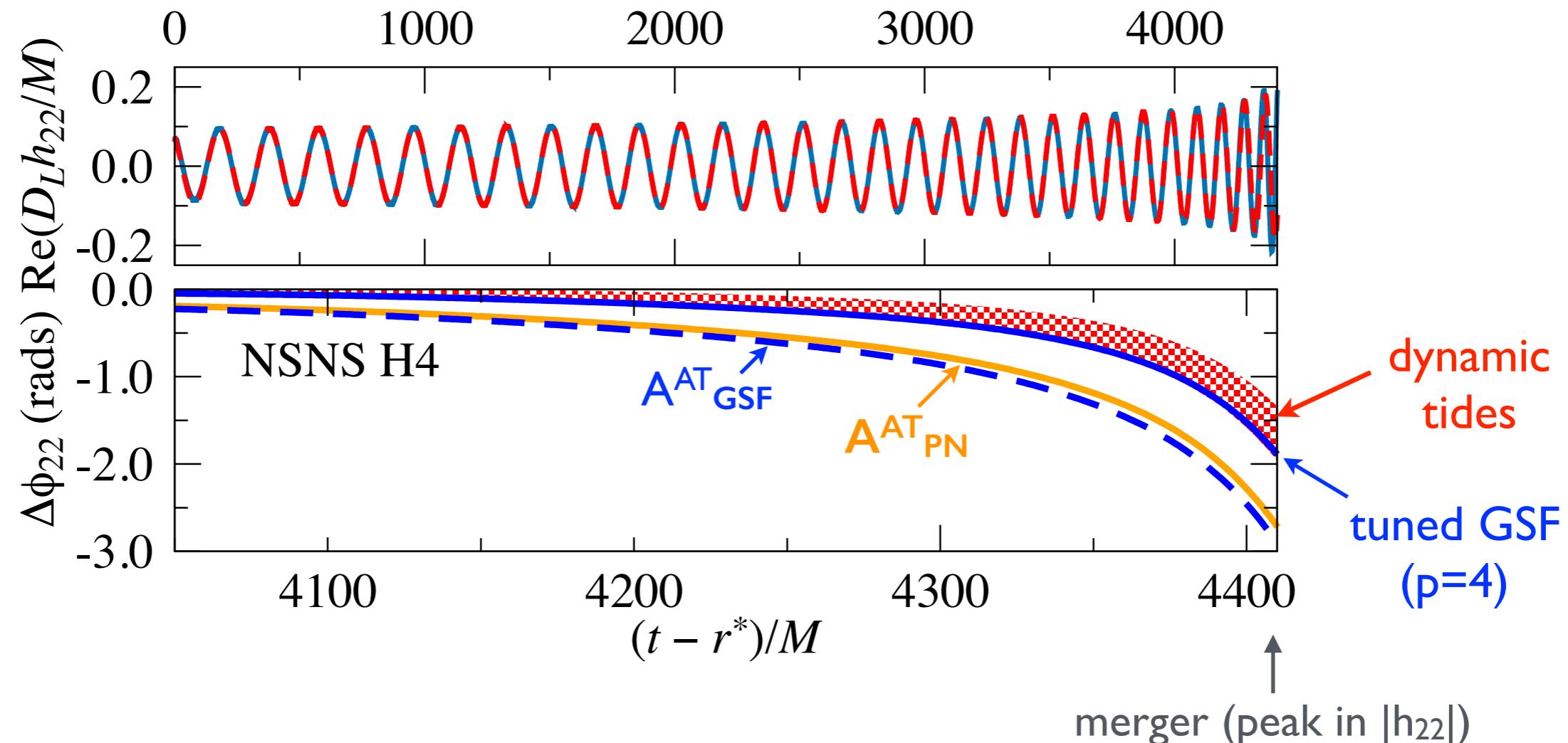
- requires evolving only the orbital variables
- tidal effects encoded in λ^{eff}

Performance of the tidal model: NS-BH



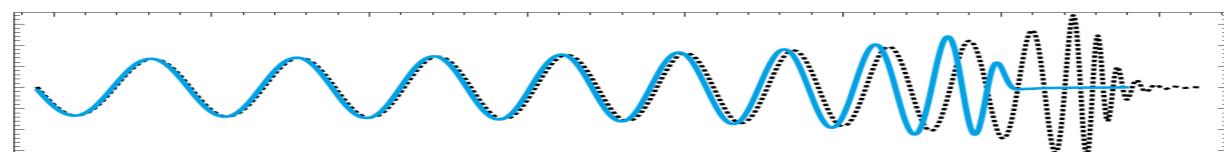
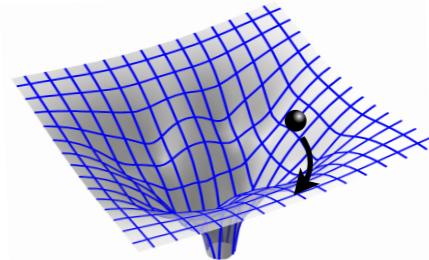
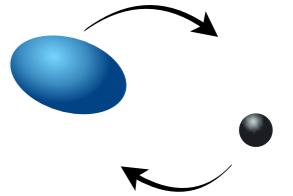
Performance of the tidal model: NS-NS

NR data from M. Shibata and K. Hotokezaka



Conclusions

- Main imprint of NS microphysics in the GWs from NS **inspirals**: tidal effects
- **Dynamic f-mode tides** can be significant, now included in **EOB**
- Further work needed to
 - **test/improve** the models and measurement potential
 - **quantify and reduce systematics**
 - **include more realistic physics**
 - optimize data analysis strategies (e.g. parameterization)
- **Accurate NR simulations** are **crucial** to inform model developments



Thank you