

Transport phenomena in superfluid matter in neutron stars

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Outline

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- ✧ EFT and superfluid phonons
- ✧ EoS for superfluid neutron star matter
- ✧ Shear viscosity and the r-mode instability window
- ✧ Bulk viscosity
- ✧ Thermal conductivity
- ✧ Summary

Manuel and Tolos, Physical Review D 84 (2011) 123007

Manuel and Tolos, Physical Review D 88 (2013) 043001

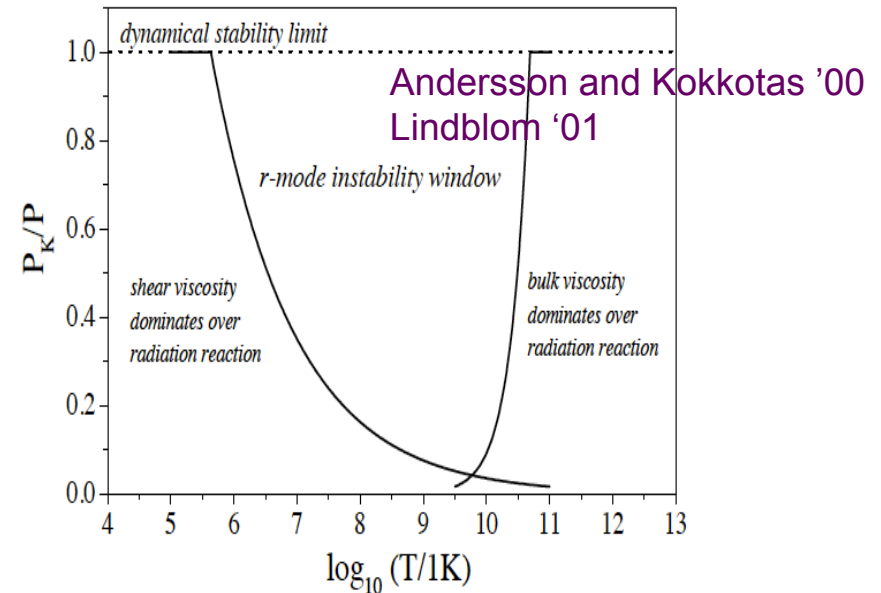
Manuel, Tarrus and Tolos, JCAP 1307 (2013) 003

Manuel, Sarkar and Tolos, Physical Review C 90 (2014) 055803

Motivation

To study transport phenomena to understand the dynamics of neutron stars

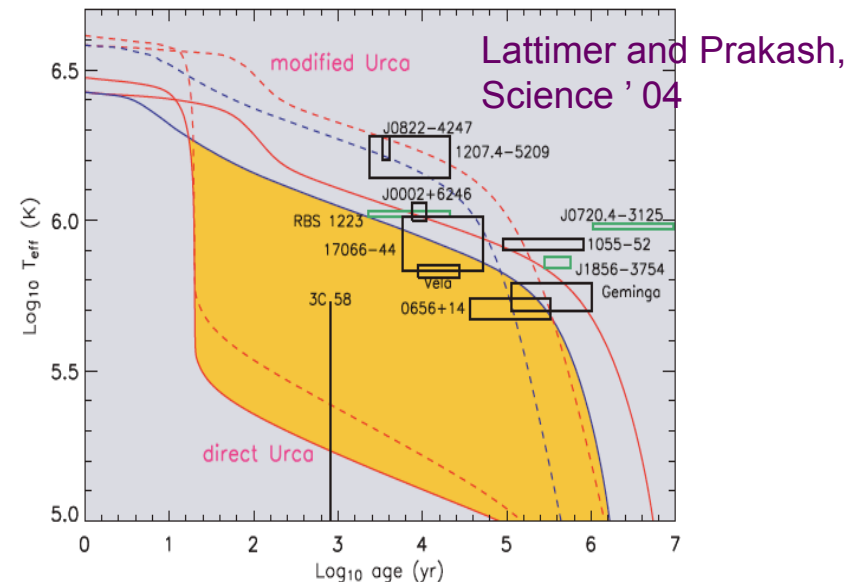
r-modes is one of the families of pulsation modes in rotating neutron stars. They **are unstable** via emission of gravitational waves. However, there are **damping mechanisms**, such as **shear and bulk viscous processes**, that may counteract the growth of an unstable r-mode



cooling of a neutron star depends on the rate of neutrino emission, the photon emission rate, the specific heat and the **thermal conductivity**

Pethick '92

Yakovlev and Pethick '04



EFT and superfluid phonon

Exploit the **universal character of EFT at leading order** by obtaining the effective Lagrangian associated to a superfluid phonon and implement the **particular features of the system**, associated to the coefficients of the Lagrangian, via the **EoS**

Son '02

Son and Wingate '06

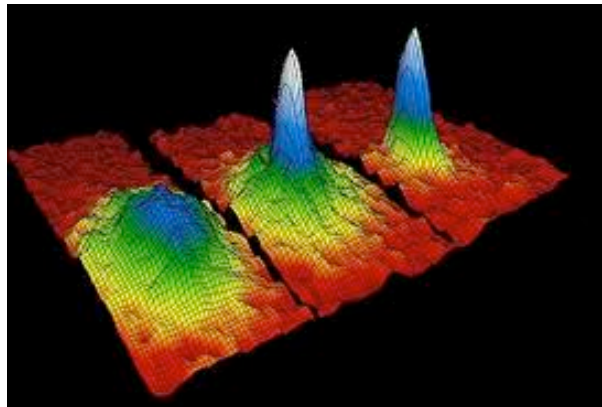
non-relativistic
case

$$\mathcal{L}_{\text{LO}} = P(X)$$

$$X = \mu - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m}$$

$P(\mu)$	pressure
μ	chemical potential
φ	phonon field
m	mass condense particles

Applicable in
superfluid systems
such as cold Fermi
gas at unitary, ^4He
or neutron stars



EoS for superfluid neutron star matter

In order to obtain the speed of sound at $T=0$ and the different phonon self-couplings one has to determine the **EoS for neutron matter in neutron stars**.

A common benchmark for nucleonic EoS is **APR98**

Akmal, Pandharipande and Ravenhall '98

which was later parameterized in a causal form Heiselberg and Hjorth-Jensen '00

$$E/A = \mathcal{E}_0 y \frac{y - 2 - \delta}{1 + \delta y} + S_0 y^\beta (1 - 2x_p)^2$$

$$y = n/n_0 \quad x_p = n/n_0$$

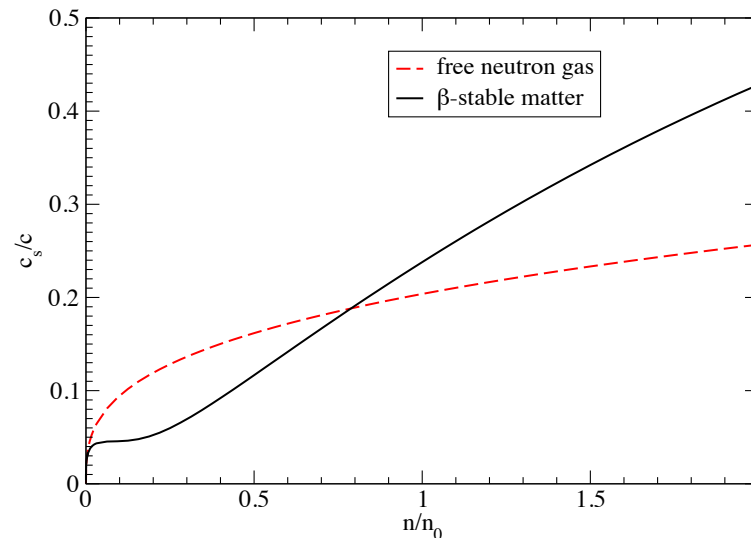
$$n_0 = 0.16 \text{ fm}^{-3}$$

$$\mathcal{E}_0 = 15.8 \text{ MeV} \quad \delta = 0.2$$

$$S_0 = 32 \text{ MeV} \quad \beta = 0.6$$

For β -stable matter made up of neutrons, protons and electrons, the speed of sound at $T=0$ is

$$\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$



Effective Lagrangian for superfluid phonon at LO

$$\begin{aligned}\mathcal{L}_{\text{LO}} = & \frac{1}{2}((\partial_t \phi)^2 - v_{\text{ph}}^2 (\nabla \phi)^2) - g((\partial_t \phi)^3 \\ & - 3\eta_g \partial_t \phi (\nabla \phi)^2) + \lambda((\partial_t \phi)^4 \\ & - \eta_{\lambda,1}(\partial_t \phi)^2 (\nabla \phi)^2 + \eta_{\lambda,2}(\nabla \phi)^4) + \dots\end{aligned}$$

with Φ the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the **speed of sound at T=0**

$$v_{ph} = \sqrt{\frac{\frac{\partial P}{\partial \mu}}{m \frac{\partial^2 P}{\partial \mu^2}}} = \sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$

and **derivatives with respect to mass density:**

Escobedo and Manuel '10

$$\begin{aligned}u &= \frac{\rho}{c_s} \frac{\partial c_s}{\partial \rho}, & w &= \frac{\rho}{c_s} \frac{\partial^2 c_s}{\partial \rho^2}, \\ g &= \frac{1 - 2u}{6c_s \sqrt{\rho}}, & \eta_g &= \frac{c_s^2}{1 - 2u}, & \lambda &= \frac{1 - 2u(4 - 5u) - 2w\rho}{24c_s^2 \rho}, \\ \eta_{\lambda,1} &= \frac{6c_s^2(1 - 2u)}{1 - 2u(4 - 5u) - 2w\rho}, & \eta_{\lambda,2} &= \frac{3c_s^4}{1 - 2u(4 - 5u) - 2w\rho}\end{aligned}$$

Results valid for neutrons pairing in 1S_0 channel and also valid for 3P_2 neutron pairing if corrections $\bar{\Delta}(^3P_2)^2 / \mu_n^2$ are ignored Bedaque, Rupak and Savage '03

Including NLO corrections in the phonon dispersion law

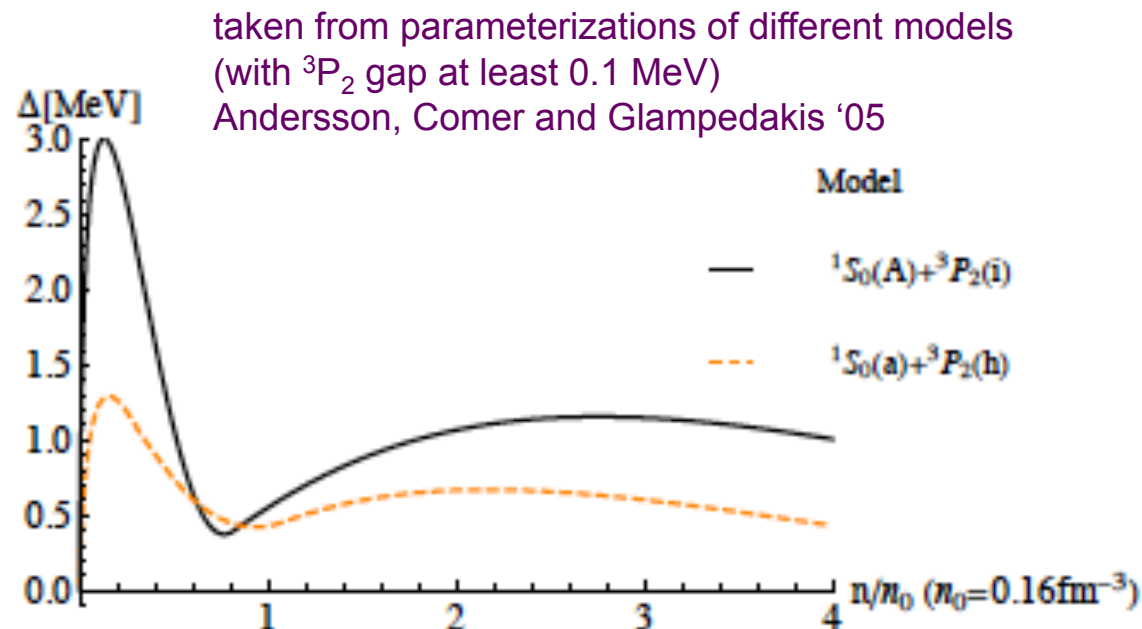
$$E_P = c_s p (1 + \gamma p^2)$$

$$\gamma = - \frac{v_F^2}{45\Delta^2}$$

v_F : Fermi velocity

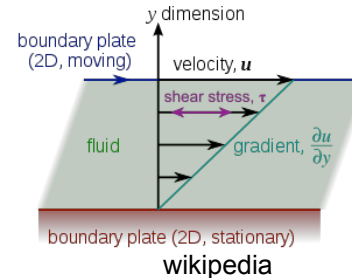
Δ : gap function

$\Upsilon < 0$: first allowed phonon scattering are binary collisions



Shear viscosity due to superfluid phonons

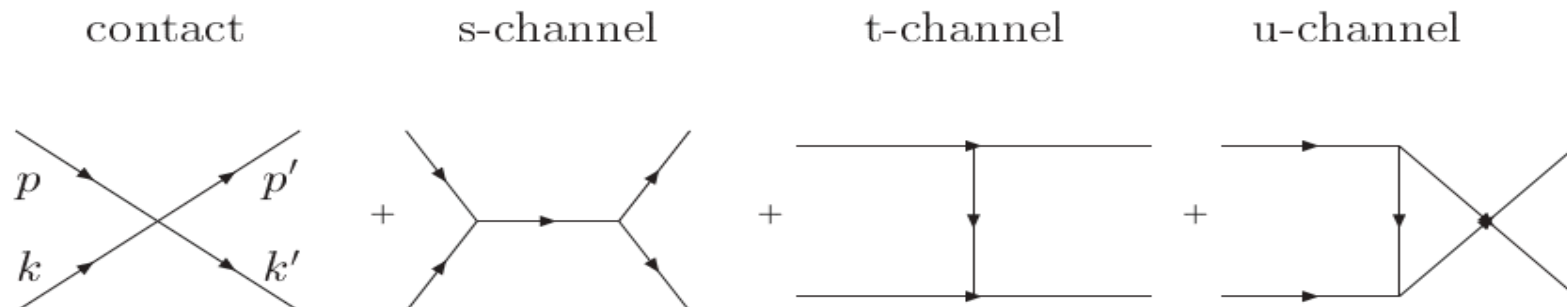
Shear viscosity



The [shear viscosity](#) is calculated using variational methods for solving the transport equation as

$$\eta = \left(\frac{2\pi}{15} \right)^4 \frac{T^8}{c_s^8} \frac{1}{M}$$

where M represents a multidimensional integral that contains the thermally weighted scattering matrix for phonons.



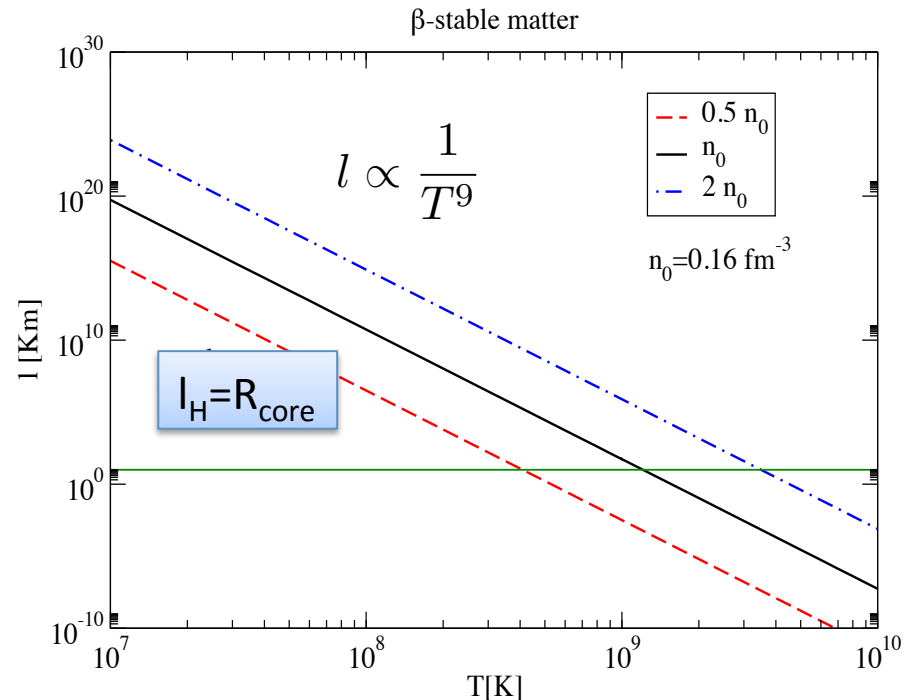
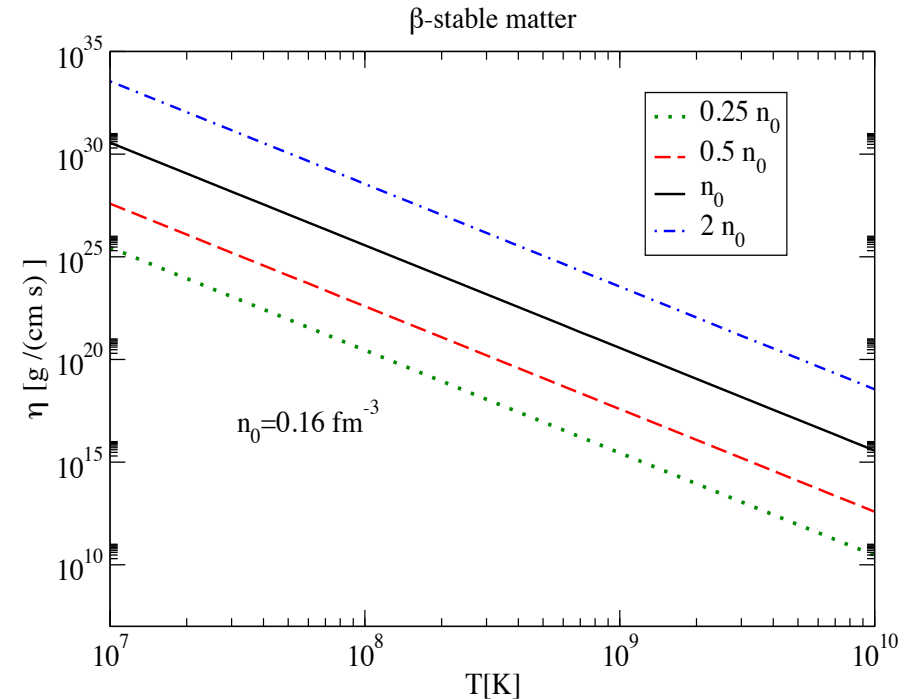
Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for ^4He and cold Fermi gas at unitary) while the coefficient **depends on EoS**.

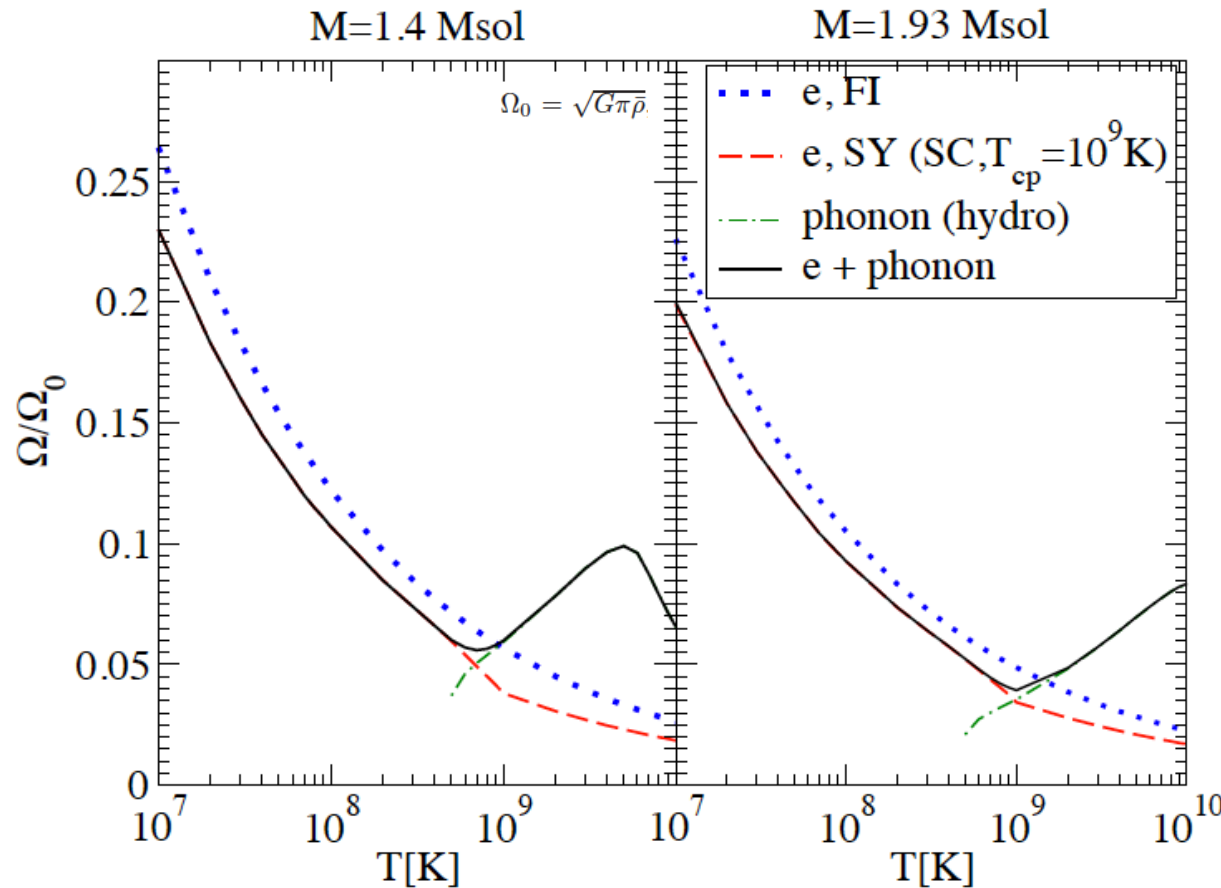
Mean free path of phonons: establish when phonons become hydrodynamic

$$l = \frac{\eta}{n \langle p \rangle}$$

$\langle p \rangle$: thermal average
 n : phonon density

Alford, Braby, and Mahmoodifar '10





r-mode instability window (only shear)

$$-\frac{1}{|\tau_{\text{GR}}(\Omega)|} + \frac{1}{\tau_{\eta}(T)_{\text{hydro}}} = 0$$

Dissipation due to superfluid phonons start to be relevant at $T \approx 7 \times 10^8$ K for $1.4 M_{\odot}$ and $T \approx 10^9$ K for $1.93 M_{\odot}$

$$\frac{1}{|\tau_{\text{GR}}(\Omega)|} = \frac{32 \pi G \Omega^{2l+2}}{c^{2l+3}} \frac{(l-1)^{2l}}{((2l+1)!!)^2} \left(\frac{l+2}{l+1} \right)^{2l+2} \int_0^R \rho r^{2l+2} dr$$

$$\frac{1}{\tau_{\eta}(T)_{\text{hydro}}} = (l-1)(2l+1) \int_{R_c}^R \eta r^{2l} dr \left(\int_0^R \rho r^{2l+2} dr \right)^{-1} \quad l=2 \text{ (dominant)}$$

Bulk viscosities due to superfluid phonons

The **bulk viscosity coefficients** are calculated from the dynamical evolution of the phonon number density* or, equivalently, by using the Boltzmann equation for phonons in the relaxation time approximation

$$\zeta_i(\omega) = \frac{1}{1 + \left(\omega I_1^2 \frac{\partial \rho}{\partial n} \frac{\partial \rho}{\partial \mu} \frac{T}{\Gamma_{ph}} \right)^2} \frac{T}{\Gamma_{ph}} C_i, \quad i = 1, 2, 3, 4$$

$$C_1 = C_4 = -I_1 I_2, \quad C_2 = I_2^2, \quad C_3 = I_1^2,$$

$$I_1 = \frac{60T^5}{7c_s^7\pi^2} (\pi^2\zeta(3) - 7\zeta(5)) \left(c_s \frac{\partial B}{\partial \rho} - B \frac{\partial c_s}{\partial \rho} \right),$$

$$I_2 = -\frac{20T^5}{7c_s^7\pi^2} (\pi^2\zeta(3) - 7\zeta(5)) \left(2Bc_s + 3\rho \left(c_s \frac{\partial B}{\partial \rho} - B \frac{\partial c_s}{\partial \rho} \right) \right),$$

NLO
corrections in
phonon
dispersion law

Three independent coefficients: $\zeta_1 = \zeta_4 \quad \zeta_1^2 \leq \zeta_2 \zeta_3 \quad \zeta_2, \zeta_3 \geq 0$

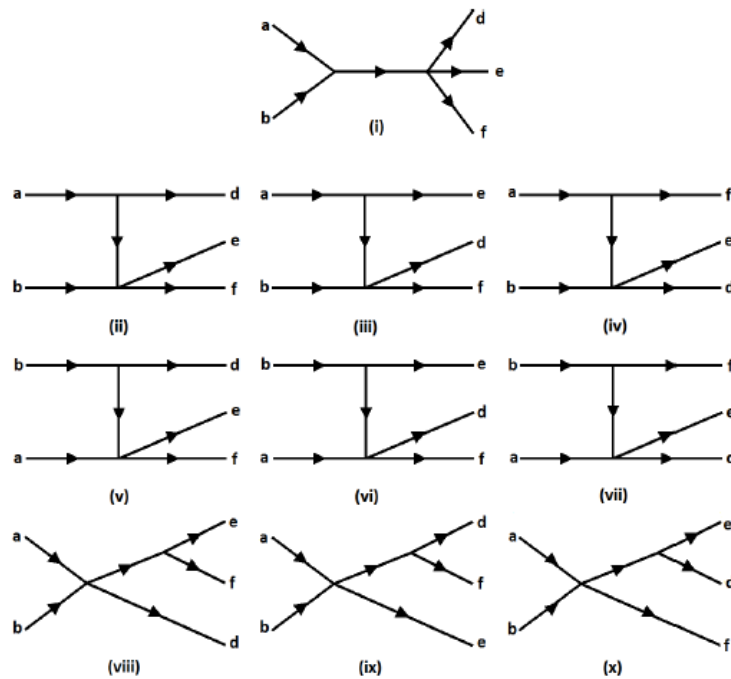
In the static limit

$$\zeta_i = \frac{T}{\Gamma_{ph}} C_i, \quad i = 1, 2, 3, 4,$$

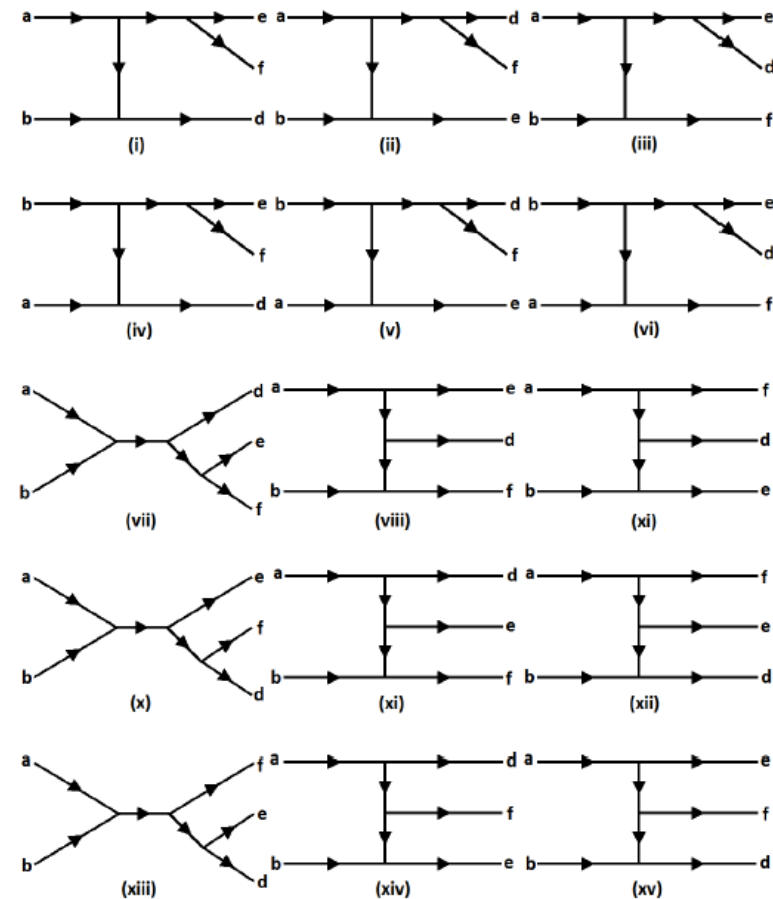
* Khalatnikov

Phonon decay rate for phonon number changing processes: $2 \leftrightarrow 3$

$$\Gamma_{ph} = \int d\Phi_5(p_a, p_b; p_d, p_e, p_f) \|\mathcal{A}\|^2 f(E_a) f(E_b) (1 + f(E_d)) (1 + f(E_e)) (1 + f(E_f))$$

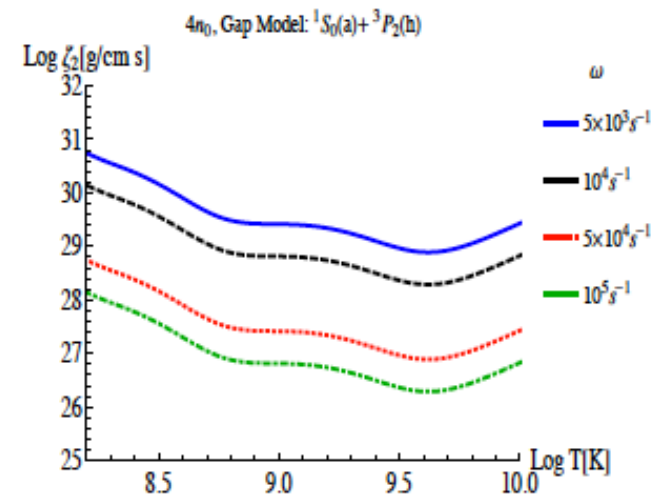
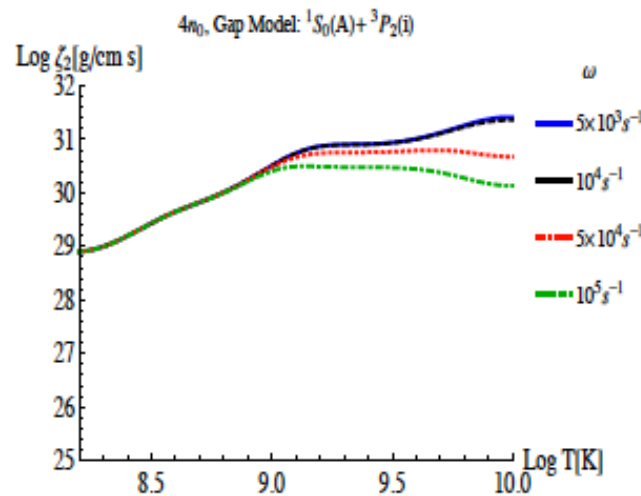


with one 4-phonon vertex
and one 3-phonon vertex

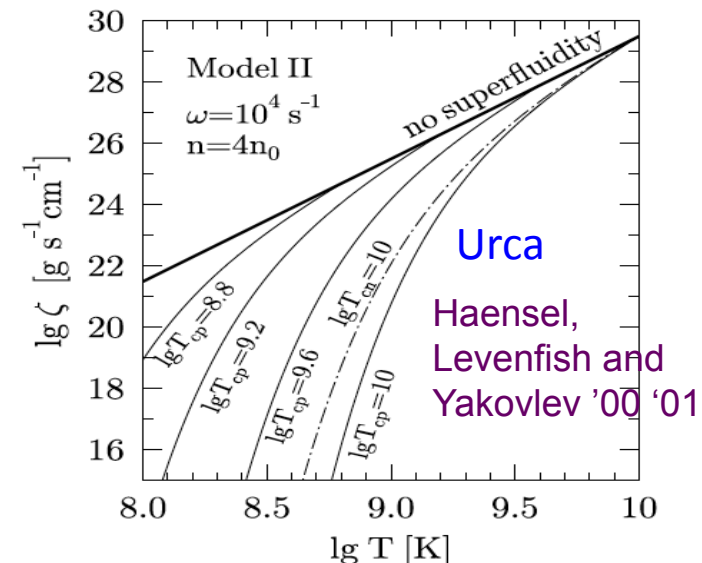


with only 3-phonon vertices

ξ_2 at $n \geq 4n_0$ is within 10% of the static value for $T \leq 10^9$ K and for the case of maximum values of the 3P_2 gap > 1 MeV, while, otherwise, the static solution is not valid. Bulk viscosity coefficients strongly depend on the gap.



Compared to the contribution of Urca (also modified Urca) processes to the bulk viscosities in neutron stars, those are dominated by phonon-phonon processes



Thermal conductivity due to superfluid phonons

The **thermal conductivity** relates the heat flux with the temperature gradient

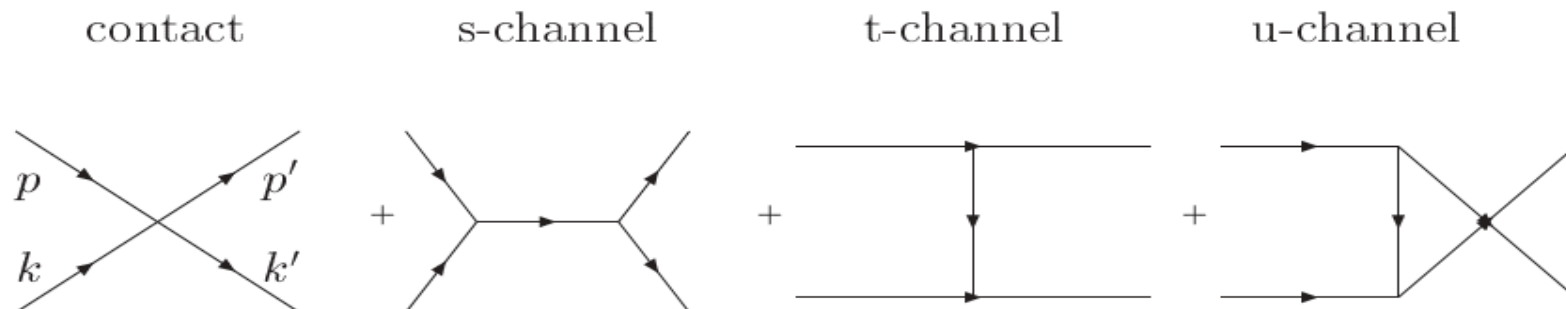
$$\mathbf{q} = -\kappa \nabla T$$

and is calculated using variational methods* for solving the transport equation as

$$\kappa \geq \left(\frac{4a_1^2}{3T^2} \right) A_1^2 M_{11}^{-1},$$

$$a_1 = \frac{4c_s^4}{15\Delta^2}, \quad A_1 = \frac{256\pi^6}{245c_s^9} T^9$$

where M_{11} is the (1,1) element of a $N \times N$ matrix (N determined by convergence). Each element is a multidimensional integral that contains the thermally weighted scattering matrix for phonons:



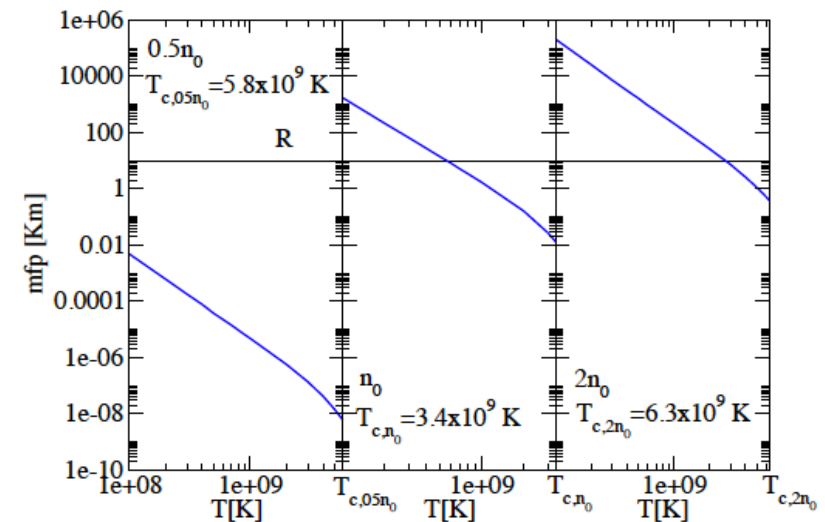
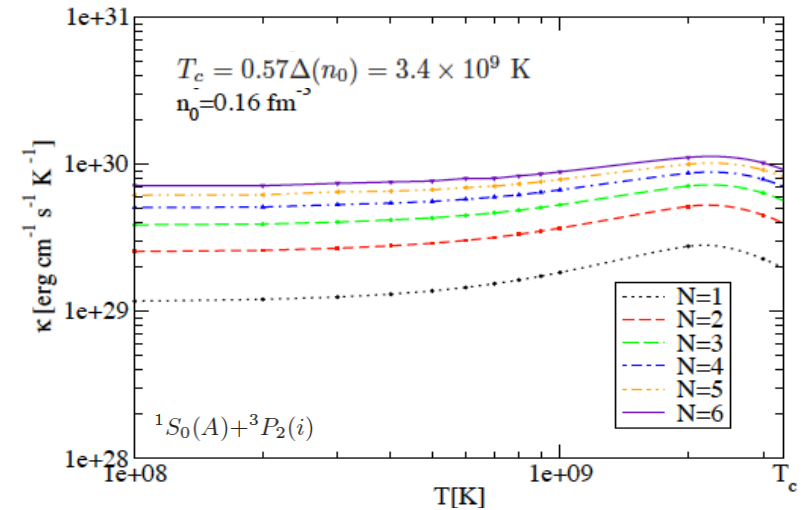
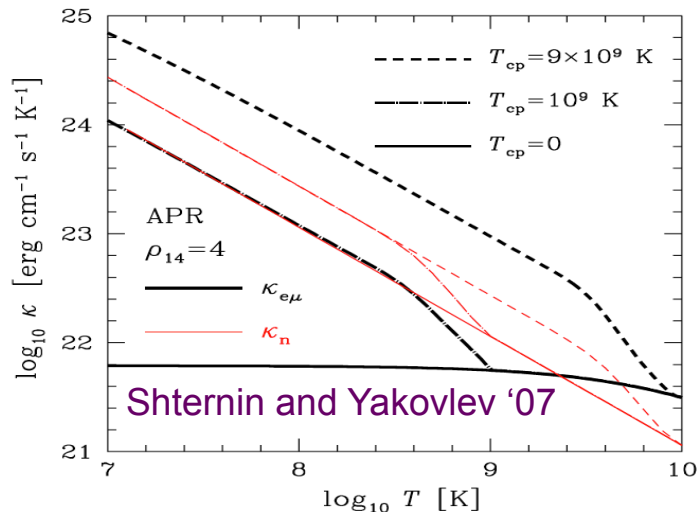
* Braby, Chao and Schafer '10

Need of **NLO corrections in phonon dispersion law** (seen for He⁴ and CFL)
 Perform a **variational calculation** up to N=6 (deviation from N=5 ≤ 10%). We find that as in CFL

$$\kappa \propto \frac{1}{\Delta^6} \quad T \lesssim 10^9 \text{ K}$$

Mean free path of phonons
 (different from shear mean free path)

$$l = \frac{\kappa}{\frac{1}{3}c_v c_s} \quad l \propto 1/T^3$$



$$10^{25} \lesssim \kappa_{ph} \lesssim 10^{32} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1}$$

Thermal conductivity in neutron stars is dominated by **phonon-phonon collisions**

Summary

Starting from a general formulation for the collisions of superfluid phonons using EFT techniques, we compute the shear and bulk viscosities as well as the thermal conductivity in terms of the EoS of the system (and the gap function)

- Binary collisions of phonons produce a **shear viscosity** that scales with $1/T^5$ (universal feature seen for ^4He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory)
r-mode window modified for $T \geq (10^8\text{-}10^9)$ K due to phonon shear viscosity
- **Bulk viscosity coefficients** strongly depend on the gap and they are dominated by phonon-phonon collisions as compared to Urca (modified Urca) processes
- **Thermal conductivity** due to phonons scales as $1/\Delta^6$ (seen for CFL), the constant of proportionality depending on the EoS. As compared to electron-muon collisions, phonon-phonon collisions dominate the thermal conductivity

Need of an accurate analysis of electron-phonon collisions Bertoni, Reddy and Rrapaj '15

Support:
NewCompStar COST action MP1304

