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Linear Interaction between Compressibility and a Gravity Field for Barotropic Fluids

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Background

Geophysical modelling have to deal with the propagation of sometimes small disturbances in compressible fluids subject to a gravity field. The gravity-compressibility interaction is governed by the parameter $H = c^2/g$ where c is the sound phase velocity and g the gravity acceleration. Sound waves are affected by gravity only when the sound wave lengths are of the order of magnitude of this parameter. On Earth, H is circa 12 km (air) and 230 km (water). This corresponds to wave periods around 35 s for air and 150 s for water.

In the Earth's atmosphere, such low frequency sounds exist. They are created by remote volcanic or man-made explosions, meteor entry, earthquakes, tsumanis, boreal aurora, mountain waves, severe weather, ocean surface waves (resulting in what is called microbaroms), aircraft or spacecraft sonic bangs. Significant levels of unknown origin are also measured. Specialists of this field call that "the infrasound zoo", cf. figure 1.

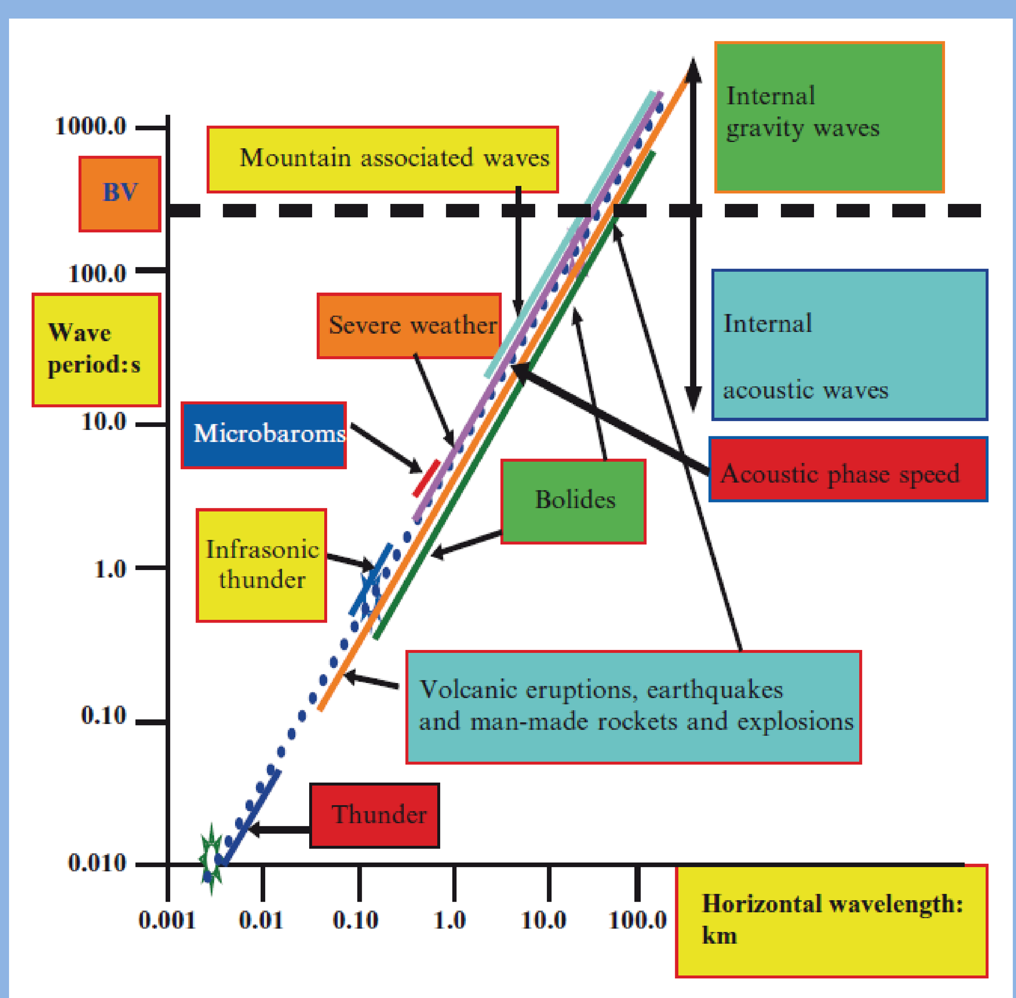

Figure 1: Atmospheric wave sources and regimes (figure 4 of [1] chapter 11)

Purpose

The purpose of this study is to investigate specifically the gravity-compressibility interaction. Free-surface, and more generally buoyancy effects, are removed from the problem.

Method

The procedure followed for this investigation is very classical:

1) Establishment of the governing equations in the acoustic approximation, i.e. of the linear potential-flow problem,
2) Solution of the fundamental function problem.

A consequence of the acoustic approximation is that the fluid has to be (considered as) barotropic, i.e. the state equation is a relation between the density and the pressure only, hence the title of this study. The variables intervening in the governing equations are then only the density ρ, the pressure p and the velocity potential φ.

Results

A curiosity encountered in this problem is that the field equations expressed for p and φ are different. A change of variables defined by Eckart [2] solve this problem: For all the new variables, the field equation is

$$\left(\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x_i^2}+\mu^2\cdot\right)(*)=0$$

(the x_i are the Cartesian coordinates and $\mu = g / 2\, c^2$). This is a Klein-Gordon equation whose time-domain fundamental solution is

$$F=-\frac{1}{4\pi\cdot c^2}\cdot\left(\frac{\delta(t-\frac{r}{c})}{r}+\mu\cdot H(t-\frac{r}{c})\cdot\frac{J_1(c\cdot\mu\cdot(t^2-\frac{r^2}{c^2})^{\frac{1}{2}})}{(t^2-\frac{r^2}{c^2})^{\frac{1}{2}}}\right).$$

This fundamental solution is the usual wave equation followed by a tail

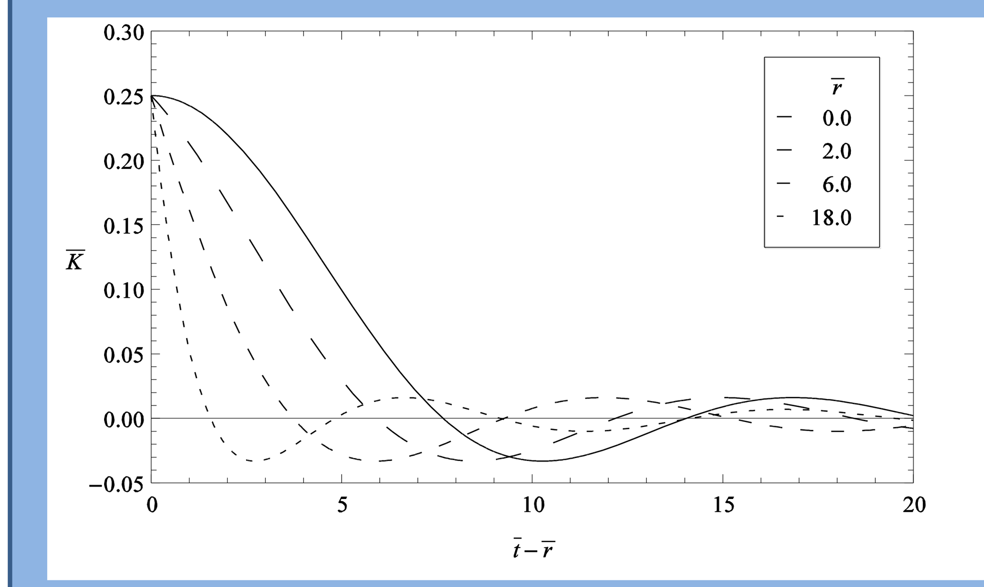

Figure 2: The tail function

shown on figure 2: The tail's shape varies with the distance from the source. In the frequency domain, this solution results in a cut-off period T_a above which sound waves do not propagate. T_a are circa 440 s for air and 1920 s for water. This does not prevent other types of waves of longer periods like gravity waves to propagate.

Is this useful?

The International Monitoring System of the Comprehensive Nuclear-Test-Ban Treaty

The Comprehensive Nuclear-Test-Ban Treaty (CTBT), opened for signature in 1996, established an International Monitoring System (IMS) to check the possible occurrence of (small) nuclear explosions. Four techniques are used in the IMS: seismological, hydroacoustic, infrasound and radionuclide measurements. The infrasound array consists of 60 stations covering the Earth surface shown on figure 3.

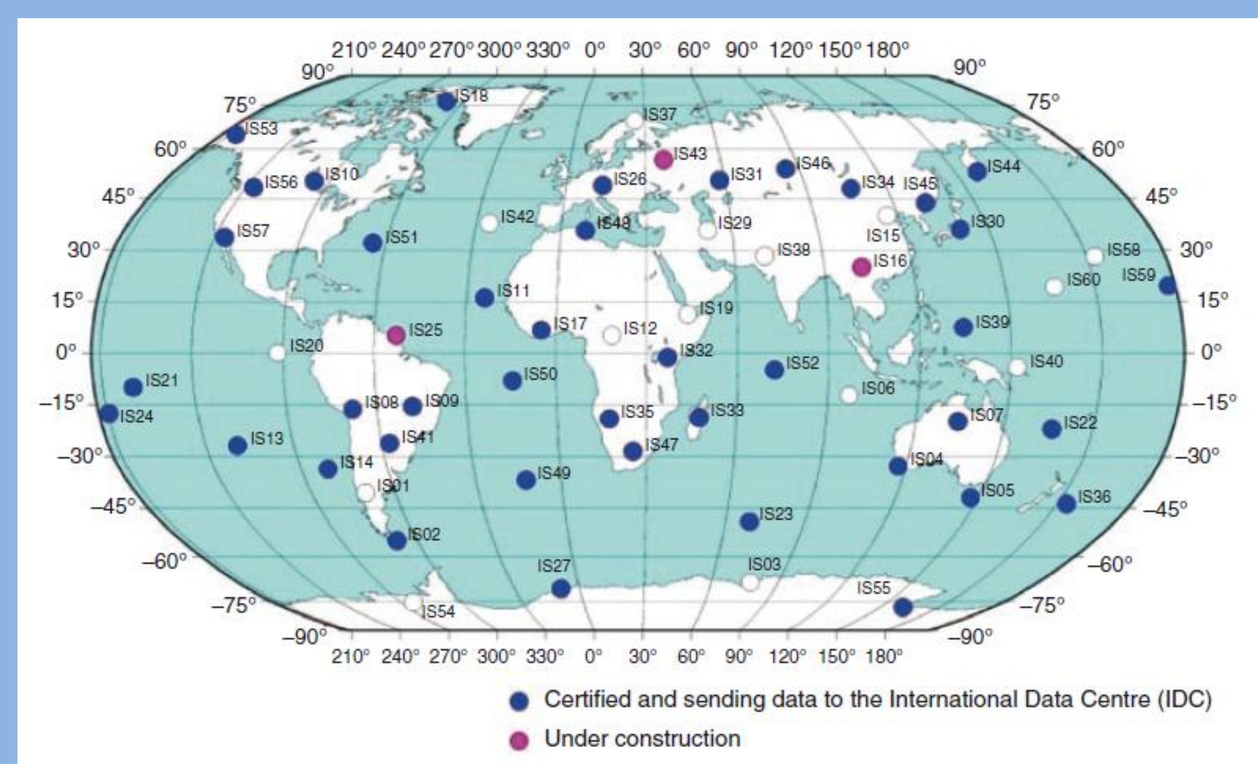

Figure 3: The 60-station International Monitoring System (IMS) infrasound network (figure 1 of [1] chapter 2)

Detection, discrimination and classification of sources are fundamental issues for the IMS. The current measurement analysis technique (PMCC) assumes that infrasounds propagate like usual sound waves. Due to the gravity-compressibility interaction, this is no longer the case for periods around and greater than 35 s. Taking into account this interaction could improve the accuracy of the analysis. As the distance from the source intervene in the shape of the time-domain fundamental solution, this analysis would also result in an estimation of the source distance.

Atmospheric Remote Sensing

Due to the high dependency of infrasound propagation on temperature and wind up to the anacoustic zone (≈ 160 km), cf. figure 4, infrasound measurements bring some information to atmospheric sensing for altitudes barely monitored. Any improvement in measurement analysis is welcomed, particularly for long period waves.

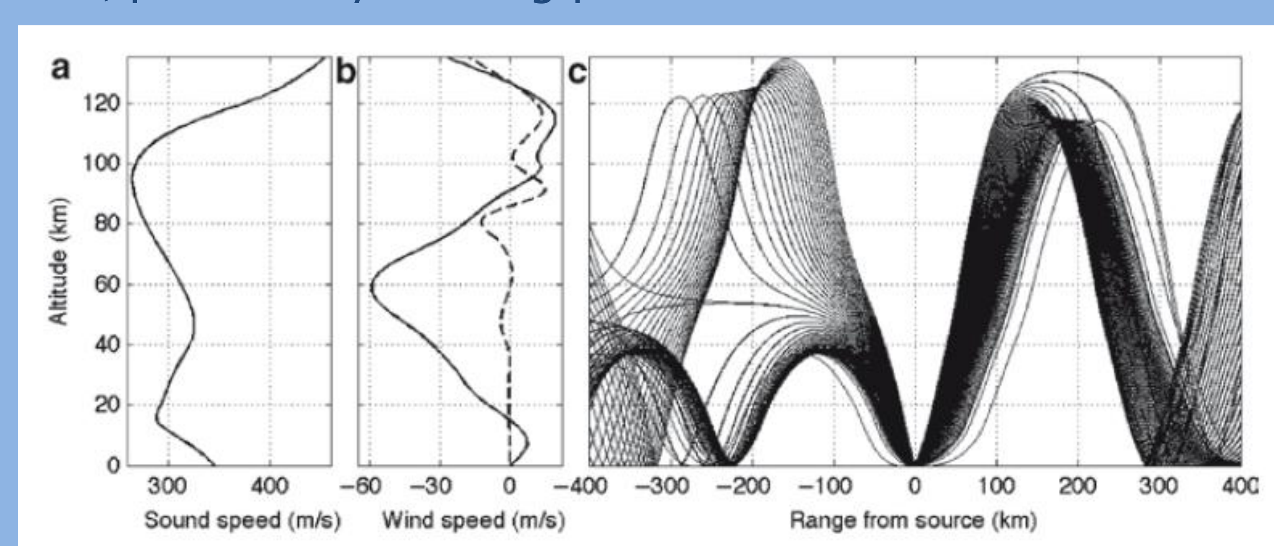

Figure 4 (a): Sound speed profile, **(b)** Zonal (black) and meridional (north-south, dotted) , **(c)** Ray diagram showing refraction of sound in an advected medium (figure 2 of [1] chapter 15)

References

[1] Le Pichon A. *et al.* (eds.) (2010) *Infrasound Monitoring for Atmospheric Studies*, DOI 10.1007/978-1-4020-9508-5_15, © Springer Science + Business Media B.V. 2010.

[2] Eckart, C. (1960) *Hydrodynamics of oceans and atmospheres*, Pergamon Press.