

## Holographic quark-hadron continuity

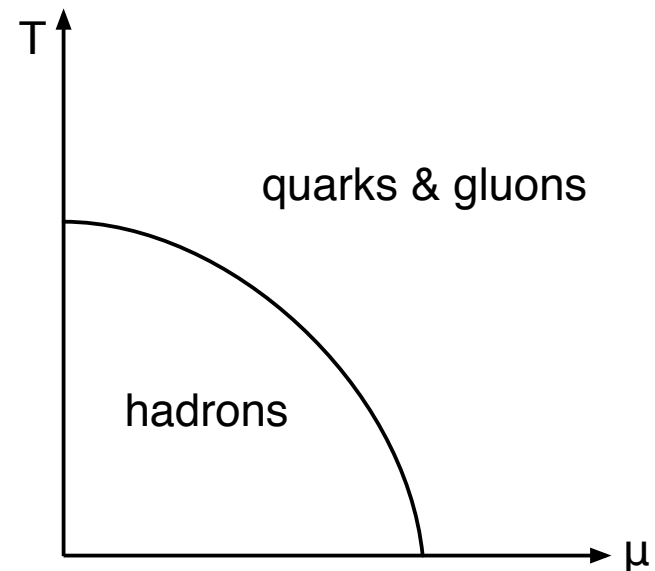
- What do we know about the quark-hadron transition?
- Why is it interesting for neutron stars?
- Holographic model for quark and hadronic phases  
K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, in preparation  
based on F. Preis and A. Schmitt, JHEP 1607, 001 (2016)  
S. w. Li, A. Schmitt and Q. Wang, PRD 92, 026006 (2015)

# Quark-hadron transition

- (1) QCD is asymptotically free  
→ free quarks and gluons at large energies  
(here: large  $T$  and/or  $\mu$ )
- (2) we live in a world of hadrons, not quarks and gluons

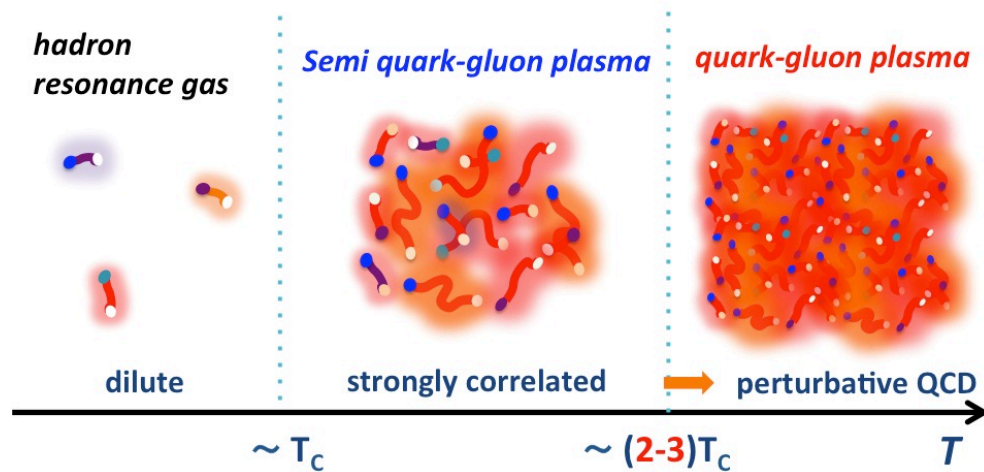
⇒ there needs to be some kind of quark-hadron transition in the QCD phase diagram

N. Cabibbo, G. Parisi, PLB 59, 67 (1975)

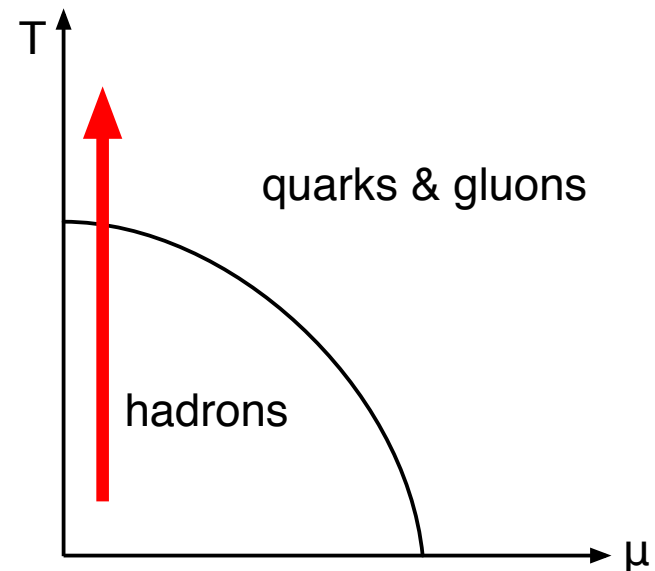


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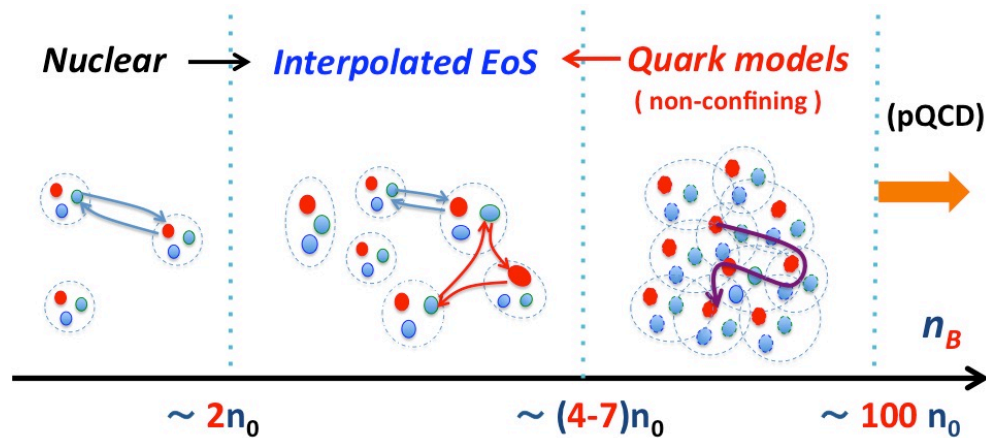


G. Baym *et al.*, Rept.Prog.Phys. 81 056902 (2018)

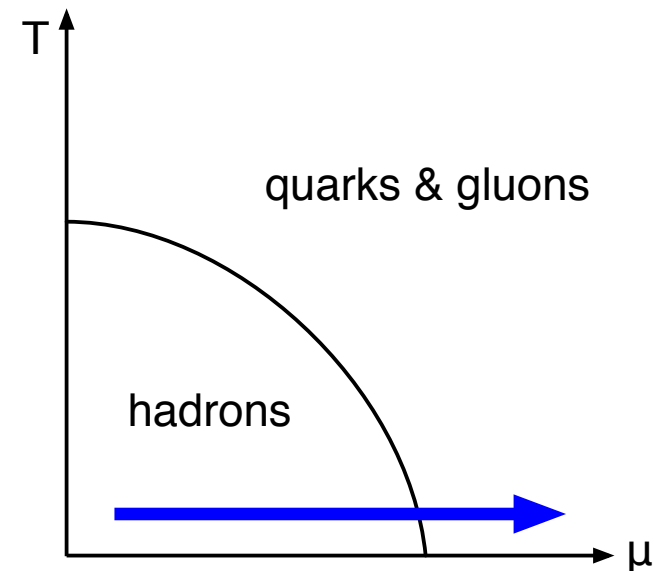


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## Is there a strict phase transition?

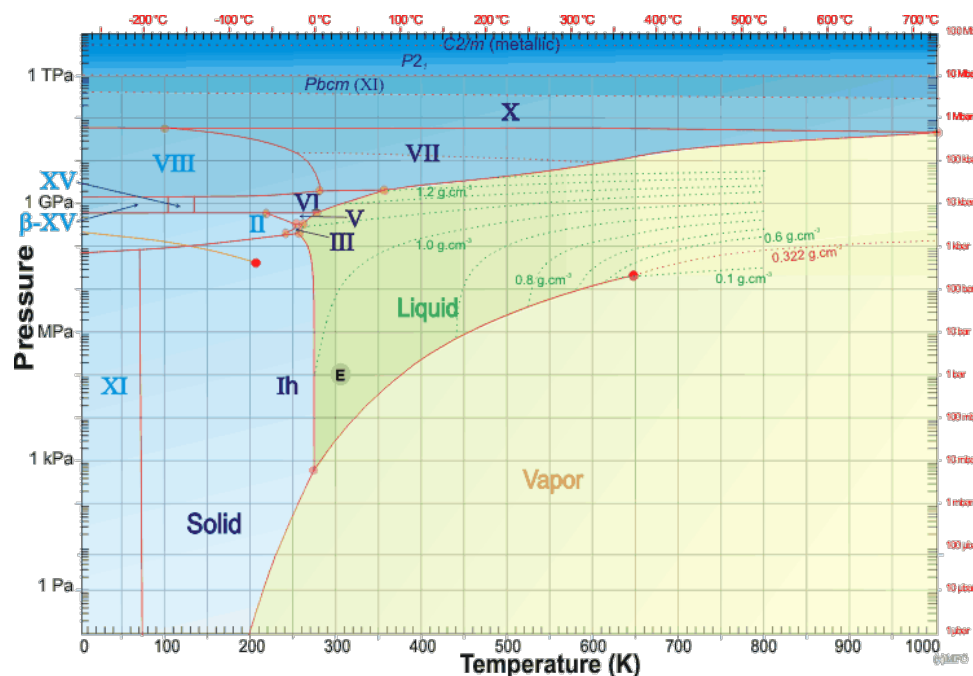
order parameter	Polyakov loop (confinement)	chiral condensate
spontaneously breaks	$\mathbb{Z}_{N_c}$	$SU(N_f) \times SU(N_f)$
symmetry exact for	pure Yang-Mills ( $m_q = \infty$ )	chiral limit ( $m_q = 0$ )

→ in real-world QCD no symmetry is broken

→ transition is allowed to be smooth (can still be first order)

Compare with water:

- **liquid-gas**: no symmetry broken, has critical point
- **liquid-solid**: translational symmetry broken, no critical point

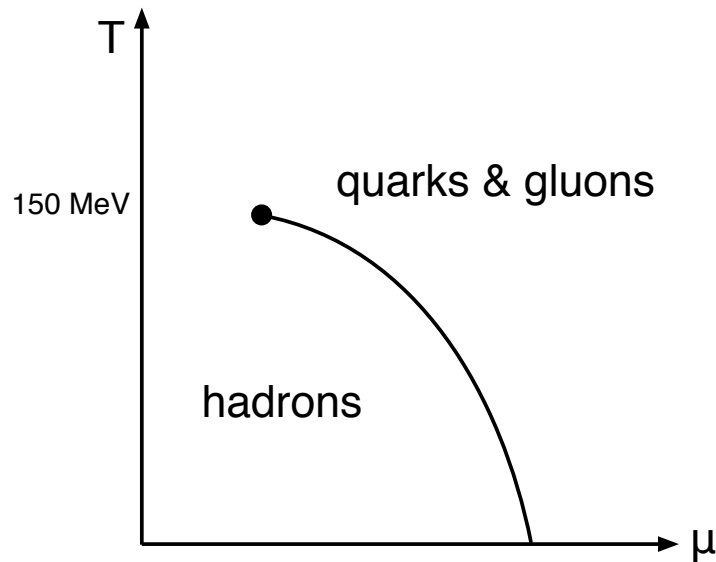
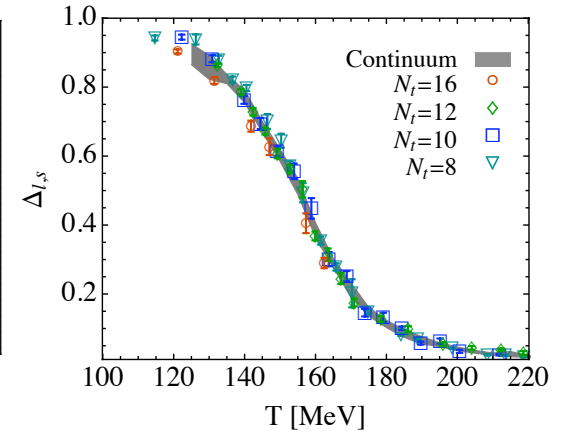
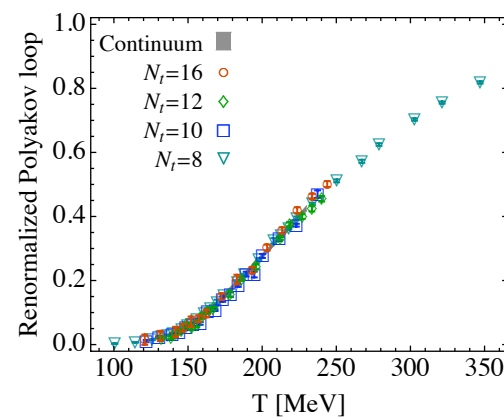


→ there is no qualitative difference between hadronic and quark matter! (ignoring Cooper pairing for now)

# Can't we simply calculate the phase diagram?

We can, with lattice gauge theory, at  $\mu = 0$

S. Borsanyi *et al.* JHEP 1009, 073 (2010)



- nonzero  $\mu$ : lattice methods don't work ("sign problem")

recent progress (reviews):

G. Aarts, J.Phys.Conf.Ser. 706, 022004 (2016)

O. Philipsen, EPJ Web Conf. 137, 03016 (2017)

Lefschetz thimbles & resurgence:

M. Cristoforetti *et al.*, PRD 88, 051501 (2013)

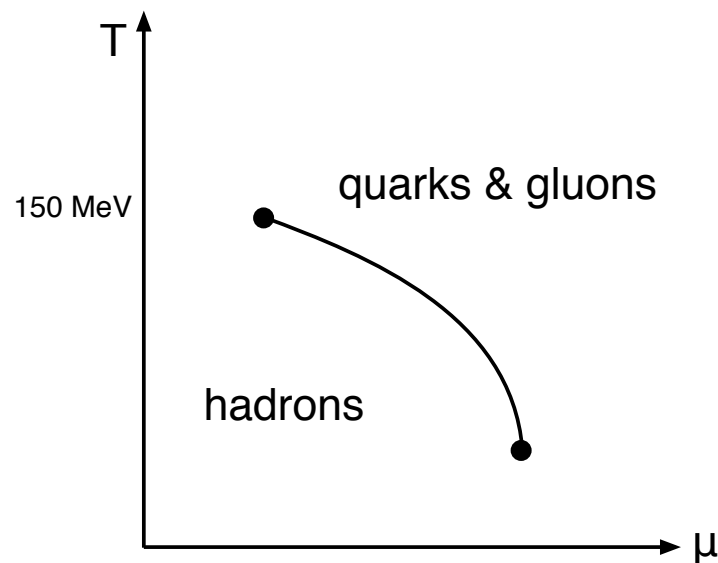
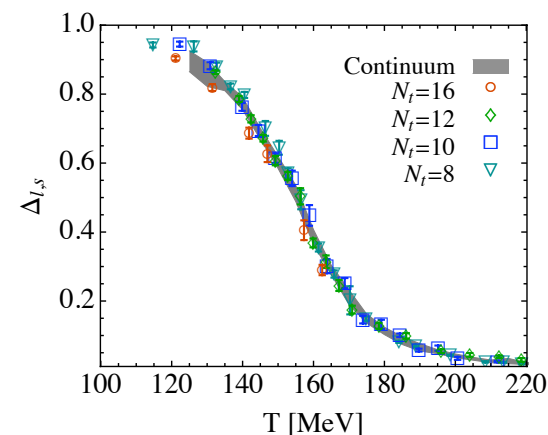
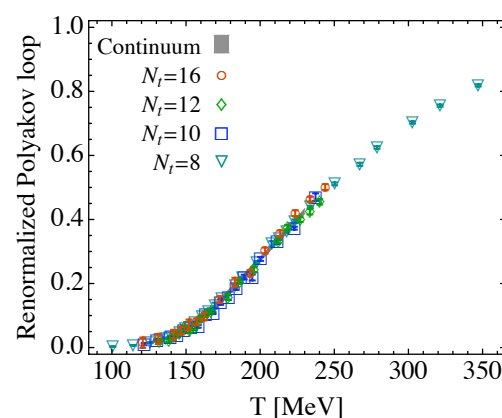
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- quark-hadron continuity at  $T = 0$ ?

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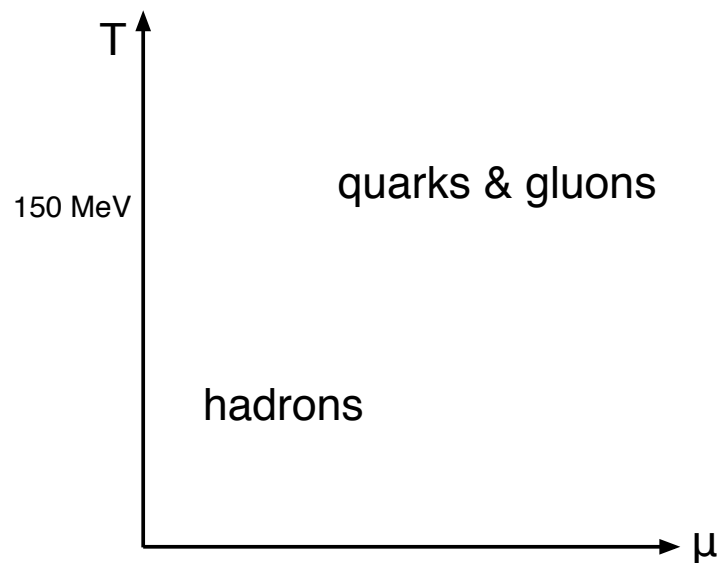
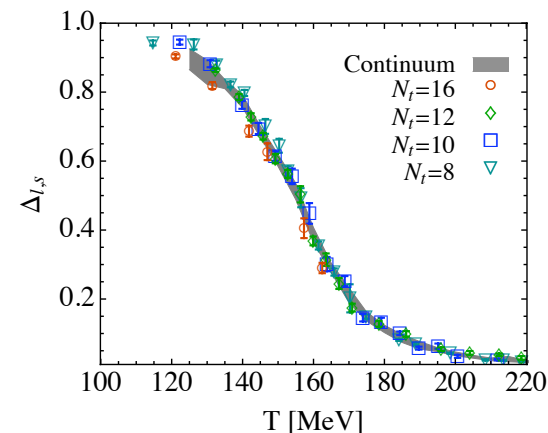
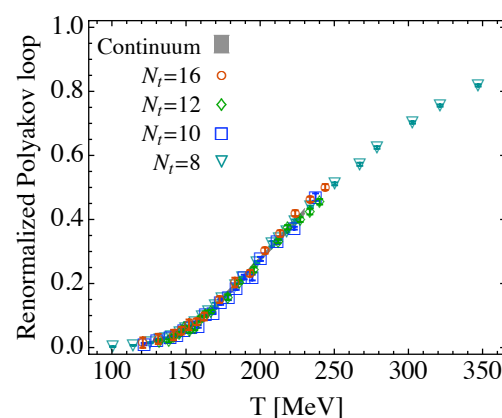


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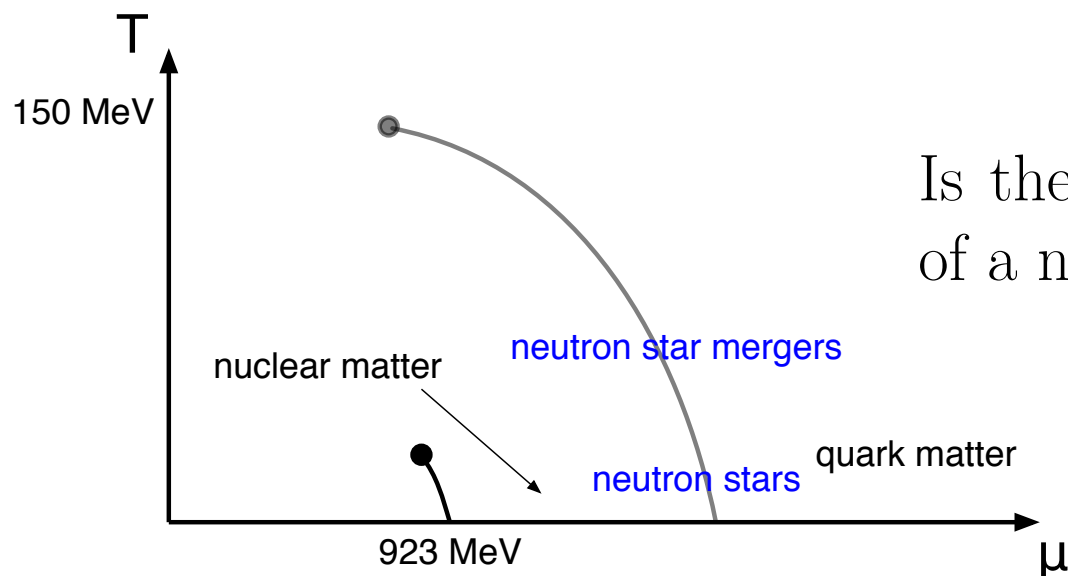
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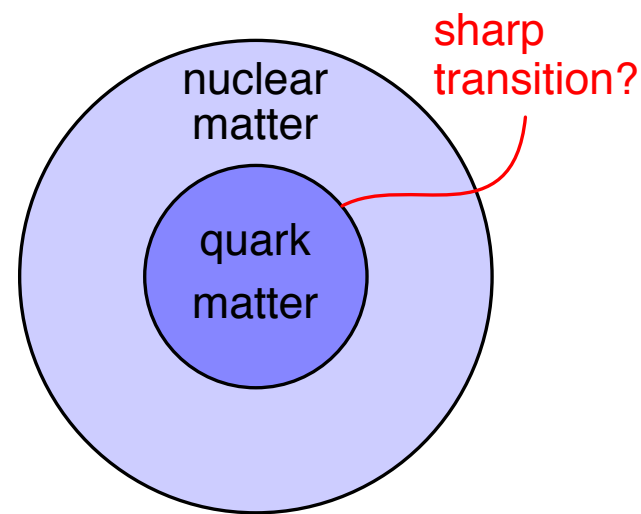
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## Relevance for neutron star physics

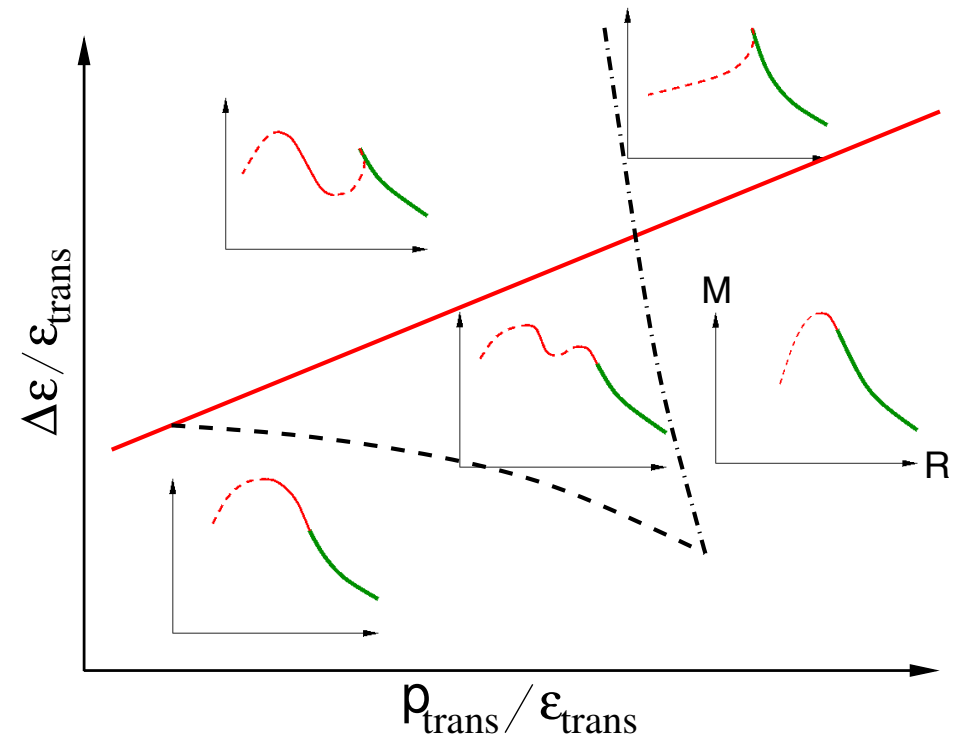


- smooth density profile?
  - first order transition:  
density jump?  
mixed quark-hadron phase?
- surface tension: E. Fraga, M. Hippert, A. Schmitt  
in preparation



## Observable consequences of first-order transition?

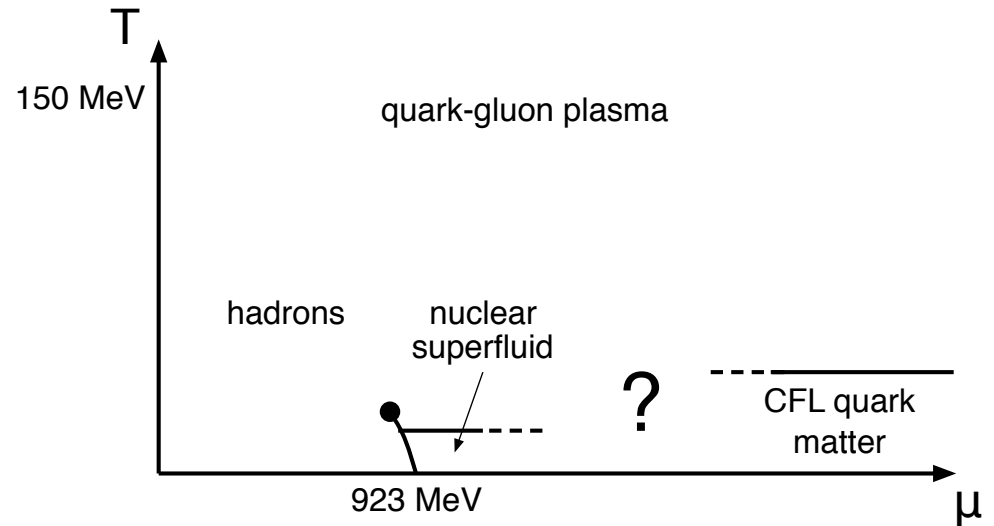
- qualitative difference in mass/radius curve  
M. G. Alford, S. Han and M. Prakash, PRD 88, 083013 (2013)
- sequential 1st-order transitions?  
M. G. Alford and A. Sedrakian, PRL 119, 161104 (2017)



- different gravitational wave signal in neutron star mergers?  
E. R. Most *et al.*, arXiv:1807.03684 [astro-ph.HE]
- gravitational wave from bubble nucleation during supernova?  
G. Cao and S. Lin, arXiv:1810.00528 [nucl-th]
- sharp interface where vortices can end?

# Effects of Cooper pairing

- quark Cooper pairing at high  $\mu$ :  
"color-flavor locking" (CFL)
- CFL breaks  $U(1)_B$



- continuity still possible  
"quark-hadron continuity" T. Schäfer, F. Wilczek, PRL 82, 3956 (1999)  
axial anomaly crucial for continuity T. Hatsuda *et al.*, PRL 97, 122001 (2006)  
A. Schmitt *et al.*, PRD 83, 045008 (2011)  
vortices across quark-hadron transition? M. G. Alford *et al.*, arXiv:1803.05115 [hep-ph]

- effects probably small for equation of state (but not for transport!)
- difficult to include in holography  
see however K. Bitaghsir Fadafan, J. Cruz Rojas and N. Evans, PRD 98, 066010 (2018)  
A. F. Faedo, D. Mateos, C. Pantelidou and J. Tarrío, arXiv:1807.09712 [hep-th]

→ ignore Cooper pairing in the following

## Theoretical and phenomenological approaches

- **QCD**: solve/circumvent sign problem (very hard)
- **models**: need both nuclear and quark matter (also hard)
- **phenomenology**: glue together two models or simply parameterize transition and compare to astrophysical data

remainder of the talk: try **holography**



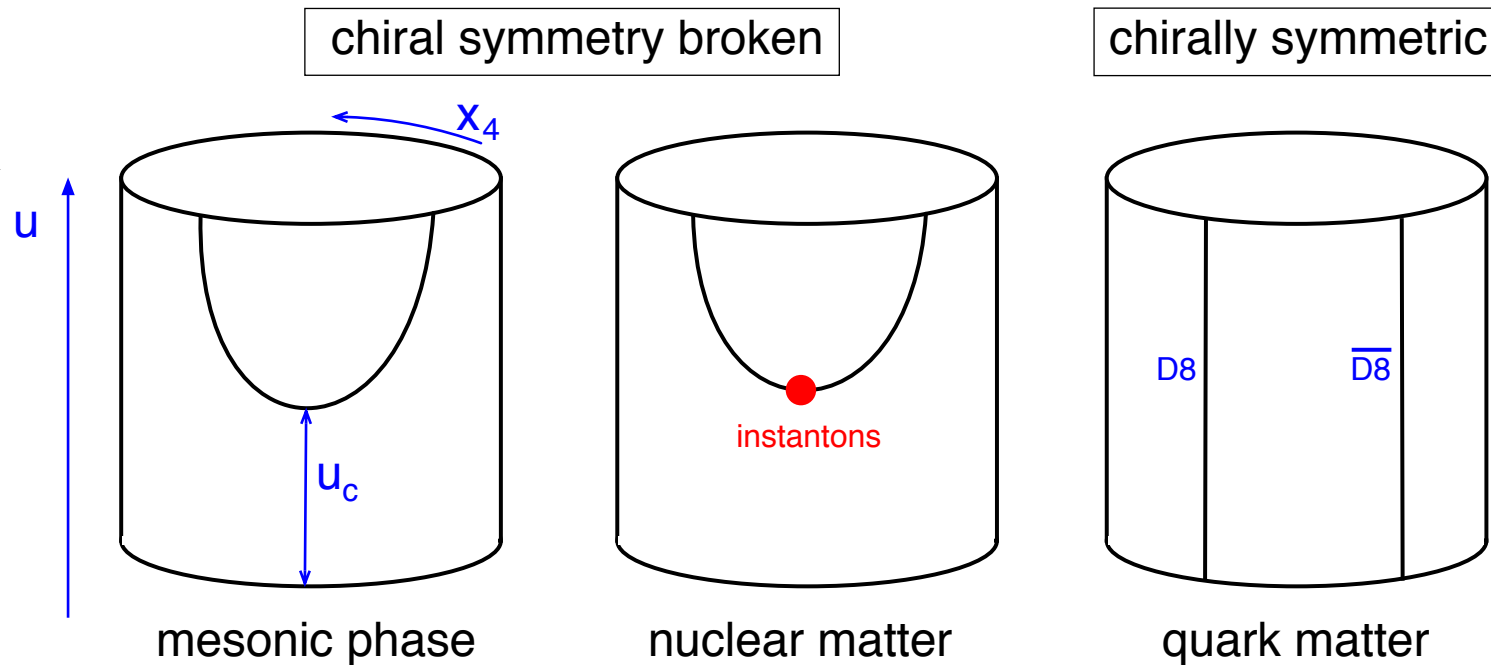
## Sakai-Sugimoto model

E. Witten, *Adv. Theor. Math. Phys.* 2, 505 (1998)

T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* 113, 843 (2005)

- "top-down" approach, dual to large- $N_c$  QCD in a certain limit
  - originally used for meson, baryon, glueball spectra
  - also employed for phase diagrams
    - "inverse magnetic catalysis" F. Preis, A. Rebhan and A. Schmitt, *JHEP* 1103, 033 (2011)
  - main difficulty: many-baryon system (need approximations)
- distorted version of QCD at best, but non-perturbative and takes into account nuclear and quark matter consistently

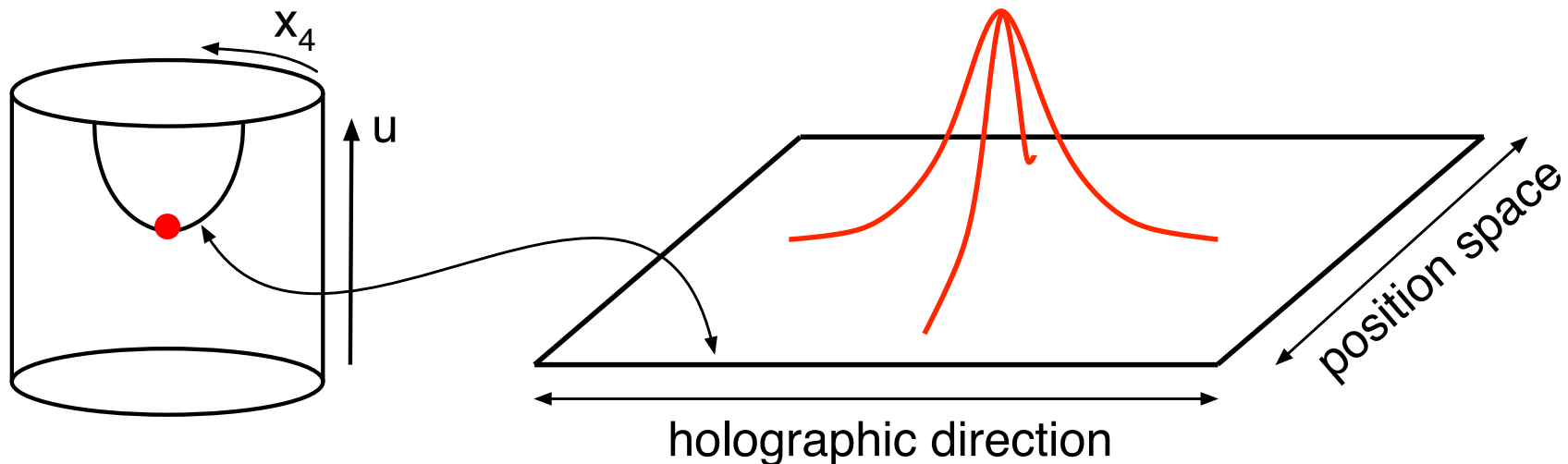
# Phases



- $U(N_f = 2)$  gauge theory in the bulk:
  - insert instanton ansatz for non-abelian  $SU(2)$  part into DBI+CS action
  - solve EOMs for abelian  $U(1)$  gauge field and embedding  $x_4(u)$
  - minimize free energy wrt  $u_c$ , parameters of ansatz etc
  - compare free energies of all three phases for all  $\mu$  and  $T$
- quark masses are neglected → chiral symmetry exact

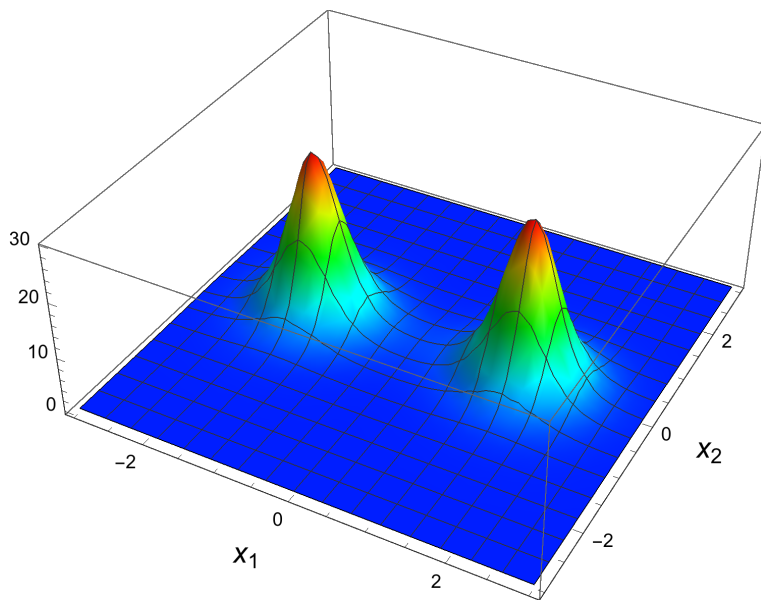
# Baryons

- baryons in AdS/CFT: wrapped D-branes with  $N_c$  string endpoints  
E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)
  - baryons in Sakai-Sugimoto:
    - D4-branes wrapped on  $S^4$
    - equivalently: instantons on D8-branes ( $\rightarrow$  skyrmions)
- T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)  
H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)

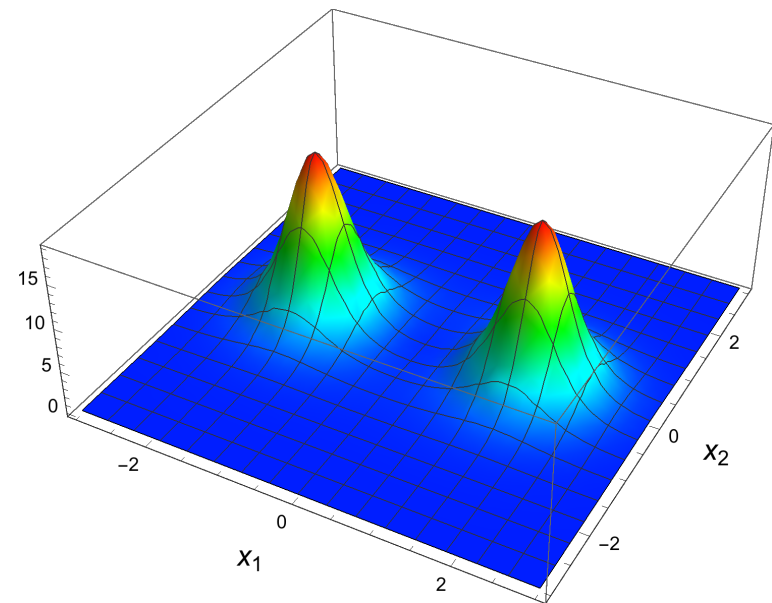


## Interaction from two-instanton solution

- exact (flat space)  $N$ -instanton solution known (ADHM)  
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K. Hashimoto, T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* 122, 427 (2009)  
 $N$ -body force (estimates) K. Hashimoto, N. Iizuka and T. Nakatsukasa, *PRD* 81, 106003 (2010)



same orientation

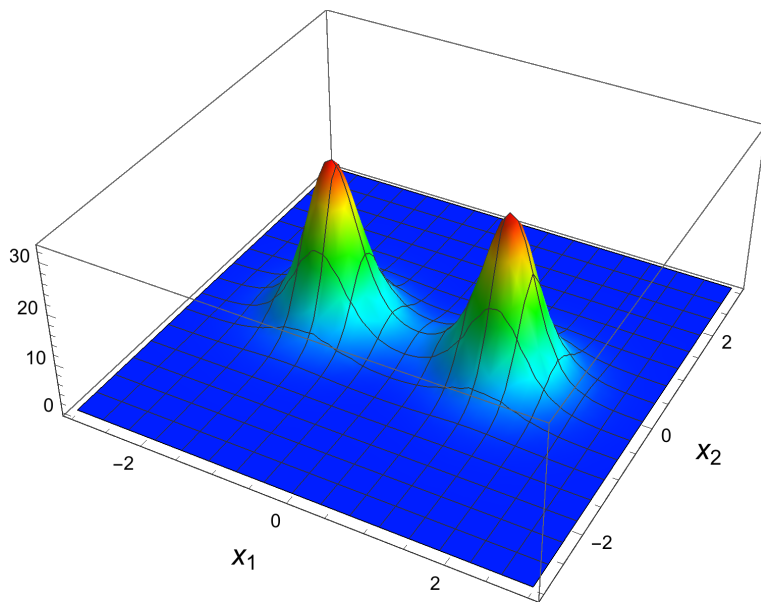


opposite orientation

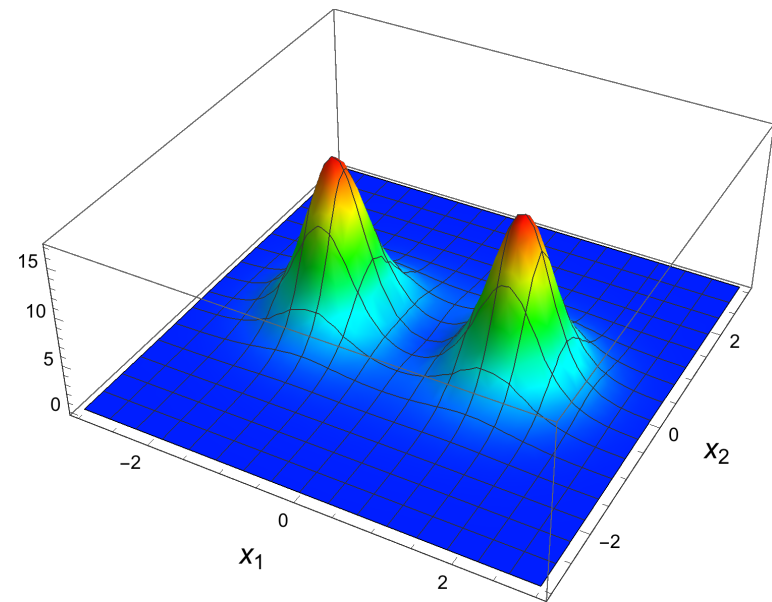
$$\ell/\rho = 3$$

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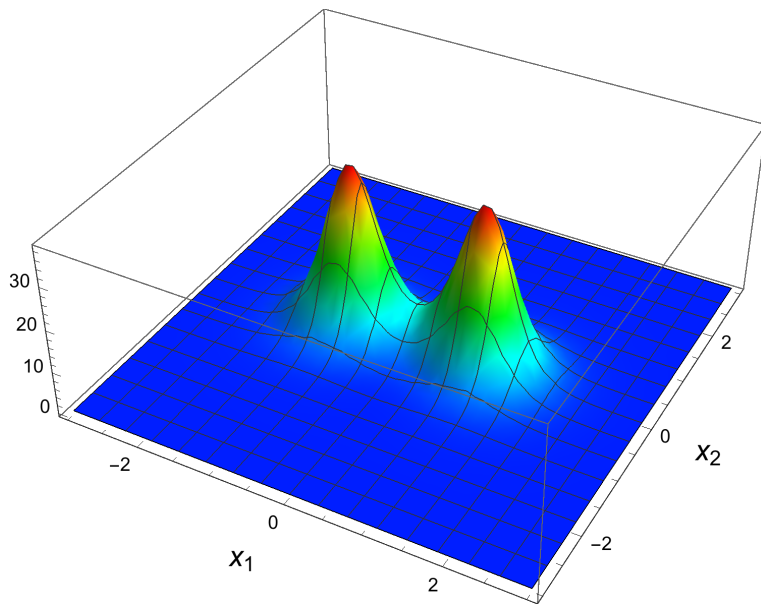


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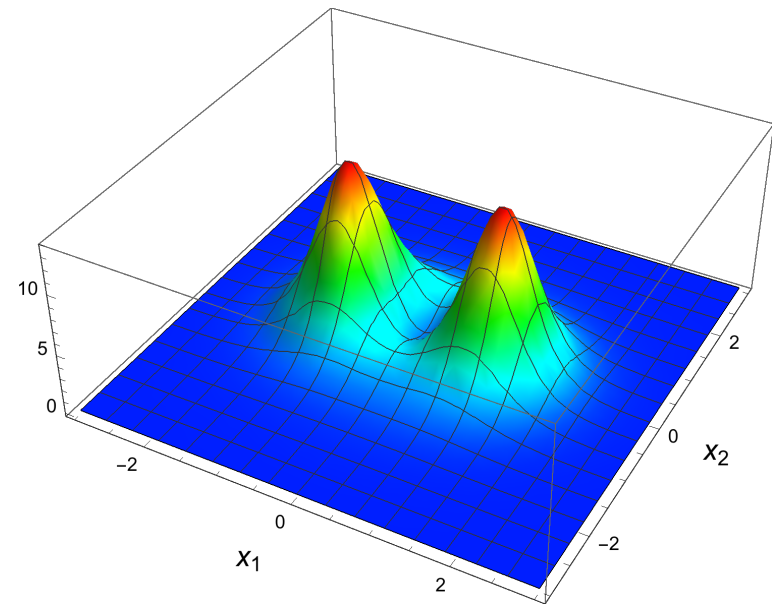
$$\ell/\rho = 2.5$$

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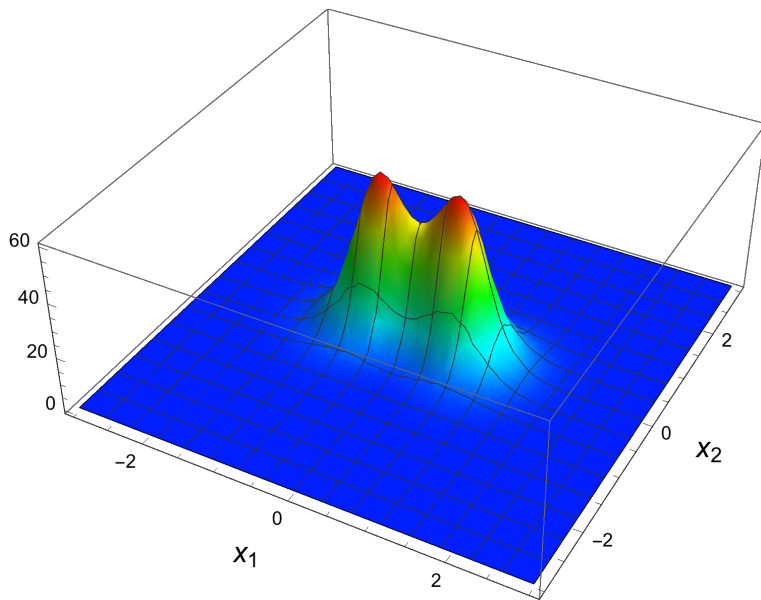


opposite orientation

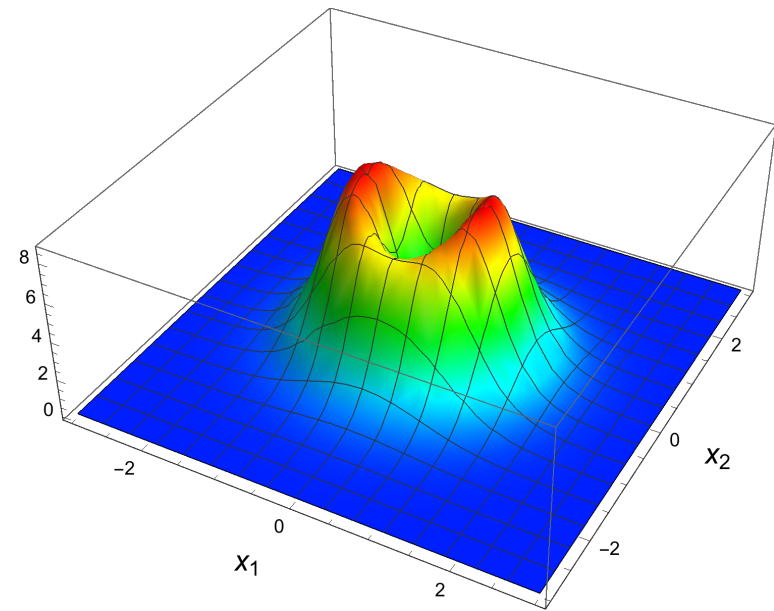
$$\ell/\rho = 2$$

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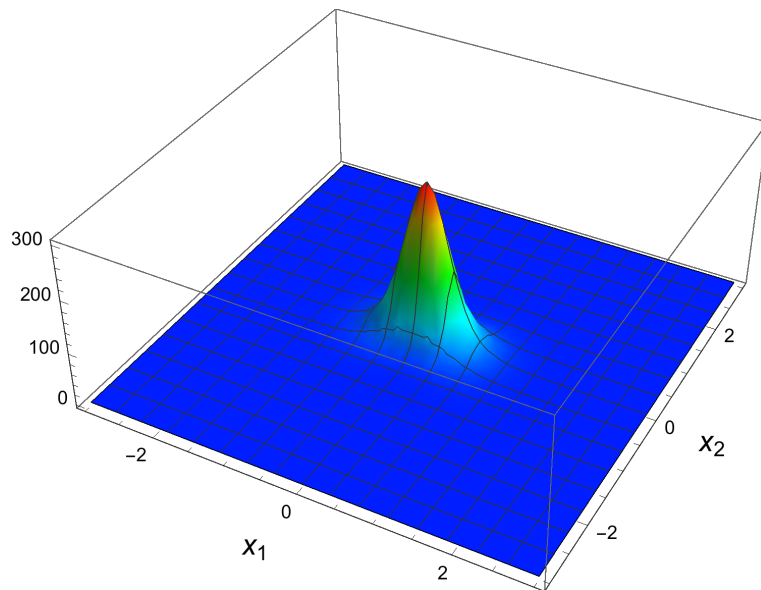


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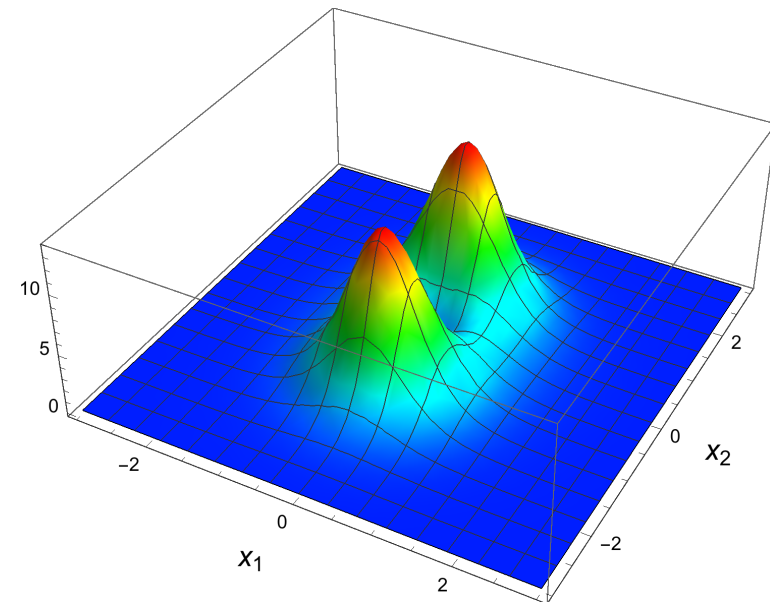
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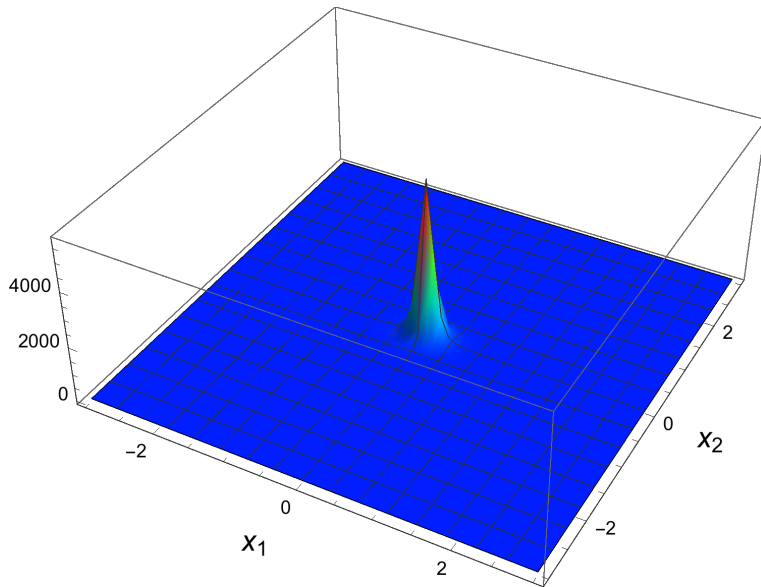
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$$\ell/\rho = 1$$

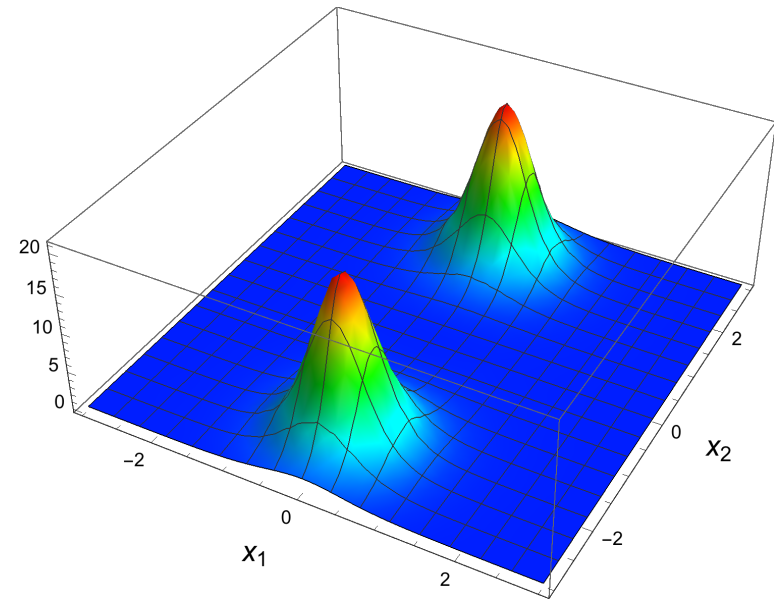


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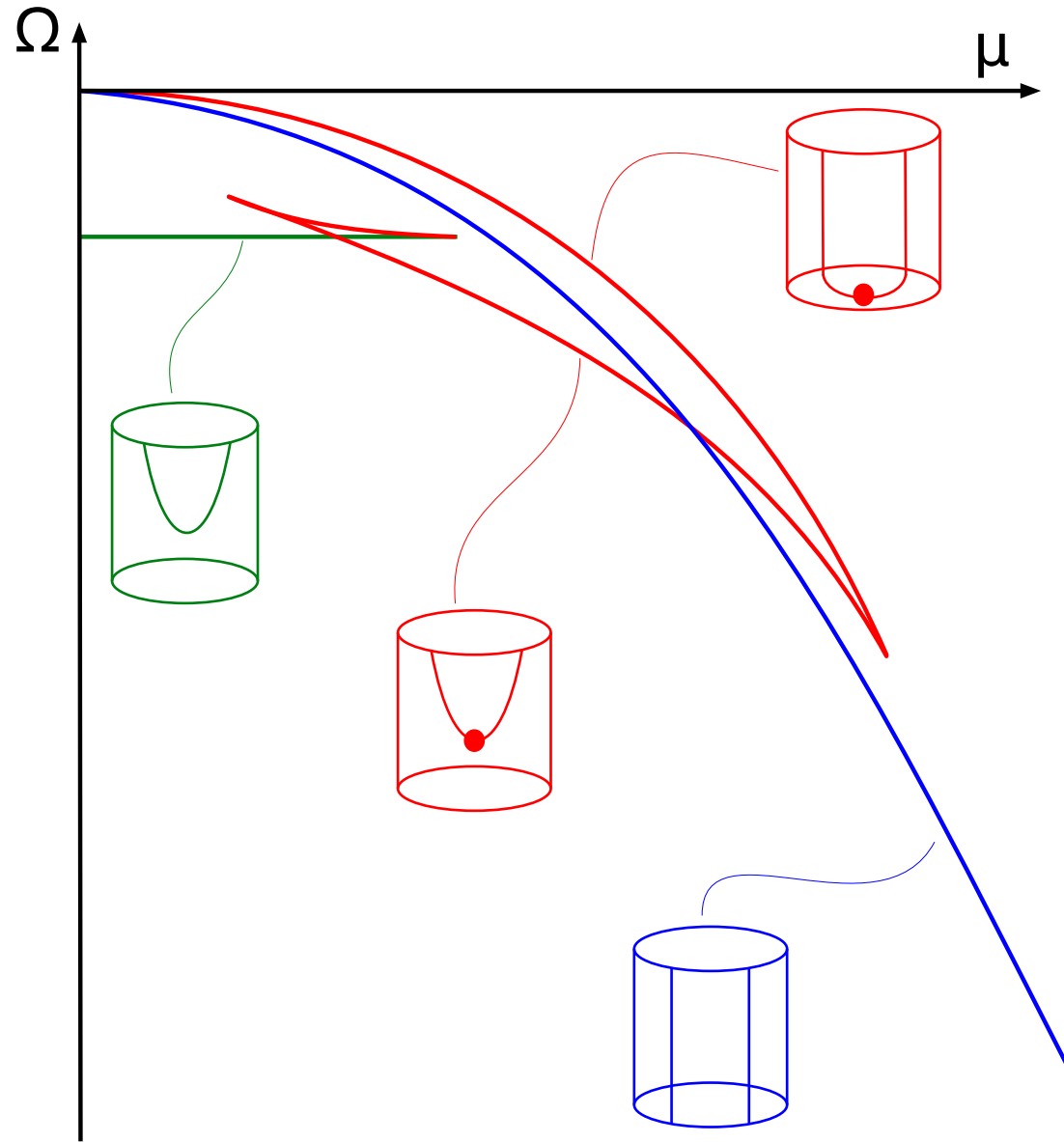
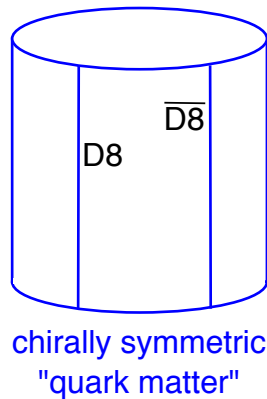
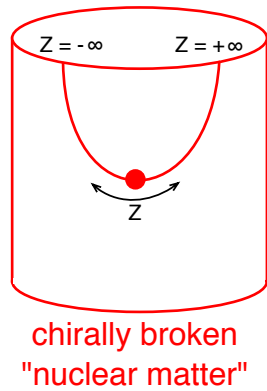
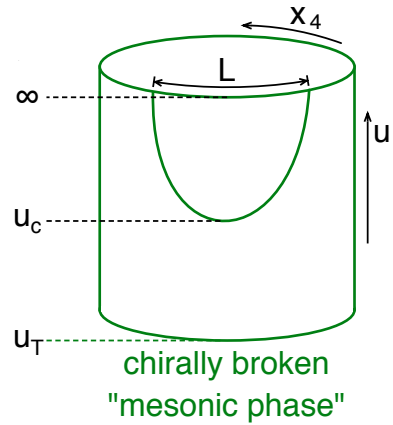
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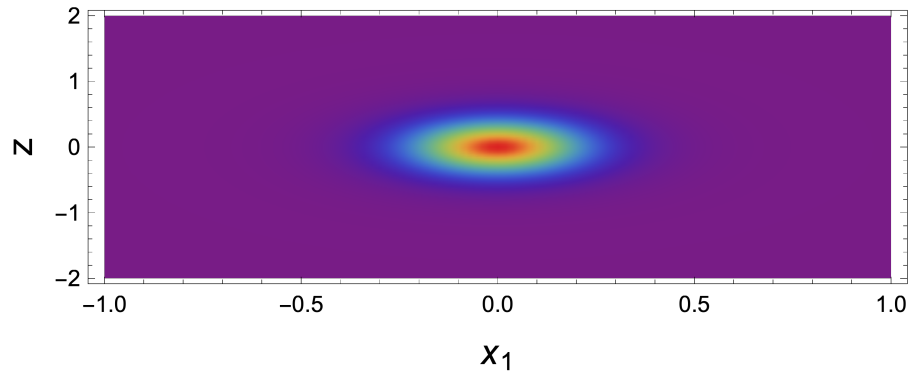
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$$\ell/\rho = 0.5$$

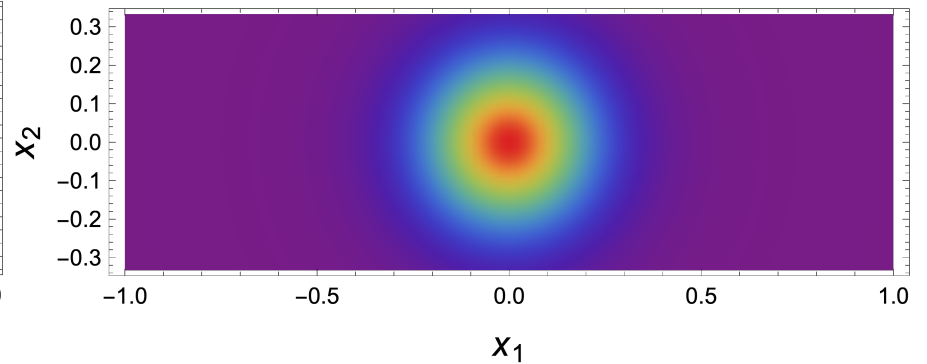
# Main result (schematically)



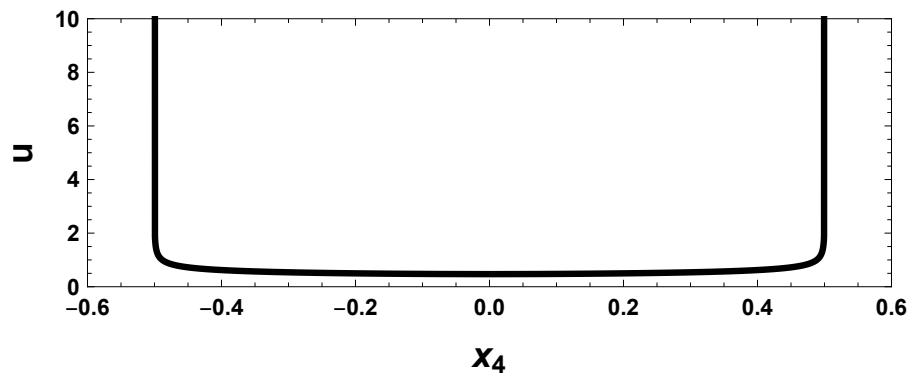
# Instanton profiles



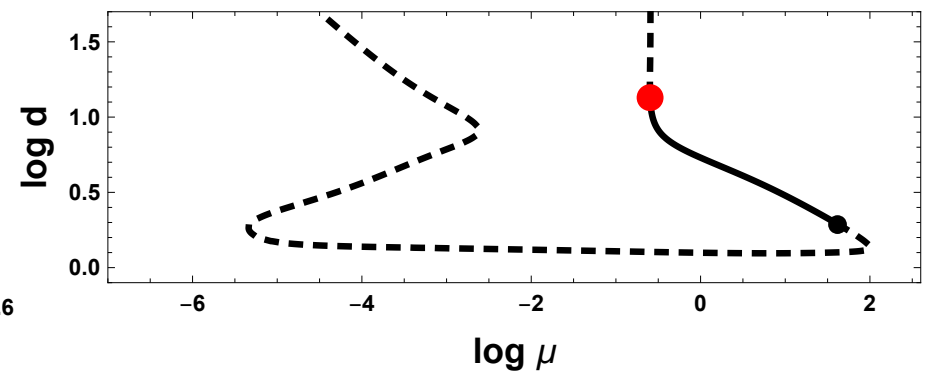
deformation from  $SO(4)$



instanton lattice

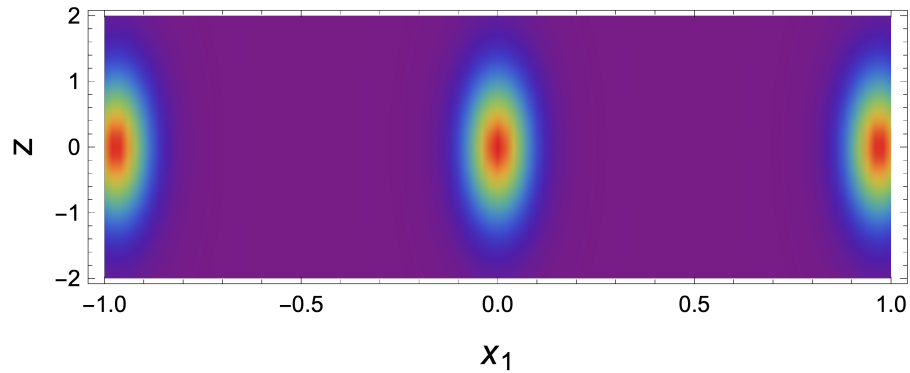


flavor brane embedding

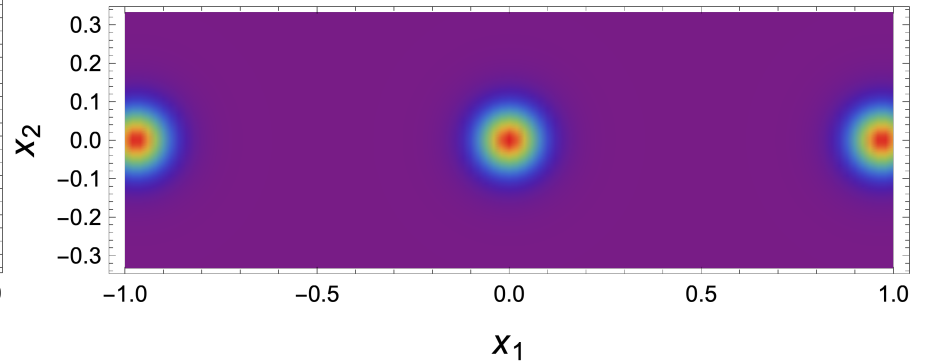


instanton overlap

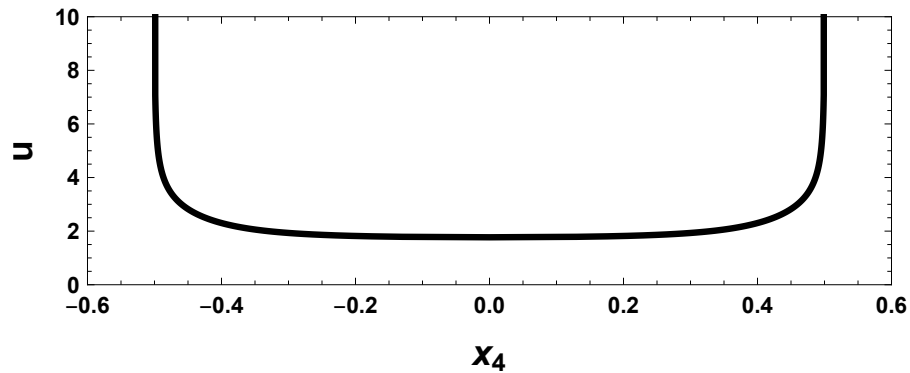
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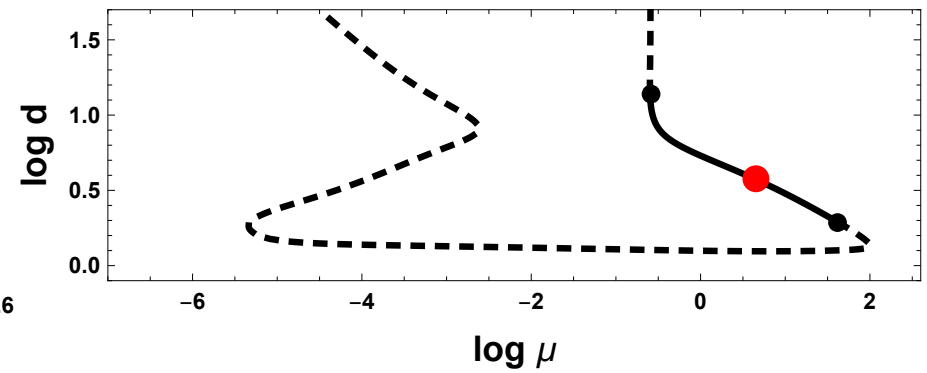
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instanton lattice

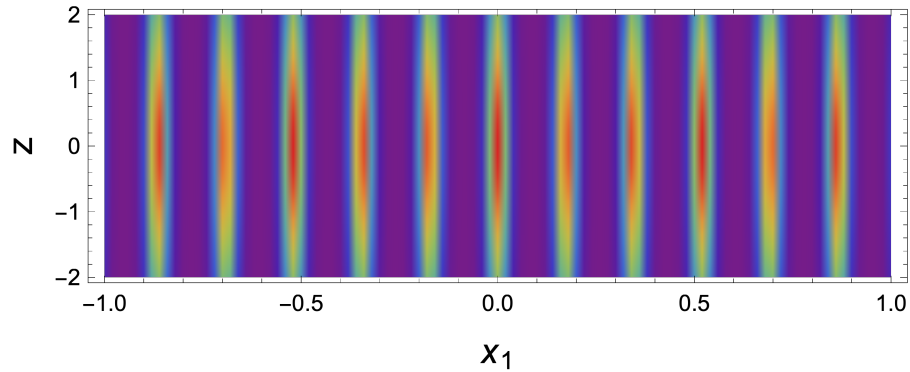


flavor brane embedding

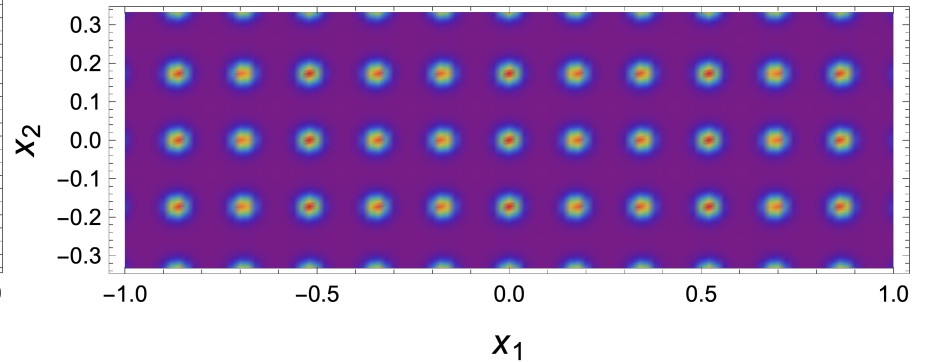


instanton overlap

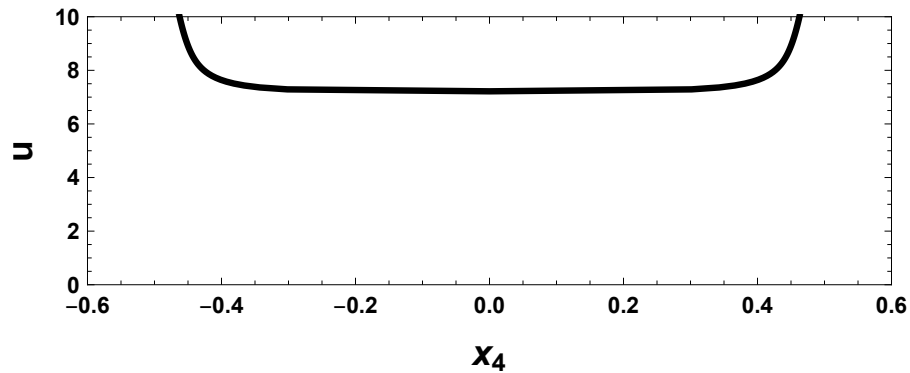
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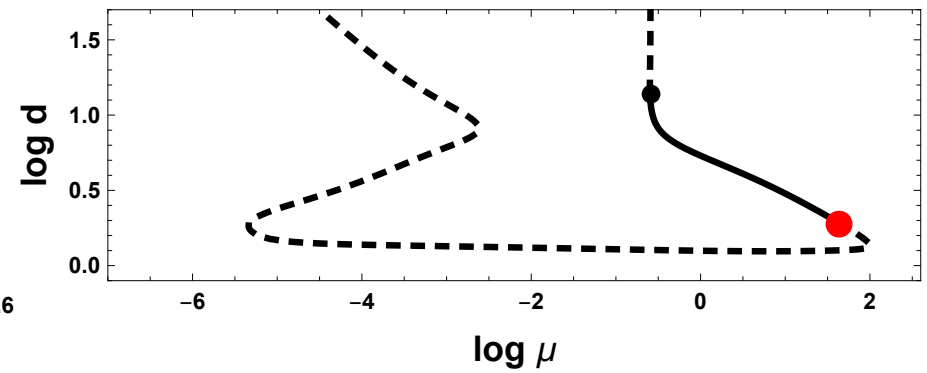
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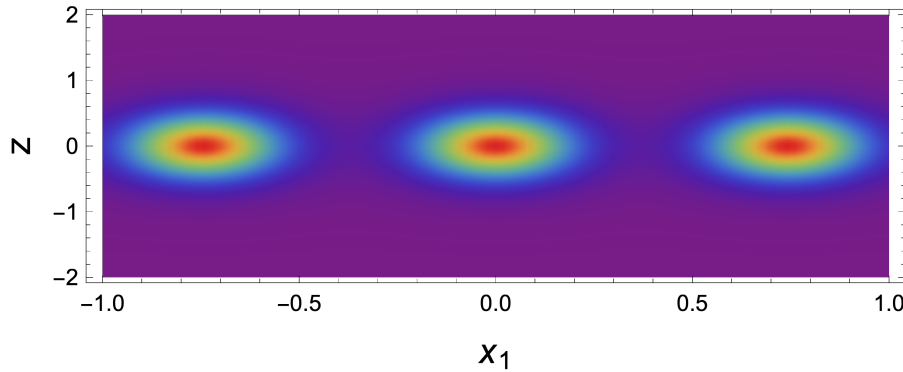


flavor brane embedding

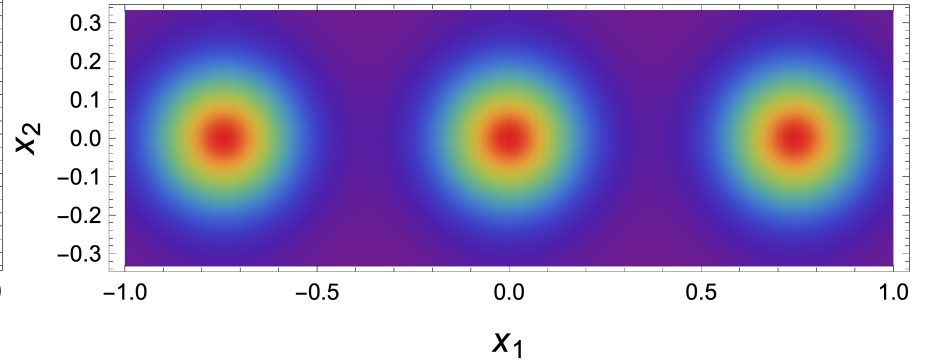


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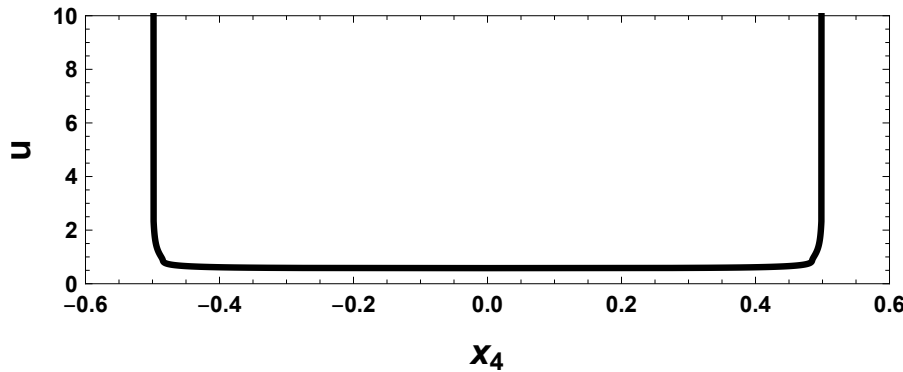
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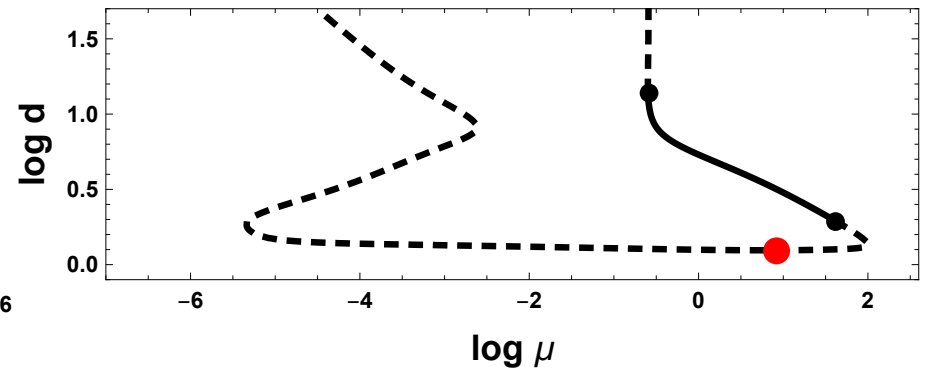
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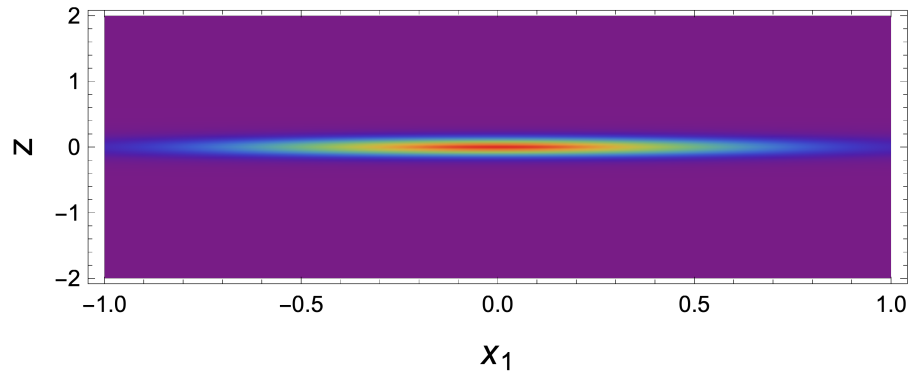


flavor brane embedding

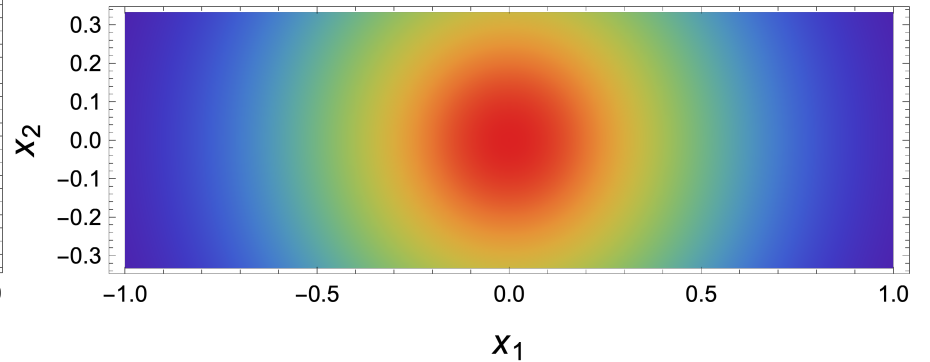


instanton overlap

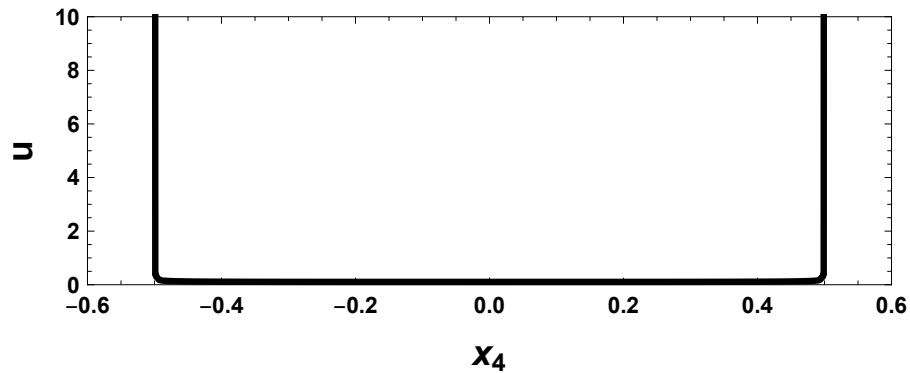
# Instanton profiles



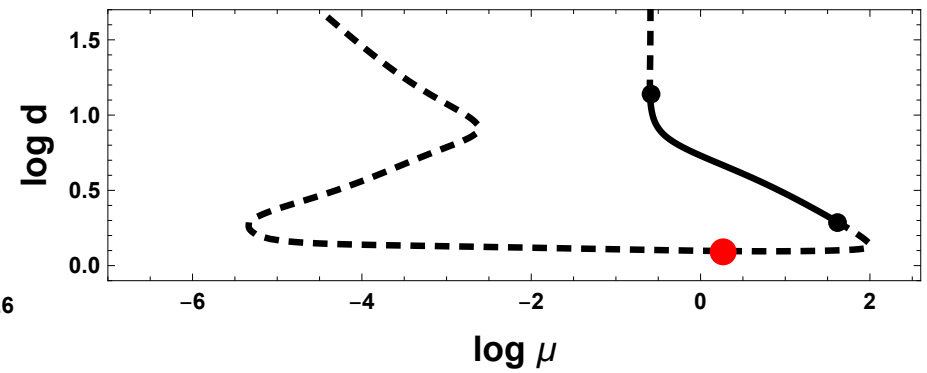
deformation from  $SO(4)$



instanton lattice

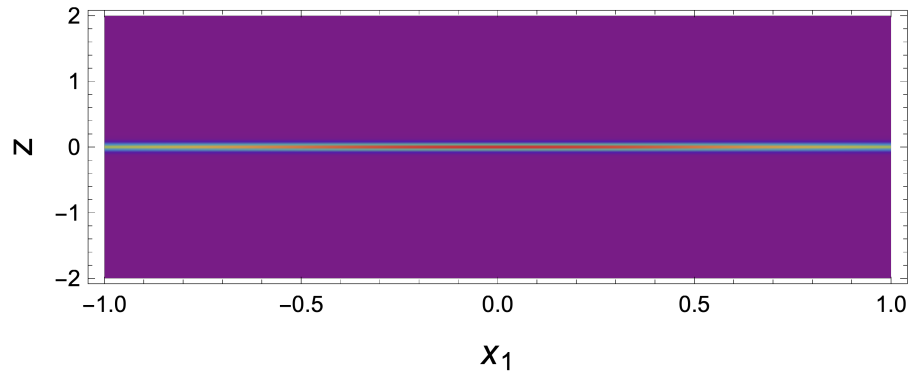


flavor brane embedding

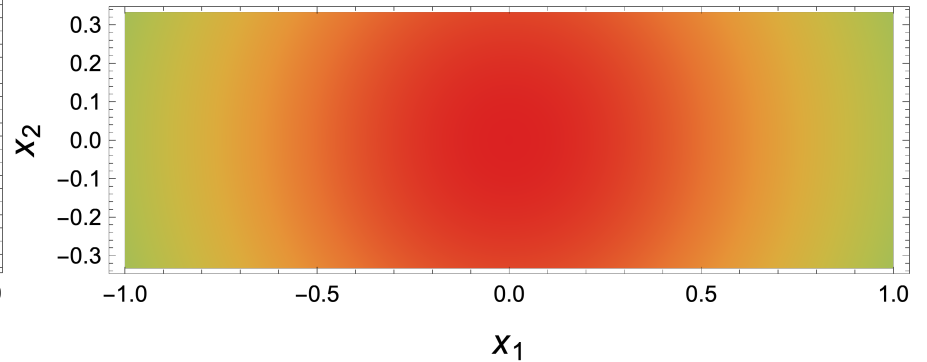


instanton overlap

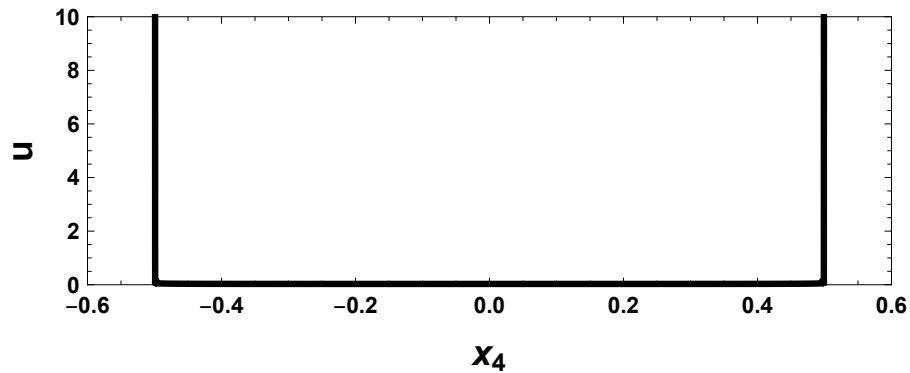
# Instanton profiles



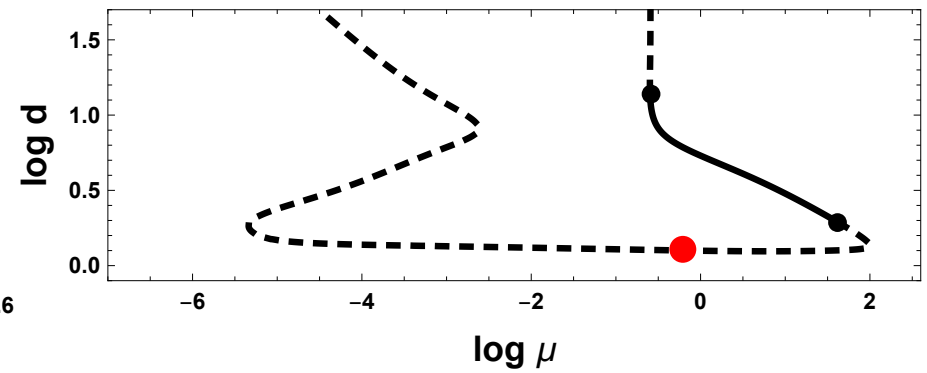
deformation from  $SO(4)$



instanton lattice



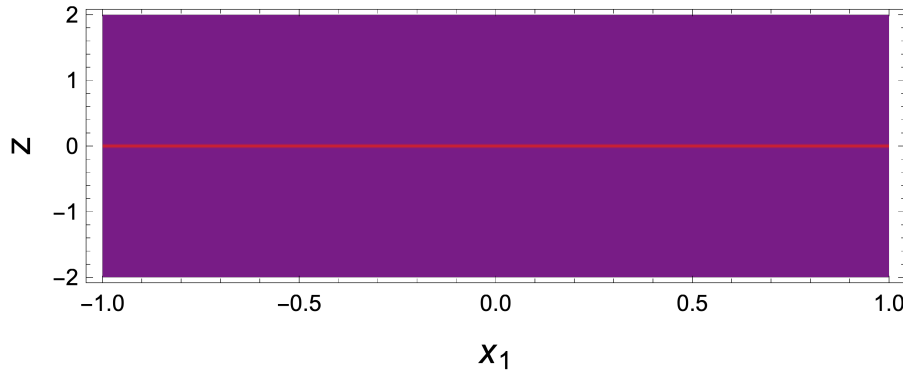
flavor brane embedding



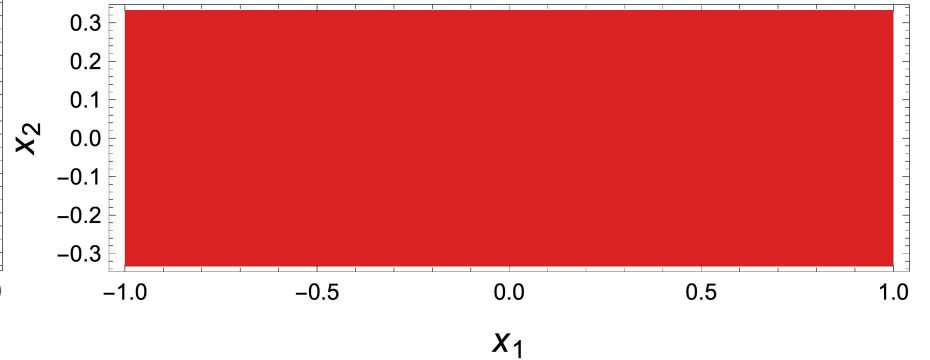
instanton overlap



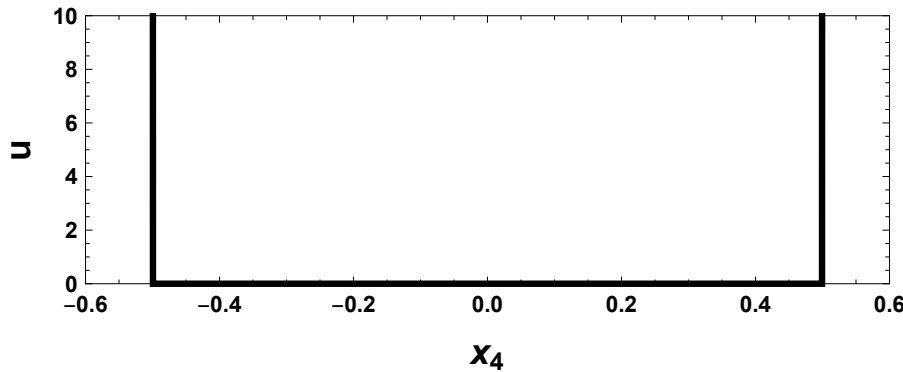
# Instanton profiles



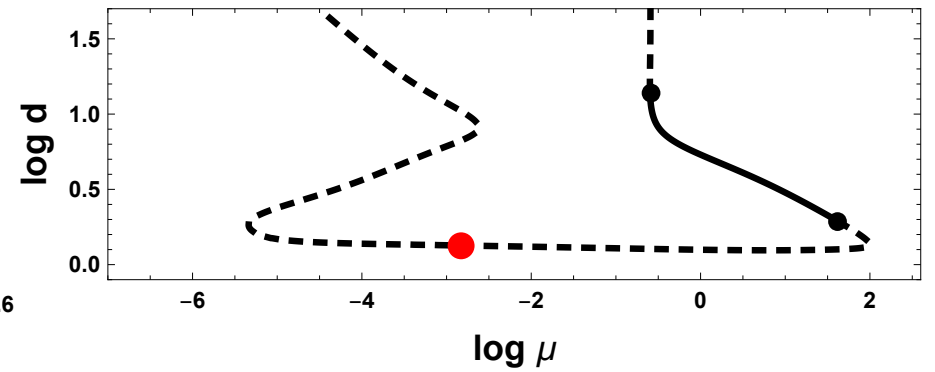
deformation from  $SO(4)$



instanton lattice



flavor brane embedding

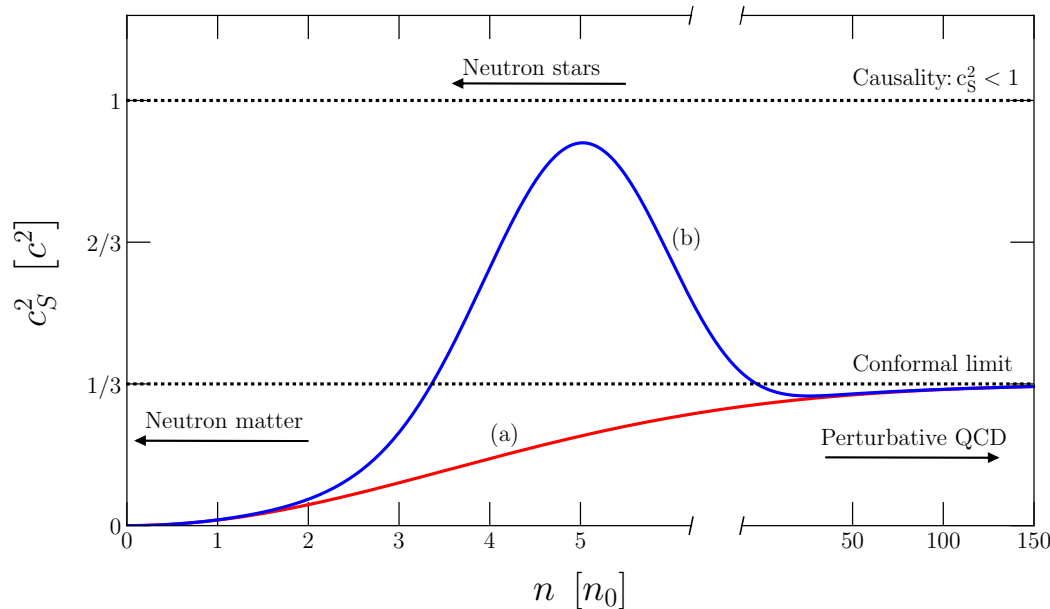


instanton overlap

# Observations

- hadron and quark phases connect continuously (instanton interactions crucial!)
  - geometrically: continuous transformation between connected and disconnected flavor branes
  - instantons smear out in spatial direction and become infinitesimally thin in holographic direction
  - continuity at zero density (analytic result available for small  $\mu$ )
- instantons avoid overlapping by becoming smaller at high density
- actual quark-hadron transition is of first order

# Speed of sound



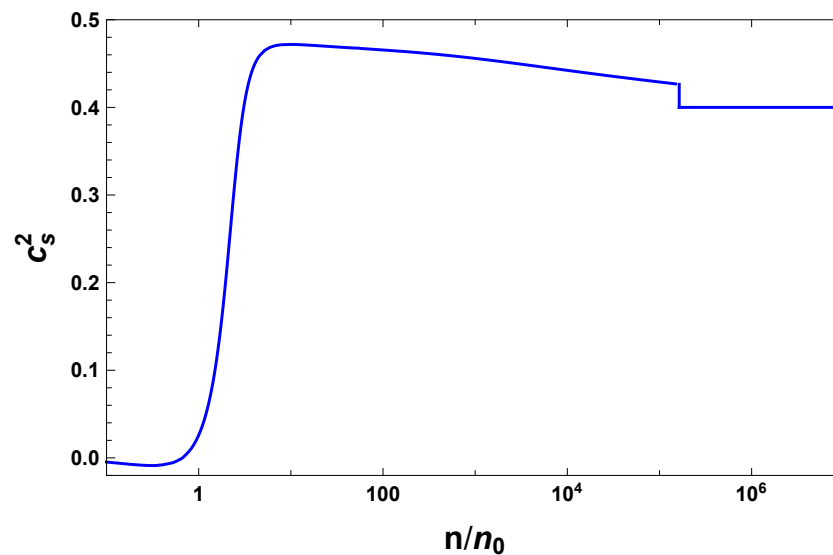
sound speed

↔ stiffness of matter

↔ neutron star masses

schematic plot from I. Tews *et al.*,  
*Astrophys. J.* 860, 149 (2018)

- fit Sakai-Sugimoto parameters to low-density nuclear matter
- non-monotonic speed of sound



## Summary

- location and nature of the quark-hadron transition at large baryon densities is unknown (sign problem)
- a potential first-order quark-hadron transition has observable consequences for neutron star physics
- holography may give some (qualitative) insights

# Outlook

- include nonzero quark masses  
worldsheet instantons K. Hashimoto *et al.*, JHEP 0807, 089 (2008)
- isospin asymmetry → from symmetric nuclear matter to neutron star matter
- nonzero temperature and/or magnetic field → phase diagrams
- multiple instanton layers in holographic direction  
V. Kaplunovsky, D. Melnikov and J. Sonnenschein, JHEP 1211, 047 (2012)  
F. Preis and A. Schmitt, JHEP 1607, 001 (2016)
- equation of state → neutron star mass/radius, deformability  
holographic quark matter C. Hoyos, *et al.*, PRL 117, 032501 (2016)  
N. Jokela, M. Järvinen and J. Remes, arXiv:1809.07770 [hep-ph]