FLUID STRUCTURE INTERACTIONS RESEARCH GROUP

UNIVERSITY OF Southampton
School of Engineering Sciences

Investigations on power flow analysis of nonlinear dynamic systems and applications

Jian Yang– jy1g09@soton.ac.uk –FSIRG, School of Engineering Sciences
Supervisors – **Dr. Ye Ping Xiong** and **Prof. Jing Tang Xing**

Background

- Predicting the dynamic responses of complex systems, such as aircrafts, ships and cars, to high frequency vibrations is a difficult task. Addressing such problems using Finite Element Analysis (FEA) leads to a significant numerical difficulty.
- Power flow approach (PFA) provides a powerful technique to characterise the dynamic behaviour of various structures and coupled systems, based on the universal principle of energy balance and transmission.
- PFA is extensively studied for linear systems, but much less for nonlinear systems, while many systems in engineering are inherently nonlinear.

Aims

- Reveal energy generation, transmission and dissipation mechanisms in nonlinear dynamic systems.
- Develop an effective PFA method for nonlinear vibrating systems .
- Apply PFA to vibration analysis and control of marine appliances, such as comfortable seat and energy harvesting device design.

Fig. 1 Nonlinear seat suspension system

Fundamental Theory

Dynamic equation for a single degree-of-freedom system:

$$m\ddot{x} + c(x,\dot{x})\dot{x} + k(x)x = f\cos\omega t$$

Equation of energy flow balance:

$$m\ddot{x}\dot{x} + c(x,\dot{x})\dot{x}^2 + k(x)x\dot{x} = f\dot{x}\cos\omega t$$

$$\dot{T} \quad + \quad \dot{D} \quad + \quad \dot{U} \quad = \quad P$$

Kinetic energy change rate | Dissipated Power | Potential energy change rate | Instantaneous input power

Corresponding energy variables: kinetic energy T, dissipated energy D, potential energy U and input energy P can be obtained by integration with respect to time. Mechanical energy and its change rate can be calculated by summing up kinetic and potential energy , kinetic and potential energy change rate, respectively.

Typical nonlinear dynamic systems

Van der Pol(VDP) oscillator -Nonlinear damping

$$\ddot{x} + \alpha(x^2 - 1)\dot{x} + x = \gamma\cos\omega t \qquad (1)$$

Duffing oscillator -Nonlinear stiffness

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma\cos\omega t \qquad (2)$$

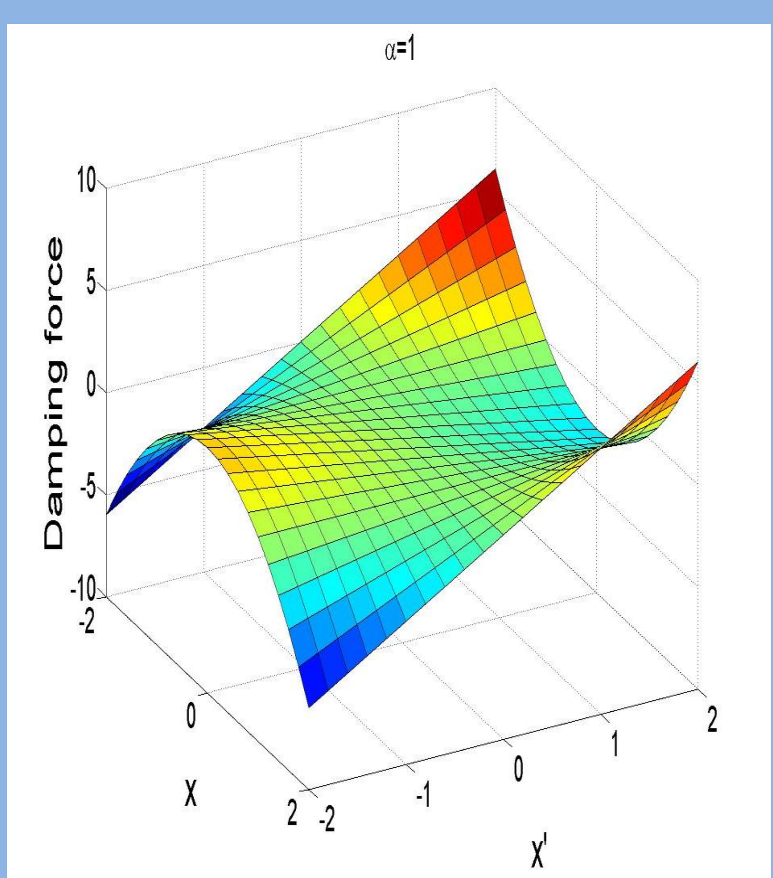

Fig.2 Nonlinear damping property of VDP oscillator

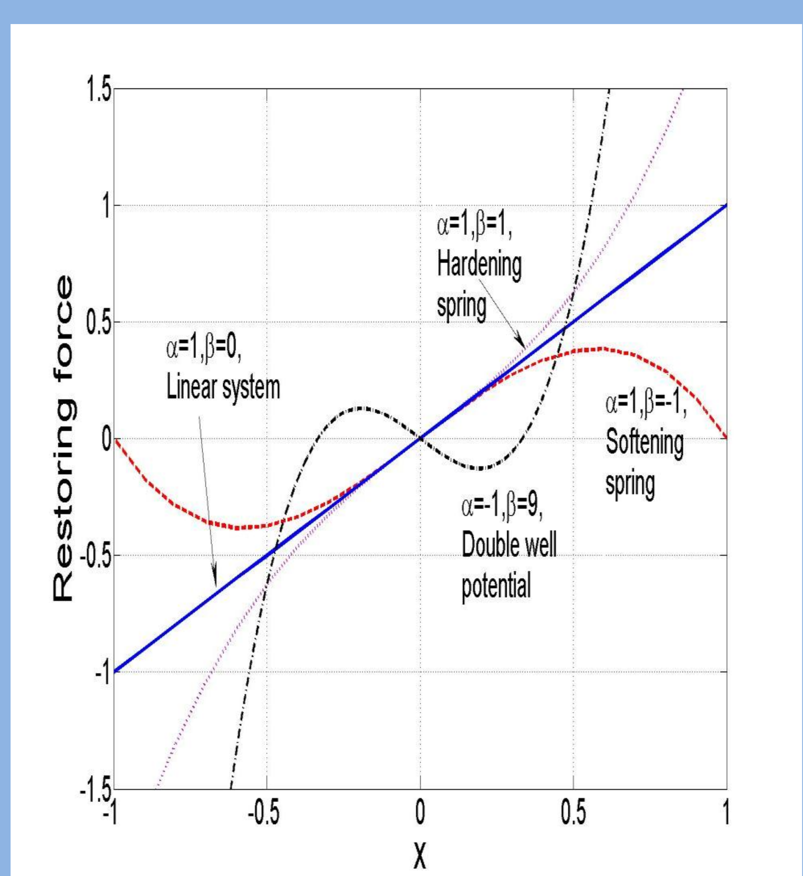

Fig.3 Nonlinear stiffness property of Duffing oscillator

Transient power flow characteristics of VDP oscillator

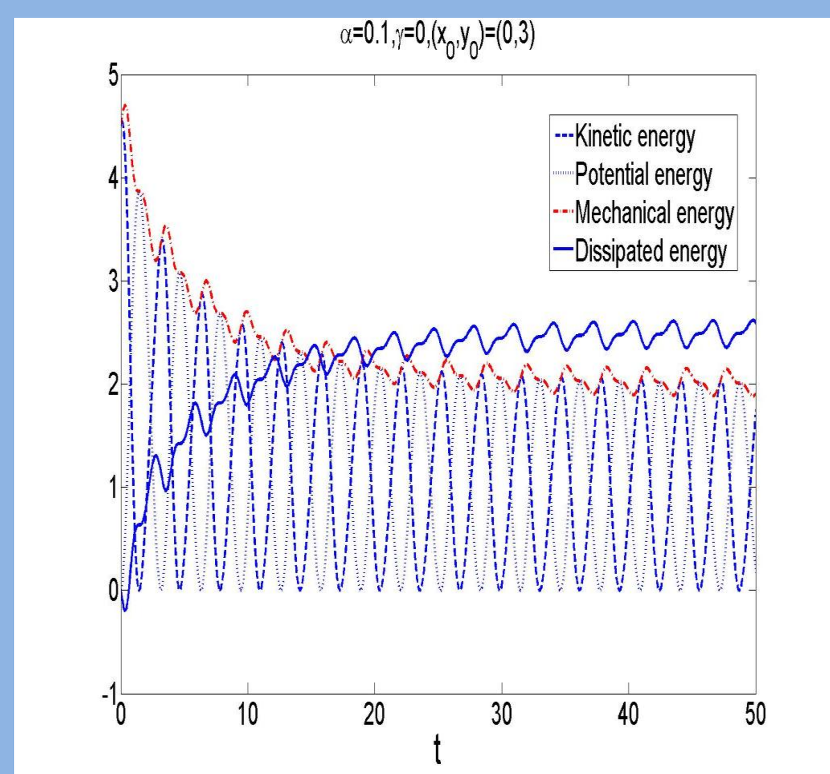

Fig.4 Energy exchange in unforced VDP oscillator

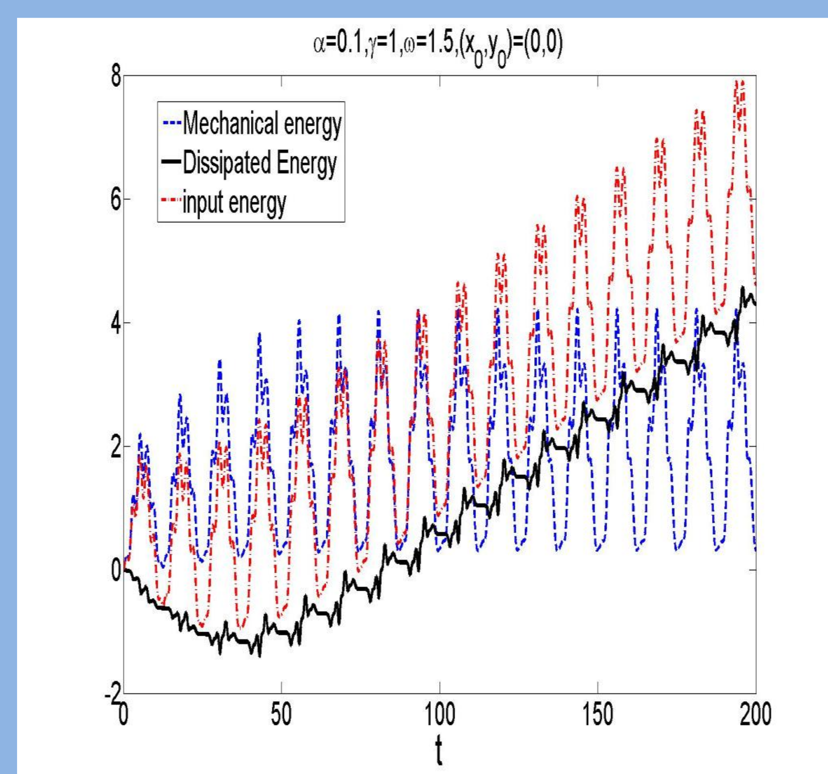

Fig.5 Energy exchange in forced VDP oscillator

Transient power flow characteristics of Duffing oscillator

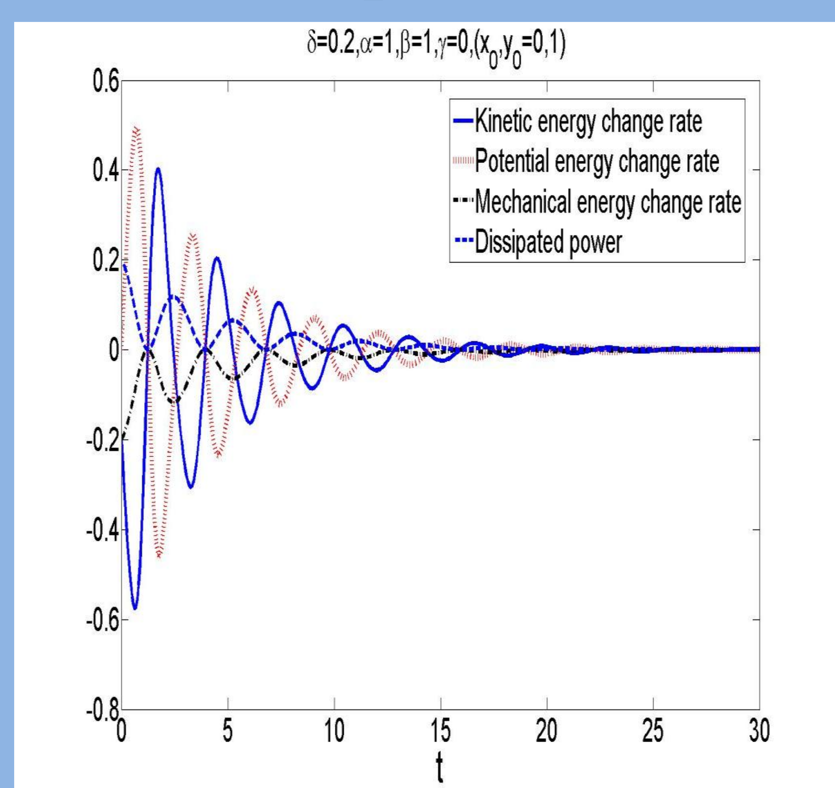

Fig.6 Energy exchange rate in unforced Duffing oscillator

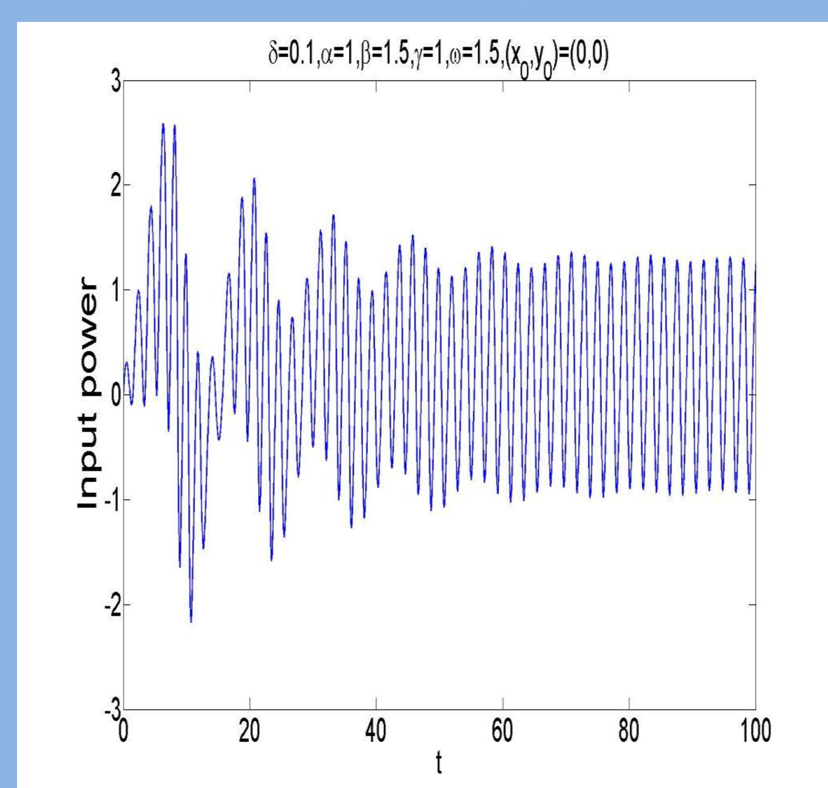

Fig.7 Input power of forced Duffing oscillator

Discussion

- Unlike unforced linear system with damping, for unforced VDP oscillator, some amount of mechanical energy remains in the system after reaching steady state.
- The energy exchange in forced VDP oscillator shows that dissipated energy can be negative, which is different from linear system.
- The time series of power variables of forced oscillation of Duffing oscillator shows the system will exhibit steady state after a relatively short initial stage. Its behaviours need further investigation.

Future work

- To study steady state power flow characteristics of nonlinear oscillators;
- To develop and simplify the power flow formulation for nonlinear systems ;
- Apply nonlinear power flow theory to engineering applications.