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Active Control of Viscoelastic Metamaterials

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Over the last decade there has been significant interest in the design and production of acoustic metamaterials with physical qualities not seen in naturally occurring media. Progress in this area has been stimulated by the desire to create materials that exhibit novel behaviour when subject to acoustic waves, such as negative refraction or the appearance of band gaps in the frequency response of the material. Proposed designs range from locally resonant phononic crystals to arrays of Helmholtz resonators within ducts and past research has investigated both passive and active materials. In this current research program a 1-dimensional active acoustic metamaterial is being investigated. The active materials is derived from a passive, Helmholtz resonator based design, where the applied control forces produce controllable double negative behaviour. A controller has been designed using both manual, heuristic techniques and using an H-infinity optimisation algorithm to attempt to maximise the region at which attenuation from the low frequency band gap occurs. The optimised controller architecture is being studied to gain insight into the structure a practical controller may take, allowing the active design process to be improved, and informing the design of high performance passive metamaterials.

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Designing the Active Metamaterial

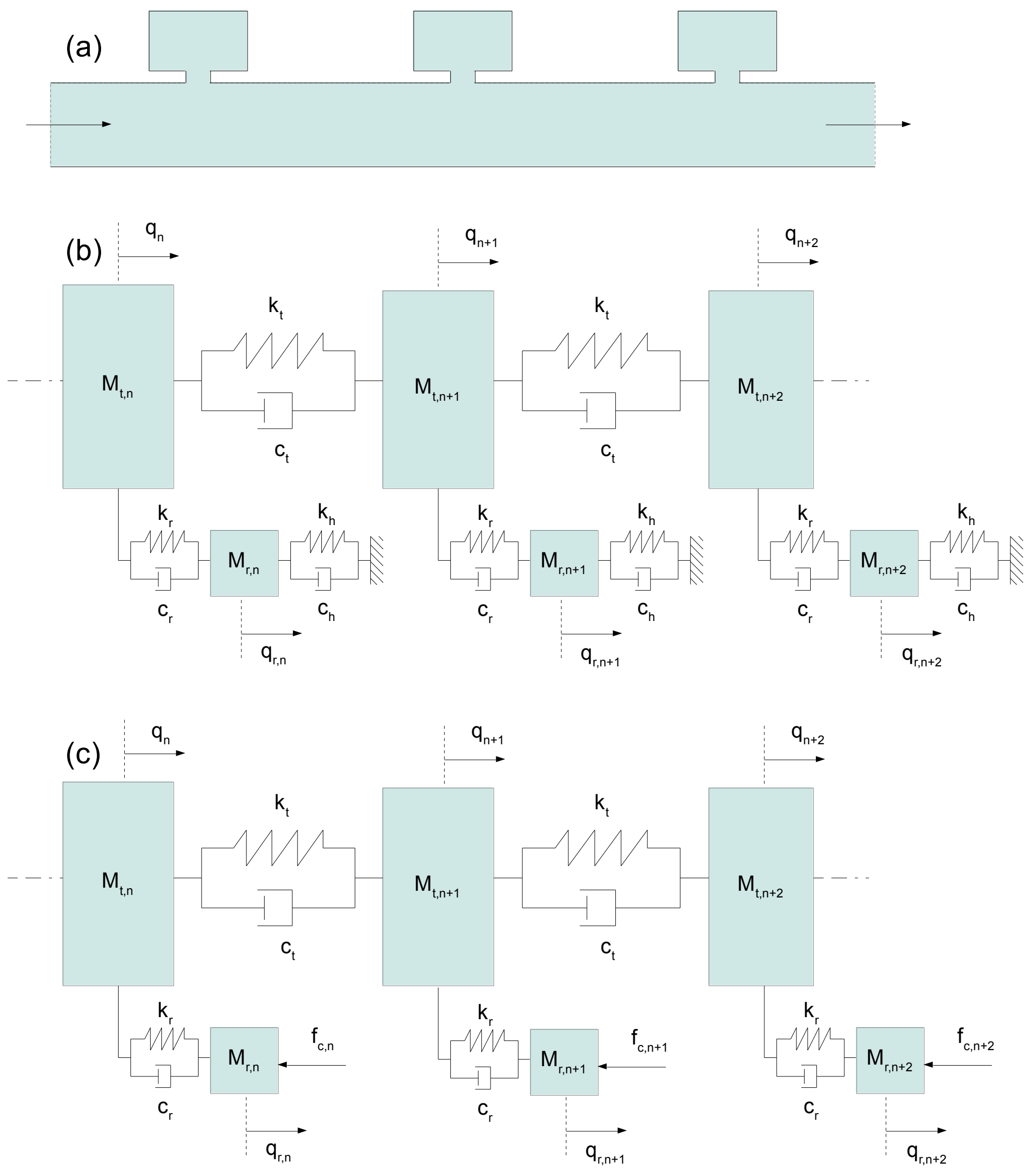

Figure 1 – An acoustic duct fitted with Helmholtz resonators (a), a lumped parameter, viscoelastic equivalent (b) and an active metamaterial based on these passive designs

- A viscoelastic metamaterial model, Figure 1(b), was designed based on an acoustic metamaterial consisting of a duct fitted with a periodic array of Helmholtz resonators, Figure 1(a), which can exhibit –ve effective mass (M_e).
- By removing the earth connections and replacing them with active feedback forces, an active metamaterial was created, Figure 1(c).
- The active metamaterial emulates the passive, material by applying a 'skyhook' active feedback force such that $f_c = kx_{r,n} + c\dot{x}_{r,n}$.
- Using a 'parallel coupled' force $f_c = k_c\left(x_{t,n-1} + x_{t,n+1} - 2x_{r,n}\right)$, the material can become 'double negative' (-ve M_e and -ve effective stiffness, K_e)

$$M_e = m_t + \frac{m_r(c_r i\omega + k_r)}{-m_r\omega^2 + (2c_c + c_r)i\omega + 2k_c + k_r}$$

$$K_e = k_t + \frac{m_r c_c c_r \omega^4 + (-m_r k_c k_r + 2c_c^2 k_r + c_r^2 k_c)\omega^2 + k_c k_r(2k_c + k_r)}{m_r^2\omega^4 + (-2m_r(2k_c + k_r) + (2c_c + c_r)^2)\omega^2 + (2k_c + k_r)^2}$$

- Double negative materials exhibit to negative refraction, and could be used in an acoustic cloak or sub-wavelength lens.
- The material has a low frequency band gap – where wave transmission is blocked – and could be used as a high performance vibration or acoustic isolator.

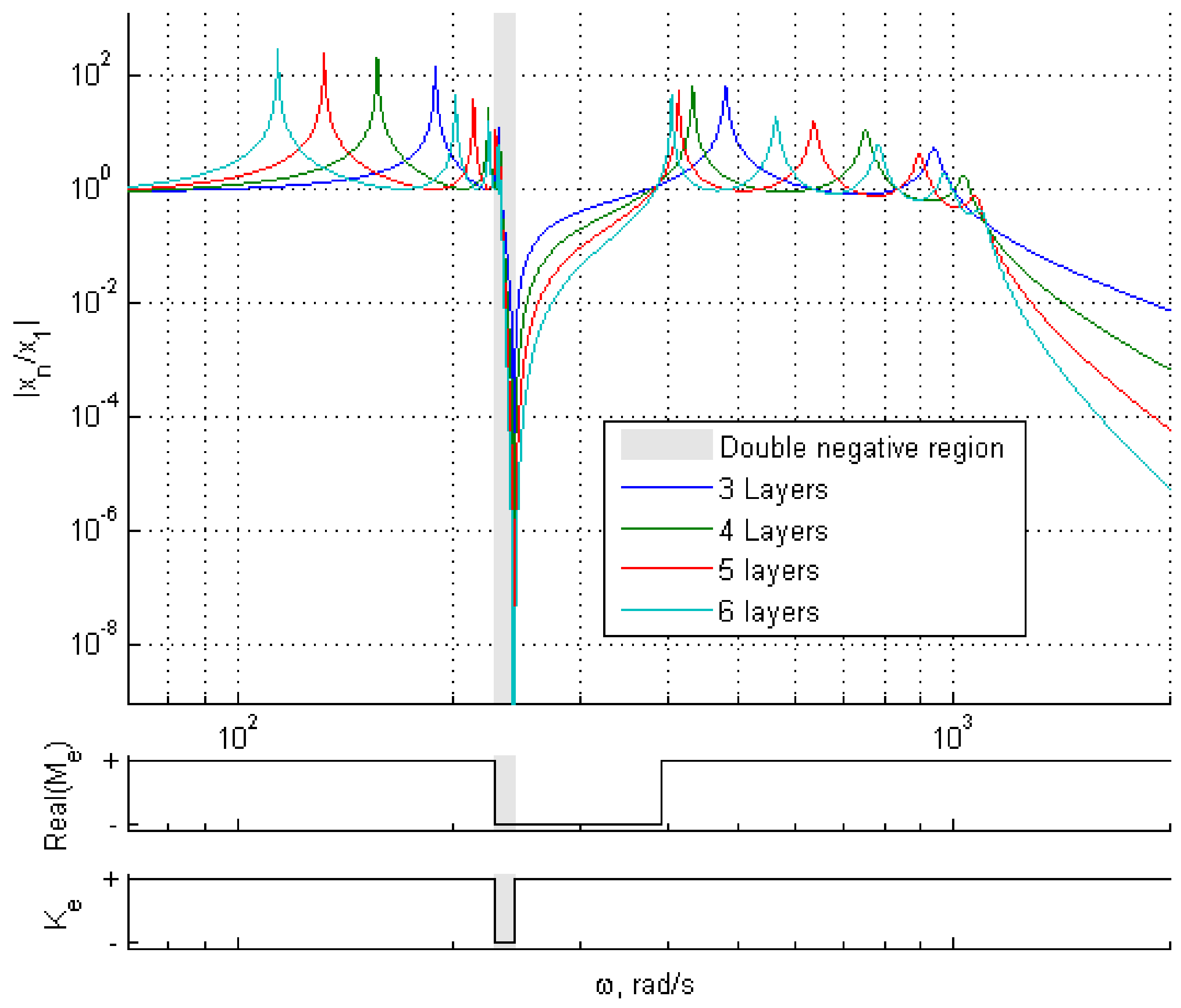

Figure 2 – The transmission response of the 'parallel coupled' active metamaterial and regions where the effective material parameters become negative

Widening the Band Gap

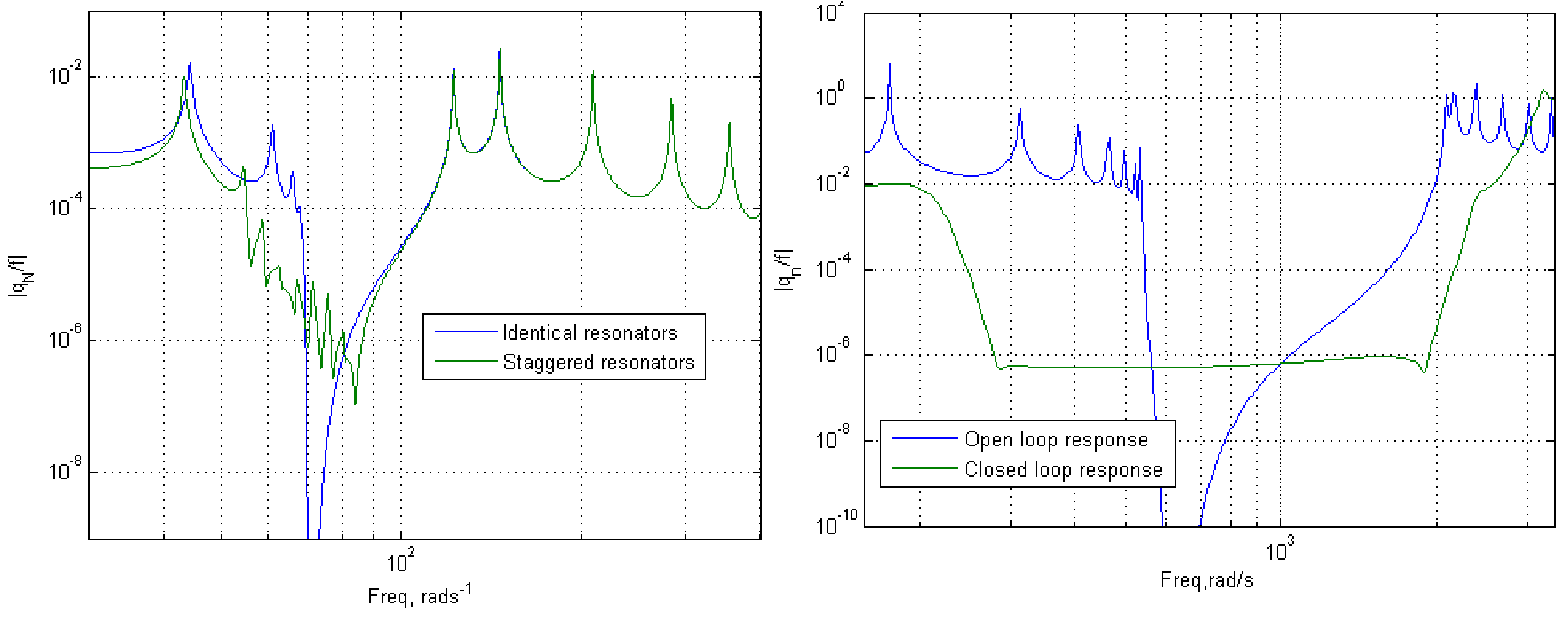

Figure 3 – (a) A 9 layer skyhook material with an enhanced bang gap width (left).(b) A nine layer material with a controller designed using H-infinity optimisation (right)

- The low frequency band gap is deep, asymmetrical and narrow, and the addition of extra layers makes the region of attenuation deeper, but not wider.
- In the 'skyhook' material the band gap can be widened by heuristically adjusting $k_{c,n}$, 'staggering' the resonators in frequency, Figure 3(a).
- Alternatively, a controller is designed by applying H-infinity optimisation to a state space model of the material to maximise the low frequency band gap performance, Figure 3(b).
- The resulting MIMO, frequency dependent gain controller provides results where the region of attenuation associated with the low frequency band gap is enhanced greatly, outperforming the local, static 'skyhook' controllers
- These preliminary results validate the efficacy of the approach, and work into investigating the structure of the optimised controllers is on going.

Analysing The Controller

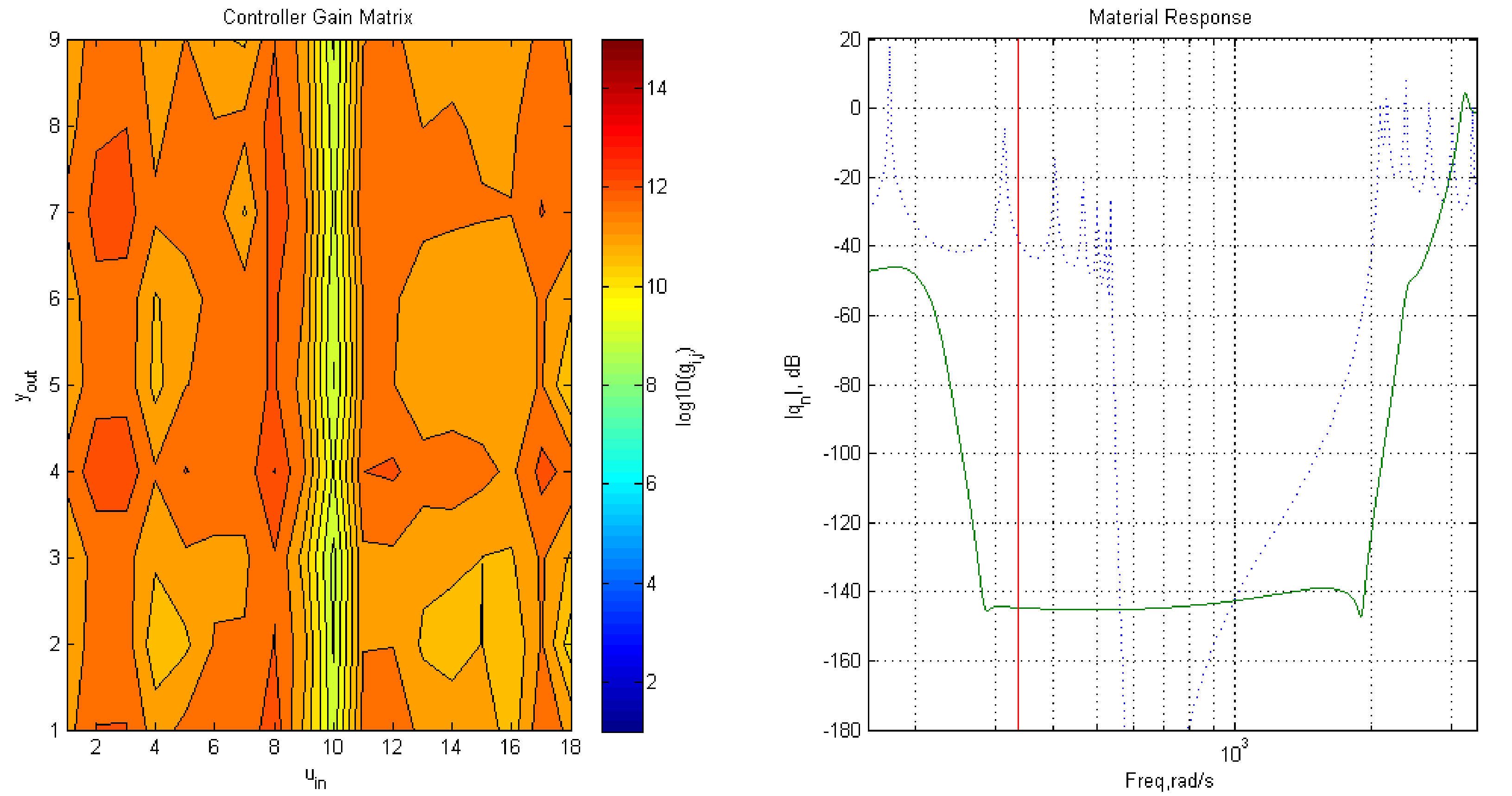

Figure 4 – A graphical representation of the 18x9 controller gain matrix (left) at a given frequency (red line on the material response, right). Hotspots denote a high degree of localisation within the optimised control loop

- Examining the controller gain matrix provides information on the degree of localisation of the controller at a given frequency
- Technique may be used to improve or simplify the controller architecture
- Can also assist the design of complex, passive metamaterials with wideband low frequency band gaps

Future Work

- Examine the structure of the optimised controllers to determine which control loops contribute most to the band gap performance, reducing the complexity of the controller and informing the design of passive materials

- Develop the optimisation technique, potentially using alternative algorithms more suited to the model, and investigate increasing the width of the double negative region.

- Develop experimental demonstrators to validate the concepts investigated thus far