

UNIVERSITY OF
Gravity seminar
Southampton

Quark-hadron continuity
under rotation

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Based on the work

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Continuity of vortices from the hadronic to the color-flavor locked phase in dense matter

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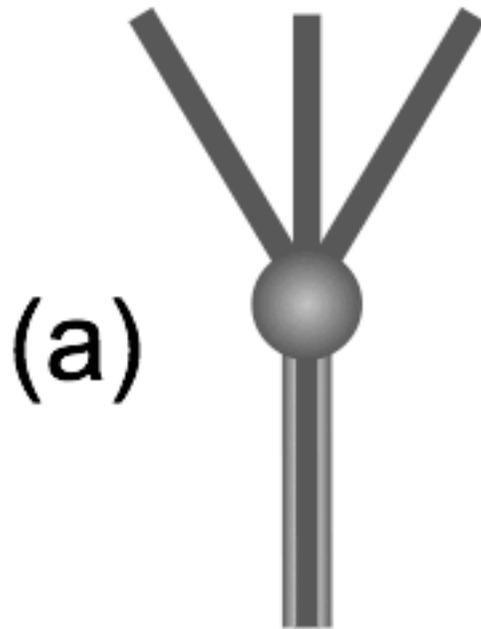
(Received 21 March 2018; published 7 February 2019)

We study how vortices in dense superfluid hadronic matter can connect to vortices in superfluid quark matter, as in rotating neutron stars, focusing on the extent to which quark-hadron continuity can be maintained. As we show, a singly quantized vortex in three-flavor symmetric hadronic matter can connect smoothly to a singly quantized non-Abelian vortex in three-flavor symmetric quark matter in the color-flavor locked phase, without the necessity for boojums appearing at the transition.

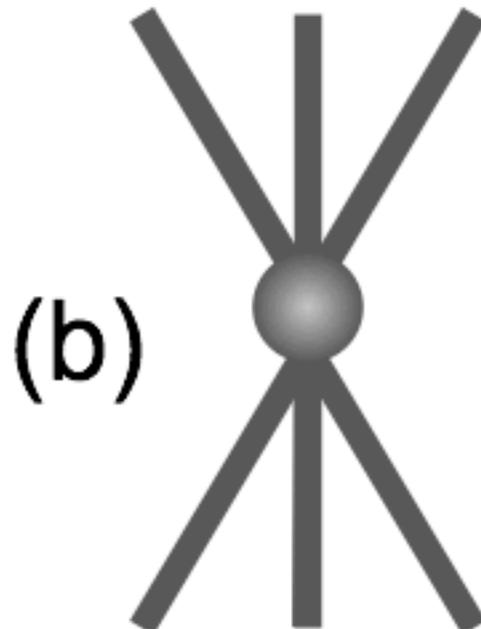
Question

Non-Abelian CFL vortices ~ Hadronic vortices

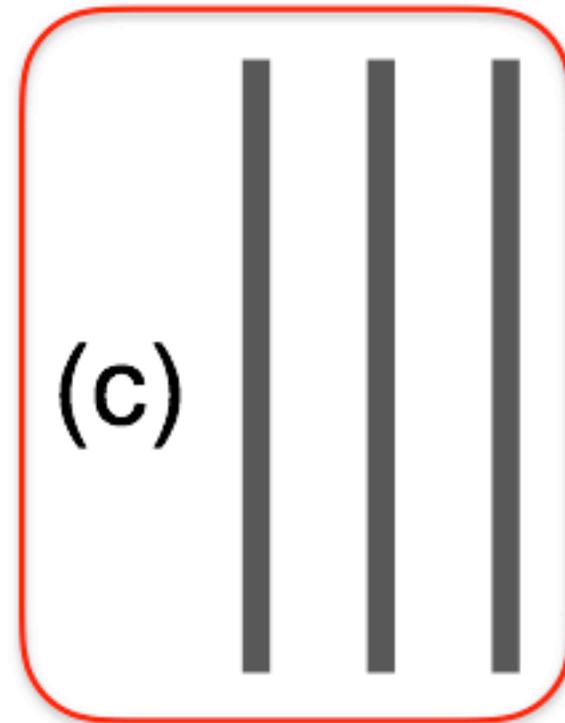
Hadronic Vortices



Abelian
Vortex



Non-Abelian
Vortices



Physics = Fertile research arena of Matters and Space-time

- ★ What matters are made of?
- ★ How matters are created and change in different conditions?
- ★ How the universe was born and will be?

Approaches: **Experiment**, **Observation**,
Theory, **Computer**

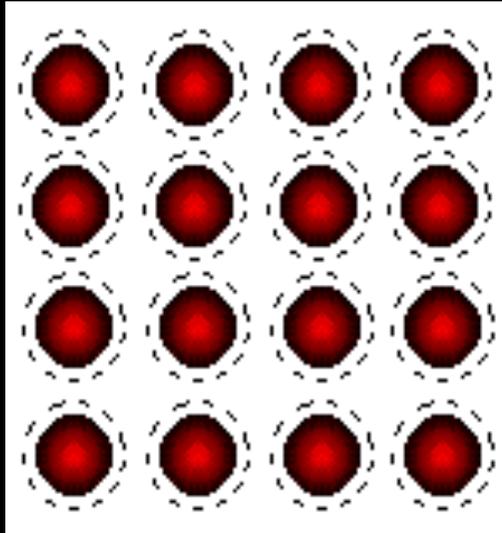
A child-like question:

What happens to matter
as we squeeze it harder and harder,
and/or make it hotter and hotter?

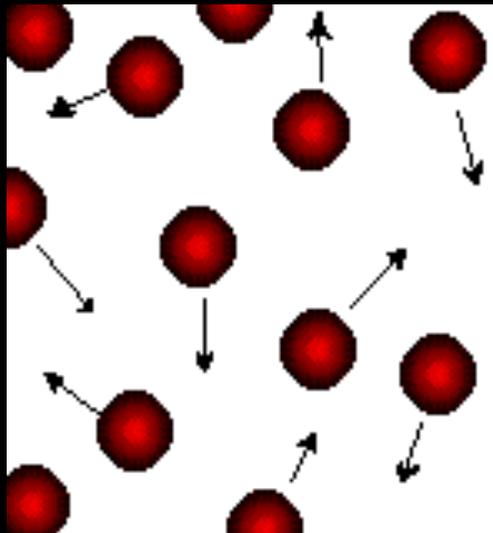


Phases of Matter

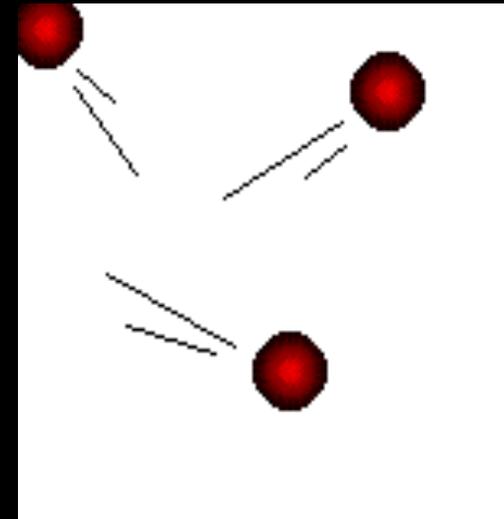
Ex.) H₂O



Solid(ice)



Liquid(water)

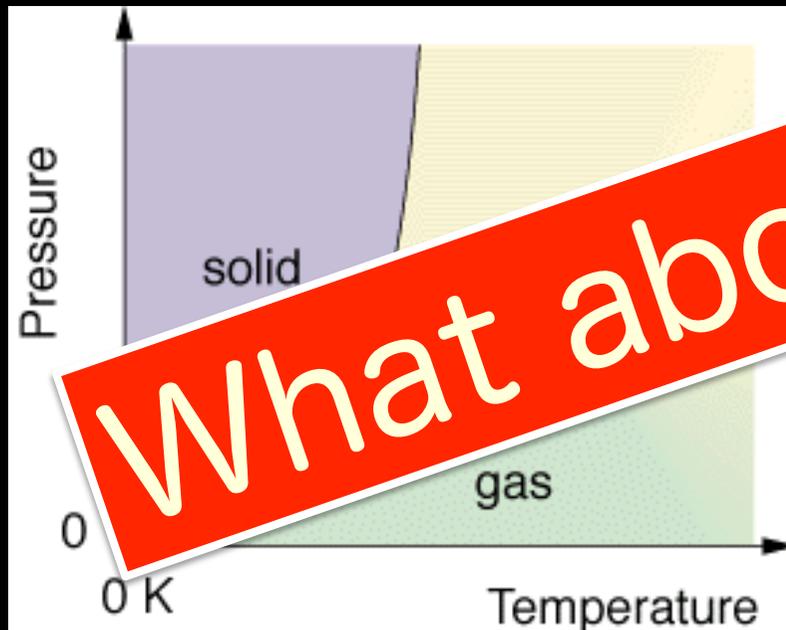


Gas(vapor)

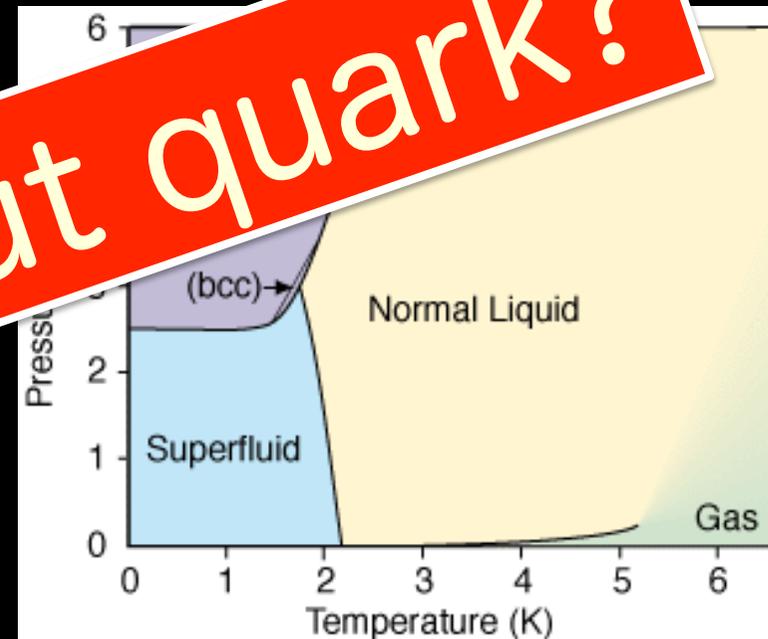
T, P

Phase diagrams

H_2O



$4He$



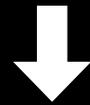
What about quark?

<http://boojum.hut.fi/research/theory/typicalpt.html>

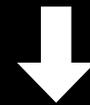
Convention

- ★ Phases are described by symmetries of underlying theory
- ★ What is the theory of quarks?
What are the symmetries?

Properties of
strongly-interacting matter



Quantum field theory
with Non-Abelian symmetry



Quantum Chromo Dynamics

QCD



QCD Lagrangian (Han-Nambu, 1965)



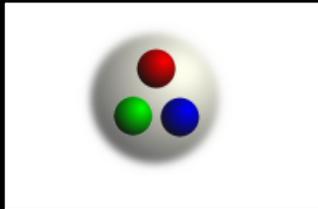
$$L_{QCD} = \bar{q}^{\alpha} \left(i\gamma_{\mu} D^{\mu} - m_q \right)_{\alpha\beta} q^{\beta} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

Just one line, but very rich in physics and math

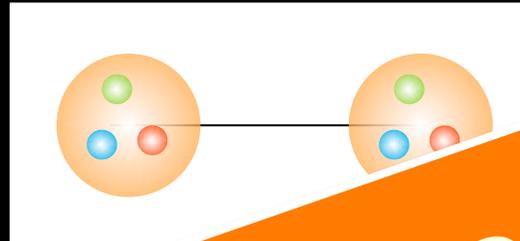
Yes! We love QCD ❤️
... kind of new ideas

Grand Challenge

- Space-time evolution of QCD matter -



Hadrons



Nuclear force

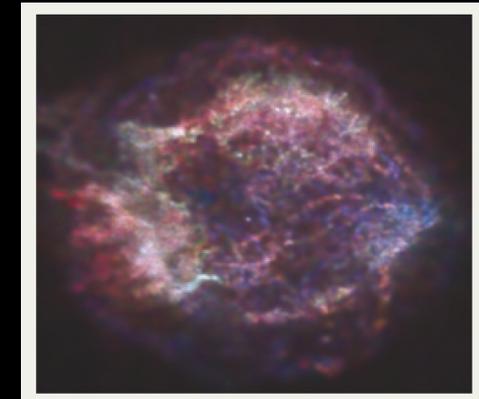
Nuclei



Early universe



Neutron/quark star



Supernovae

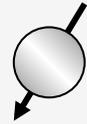
No QCD, no life

The answer to the ultimate question

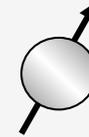
“Why the matter of our universe can be stable?”

■ Symmetries of QCD and their breaking patterns

Chiral basis :



$$q_L = \frac{1}{2}(1 - \gamma_5)q, \quad q_R = \frac{1}{2}(1 + \gamma_5)q$$



QCD Lagrangian :

$$\mathcal{L}_{\text{cl}} = \mathcal{L}_{\text{cl}}(q_L, A) + \mathcal{L}_{\text{cl}}(q_R, A) - (\bar{q}_L m q_R + \bar{q}_R m q_L)$$

classical QCD symmetry (m=0)

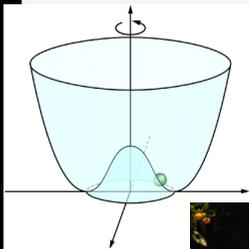
$$\mathcal{G} = SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times U(1)_A$$



Quantum QCD vacuum (m=0)

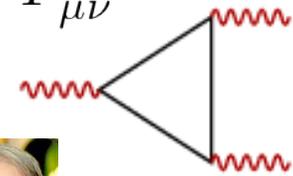
**Chiral condensate :
spontaneous mass generation**

**Axial anomaly :
quantum violation of $U(1)_A$**



$$\langle \bar{q}_R q_L \rangle \neq 0$$

$$\partial_\mu J_A^\mu = -2N_f \frac{\alpha_s}{8\pi} F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a$$

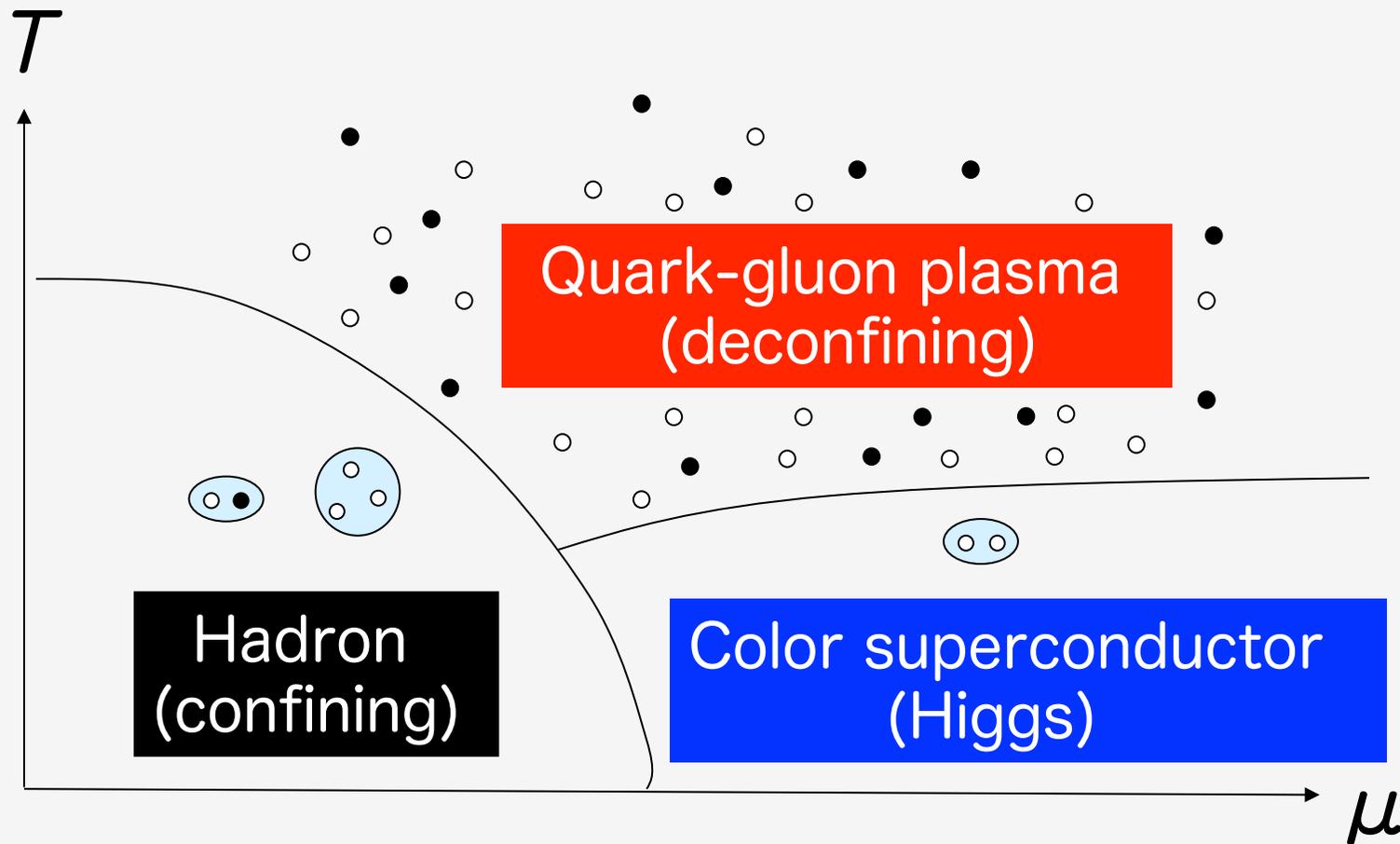


$$SU(3)_C \times SU(N_f)_{L+R} \times U(1)_B$$

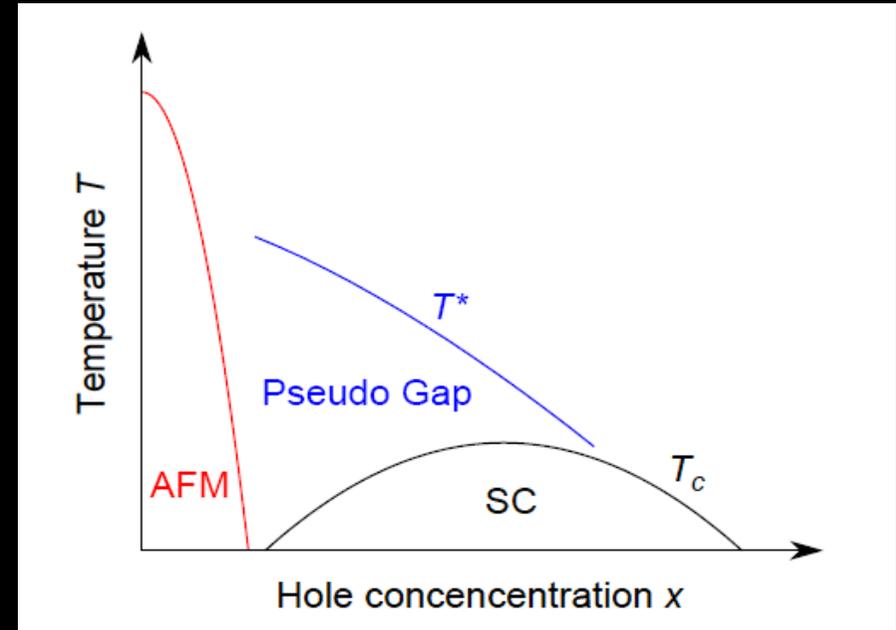
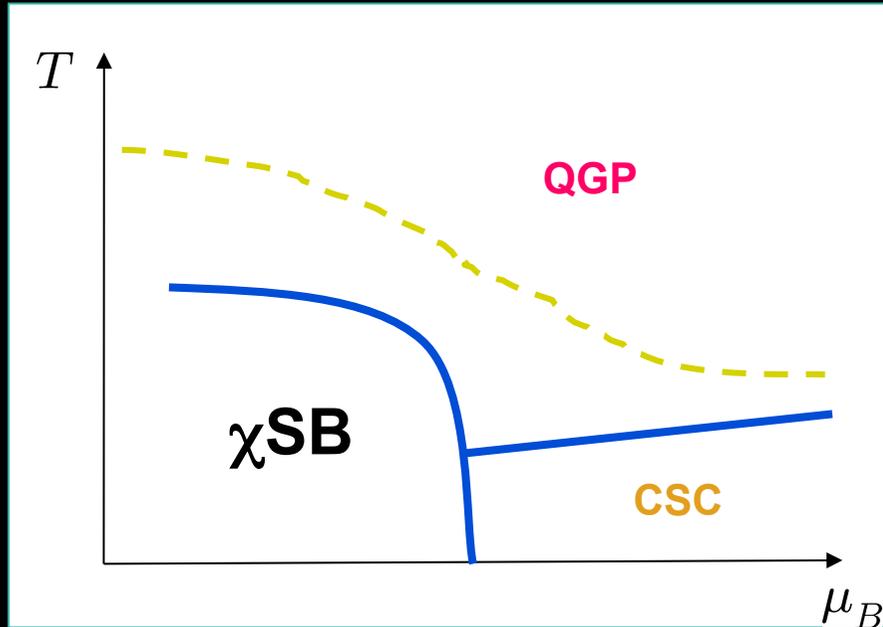


QCD phase diagram via Occam's razor

“Schematic phase diagram”



Similarity between QCD and High T_c Superconductor



Common features in QCD, HTS, and ultracold atoms

1. Competition among different orders
2. Strong coupling/correlation

Idea of Quark-Hadron continuity

hep-ph/9811473

3 flavor hadron matter
is smoothly connected to
a quark matter
as baryon density is increased

Impacts on NS physics?

Table of Quark-Hadron continuity

Phase	Hadronic (confinement)	Color-flavor locked(Higgs)
Symmetry breaking Pattern	$SU(3)_L \times SU(3)_R \times U(1)_B$ $\rightarrow SU(3)_{L+R}$	$SU(3)_L \times SU(3)_R \times SU(3)_C \times U(1)_B$ $\rightarrow SU(3)_{L+R+C}$
Order parameter	chiral condensate	diquak condensate
$U(1)_B$	broken in the dibaryon channels	broken by d
Elementary Excitations	Pseudo-scalar mesons (π etc)	NG bosons
	vector mesons (ρ etc)	massive gluons
	baryons	massive quarks (CFL gap)

A realization of Fradkin-Shenker complementarity

Diquark condensations

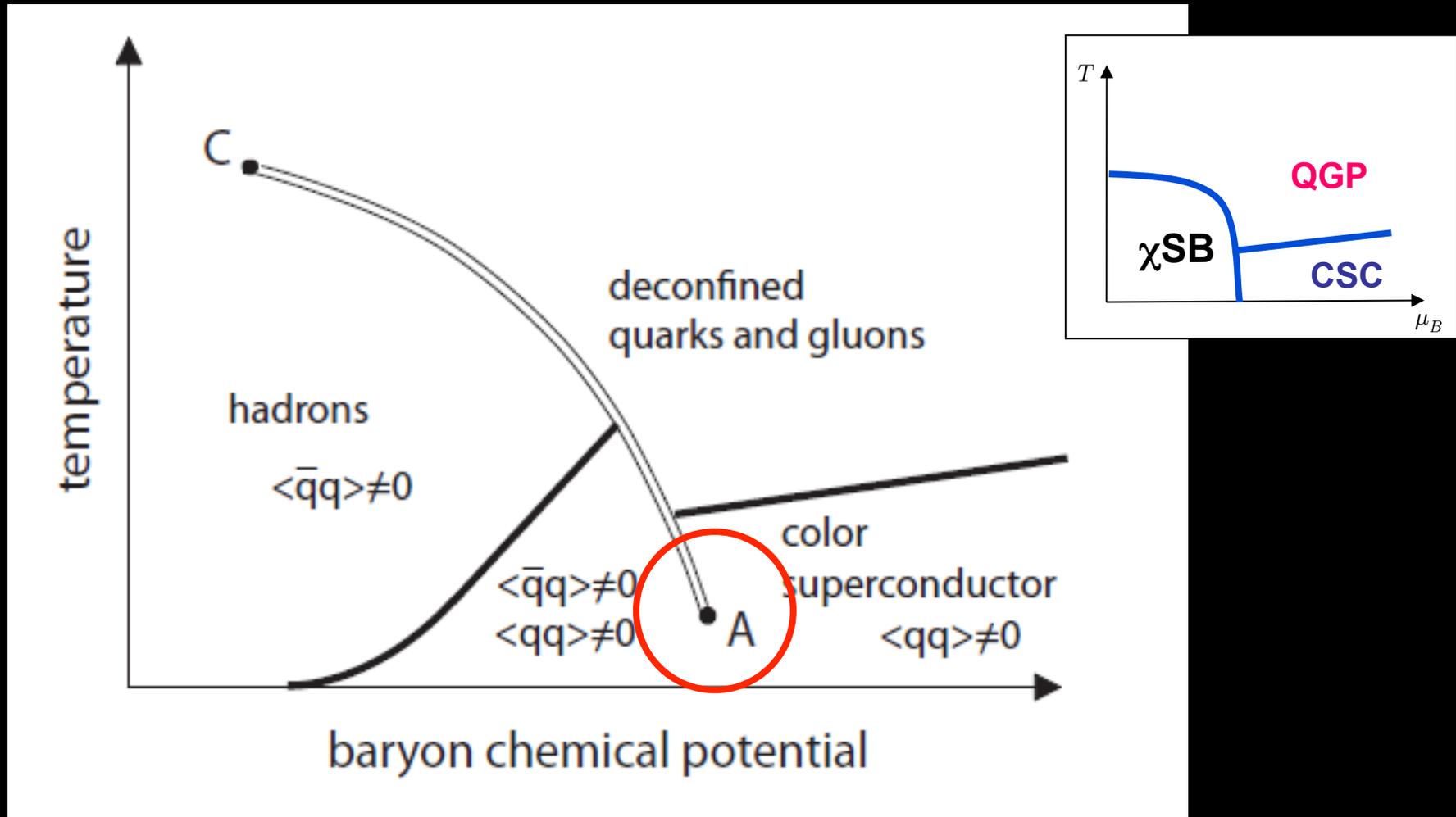
$N_f=2$ (2-flavor color superconductivity [2SC])

$$\langle q_{\alpha i} C \gamma_5 q_{\beta j} \rangle \propto \Delta_{2SC} \epsilon^{\alpha\beta 3} \epsilon^{ij}$$

$N_f=3$ (Color-flavor locking [CFL])

$$\langle q_{\alpha i} C \gamma_5 q_{\beta j} \rangle = \Delta_{CFL} \epsilon_{\alpha\beta I} \epsilon^{ijI} \propto \Delta_{CFL} \left(\delta_{\alpha}^i \delta_{\beta}^j - \delta_{\alpha}^j \delta_{\beta}^i \right)$$

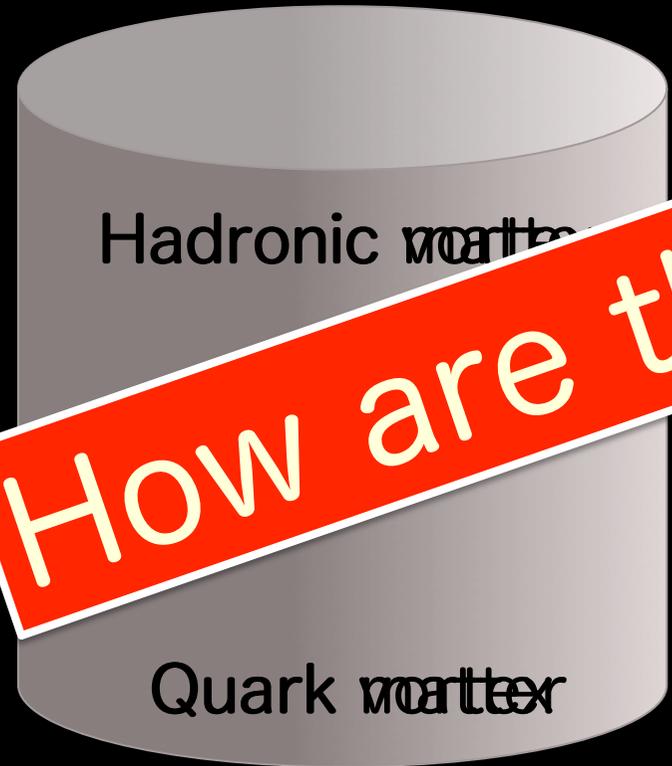
“Anomaly driven critical point in high density QCD as a realization of quark-hadron continuity”



Yamamoto, Hatsuda, Baym & Tachibana, PRL 97 ('06)

Thought experiment

Pour quarks
into



How are they connected?

Upper: Hadronic matter
Lower: Quark matter

Then, rotate the bucket

Upper: Hadronic vortex
Lower: Quark vortex

Notations

$$\Phi(\vec{r}, t) = |\Phi(\vec{r})| e^{i\phi(\vec{r}) - i\mu t} : \text{complex scalar}$$

$$\phi(\vec{r}, t) = p_\nu x^\nu = \vec{p} \cdot \vec{r} - \mu t, \quad \vec{v} = \frac{\vec{p}}{\mu}$$

$$C \equiv \oint_C \vec{v} \cdot d\vec{l} = 2\pi \frac{\nu}{\mu} \quad \nu : \text{winding \#}$$

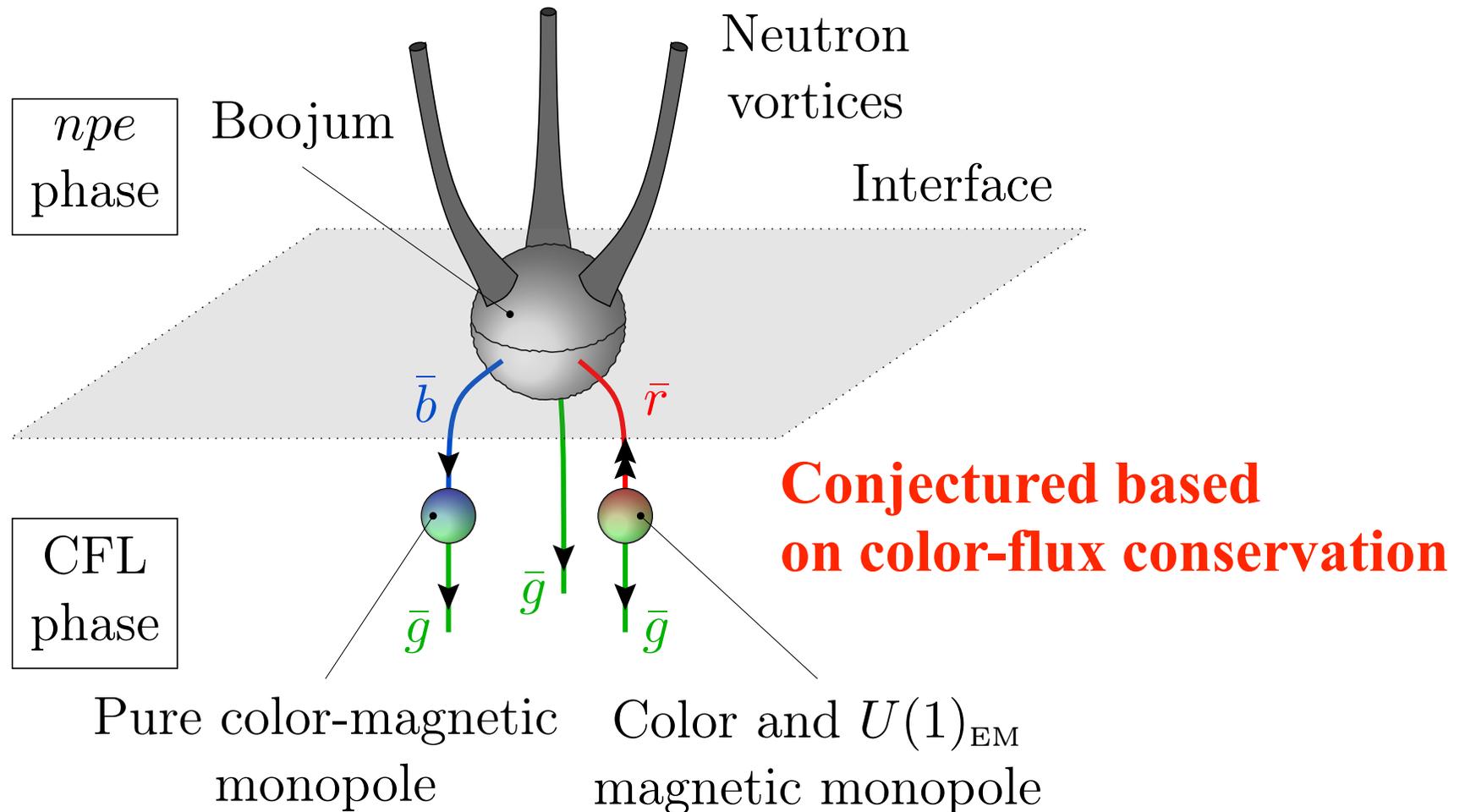
Circulation matching

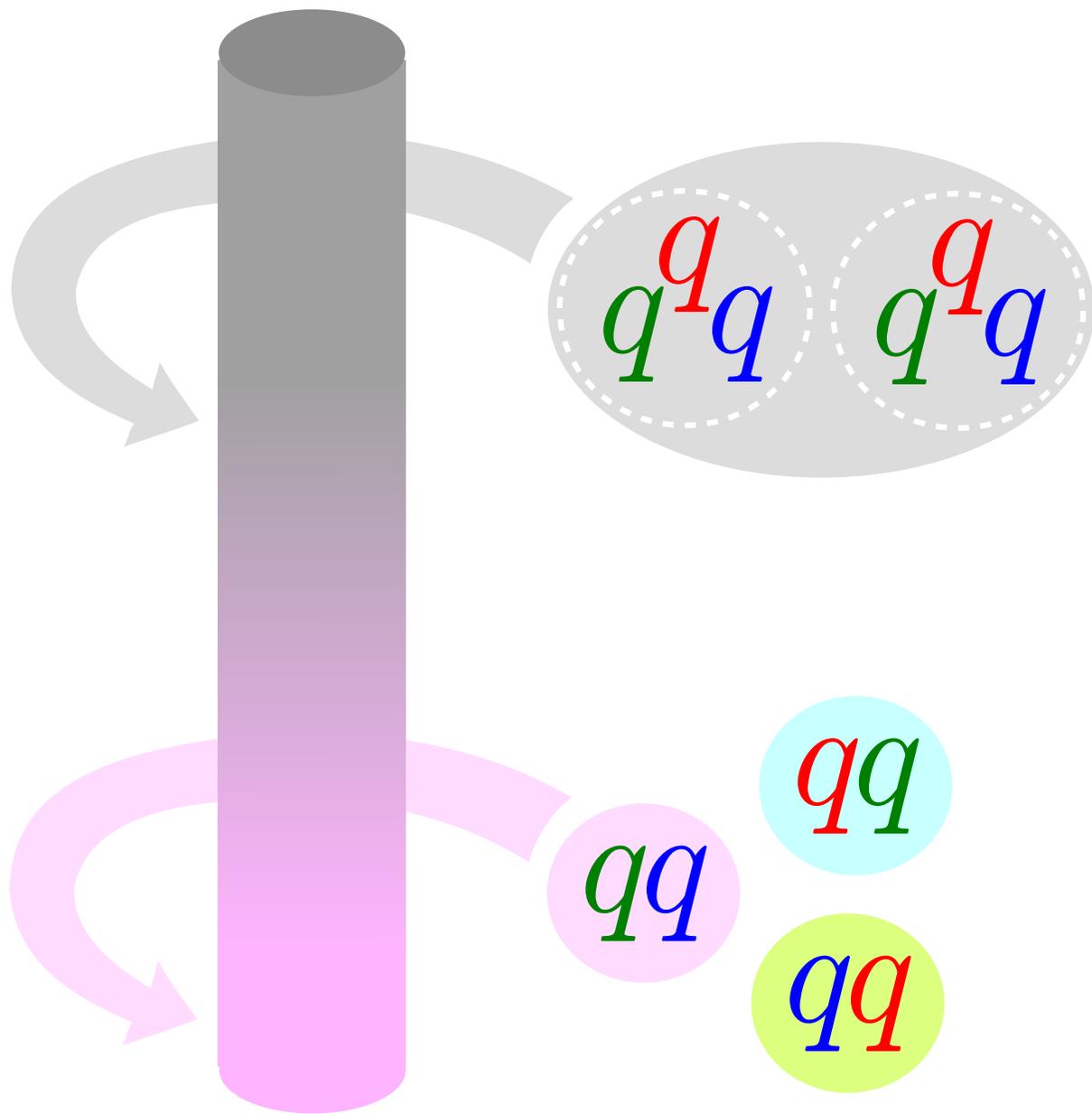


Boojum
(Wikipedia)

Non-Abelian CFL vortex

Cipiriani-Vinci-Nitta (2012)





Some more details

Color and flavor 3plet diquark order parameter

$$\hat{\Phi}^{\alpha i} \propto \varepsilon^{\alpha\beta\gamma} \varepsilon^{ijk} q_{\beta j} C \gamma_5 q_{\gamma k}$$

Φ diagonalized via color-flavor space rotation

$$\Phi = \begin{pmatrix} \Phi^{\bar{r}\bar{u}} & 0 & 0 \\ 0 & \Phi^{\bar{g}\bar{d}} & 0 \\ 0 & 0 & \Phi^{\bar{b}\bar{s}} \end{pmatrix}$$

Some more details

In this case,

$$C_{NA} = 2\pi \frac{\frac{1}{3} v_{NA}}{2\mu_q} = 2\pi \frac{v_{NA}}{2 \cdot 3\mu_q} = 2\pi \frac{v_{NA}}{2\mu_B}$$



$$v_H = v_{NA}$$

Some more details

What we (Alford *et al.*) discussed:

$$\begin{aligned}\hat{\Upsilon}^{ijk}(\vec{r}) &\equiv \frac{1}{6} \epsilon_{\alpha\beta\gamma} \hat{\Phi}^{\alpha i} \hat{\Phi}^{\beta j} \hat{\Phi}^{\gamma k} \\ &= \frac{1}{3} \epsilon^{kmn} (C\gamma_5)_{ab} \hat{B}_m^{i a} \hat{B}_n^{j b}\end{aligned}$$

6 quark objects = 3 diquarks = 2 baryons

Quantum numbers match

Non-Abelian vortices = Flavor singlet + Non-singlets
($\sim \Lambda\Lambda$)

Other topics

1. Flavor symmetry (breaking)

Flavor structure of vortices
(e.g. Neutron superfluidity)

MWHC theorem

2. Vortex interaction

Ginzburg-Landau analysis

Summary of today's talk

- ① Quark-hadron continuity hypothesis
(Fradkin/Shenker=Schafer/Wilczek)
- ② Study of superfluid vortices in light
of QH continuity
- ③ Possible applications of our findings

Thank you !

感謝！