Black Hole from Colors

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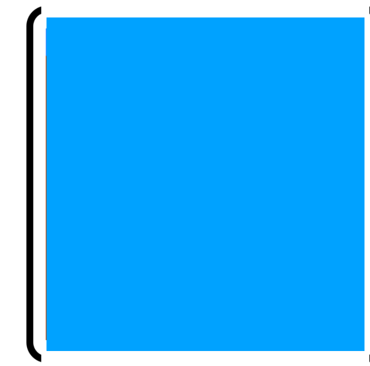
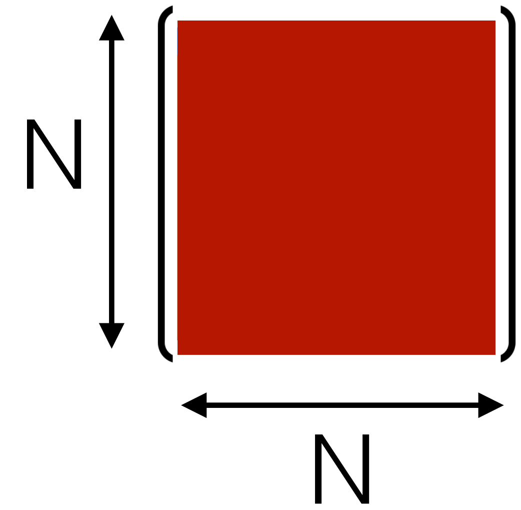
M.H.-Maltz, 1608.03276
M.H.-Ishiki-Watanabe, 1812.05494
M.H.-Jevicki-Peng-Wintergert, 1909.09118
+ work in progress

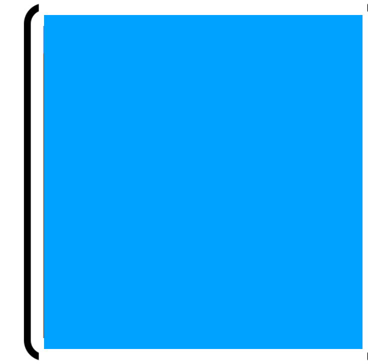


- Confinement phase: $E \sim N^0$



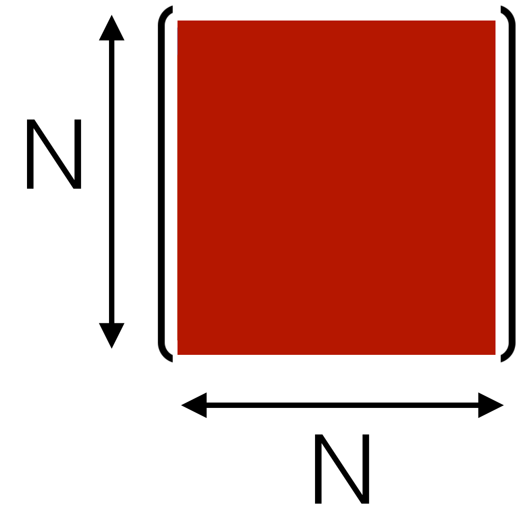
- Deconfinement phase: $E \sim N^2$

⟷ Black Hole



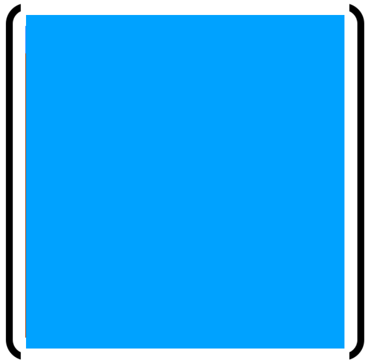
- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$

↔ Black Hole

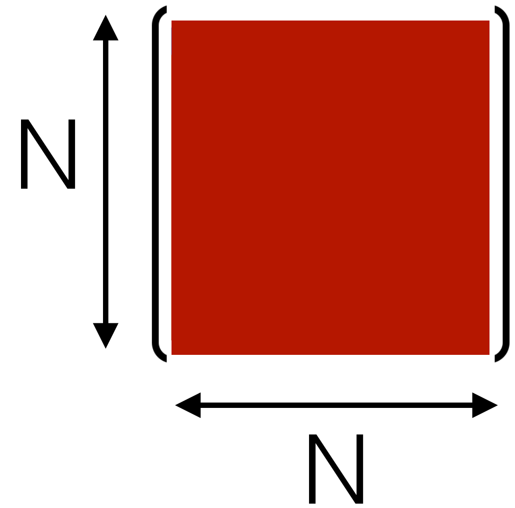


What if $E \sim N^2/100$?

- Confinement phase: $E \sim N^0$

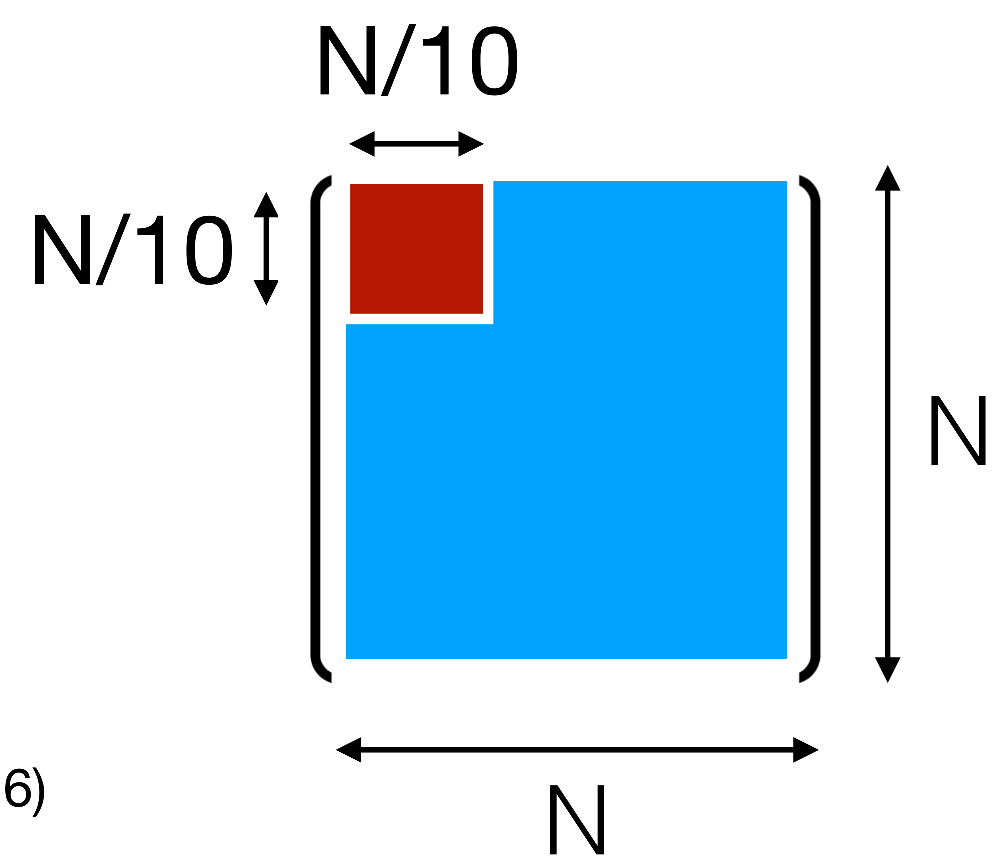


- Deconfinement phase: $E \sim N^2$



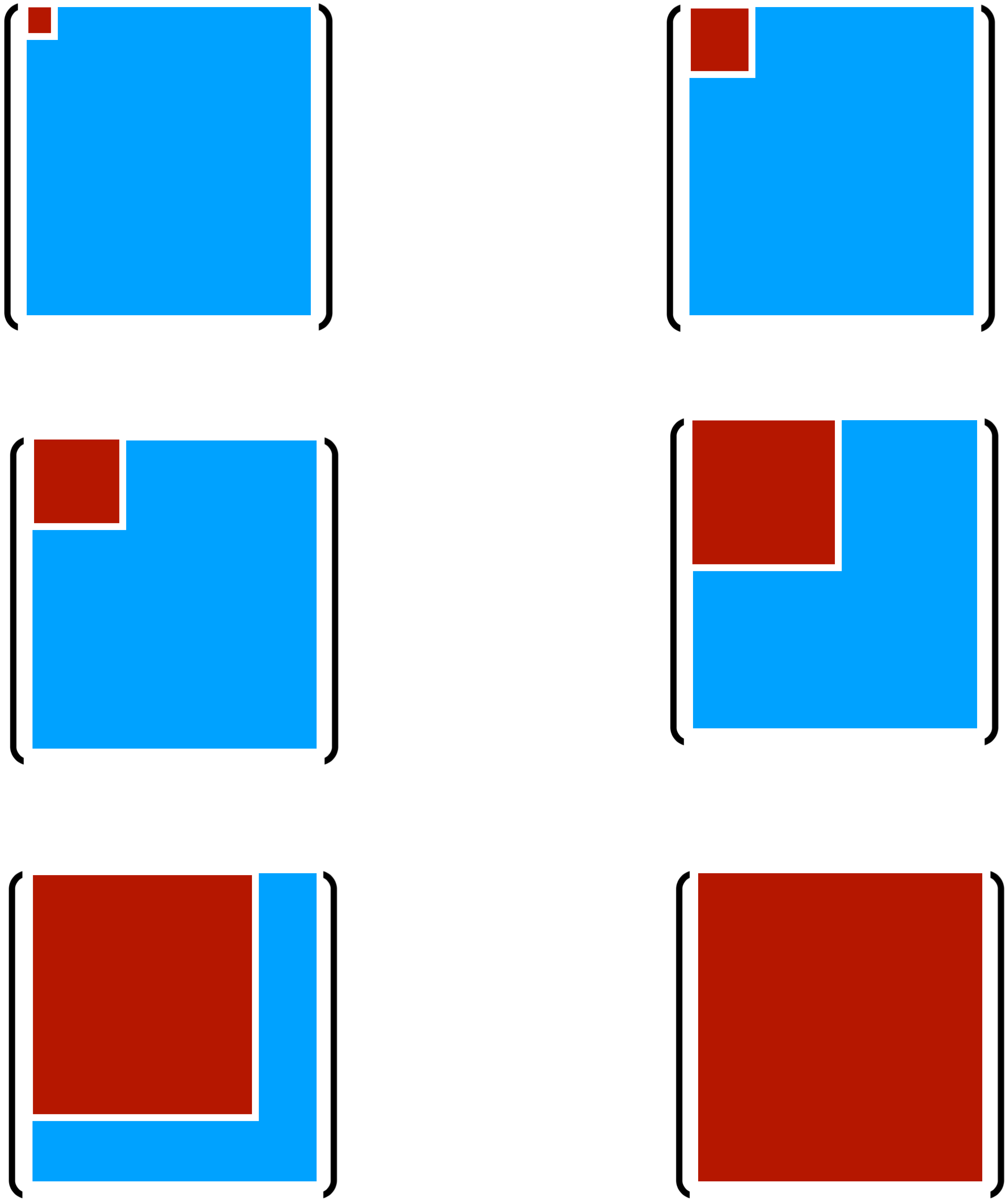
↔ Black Hole

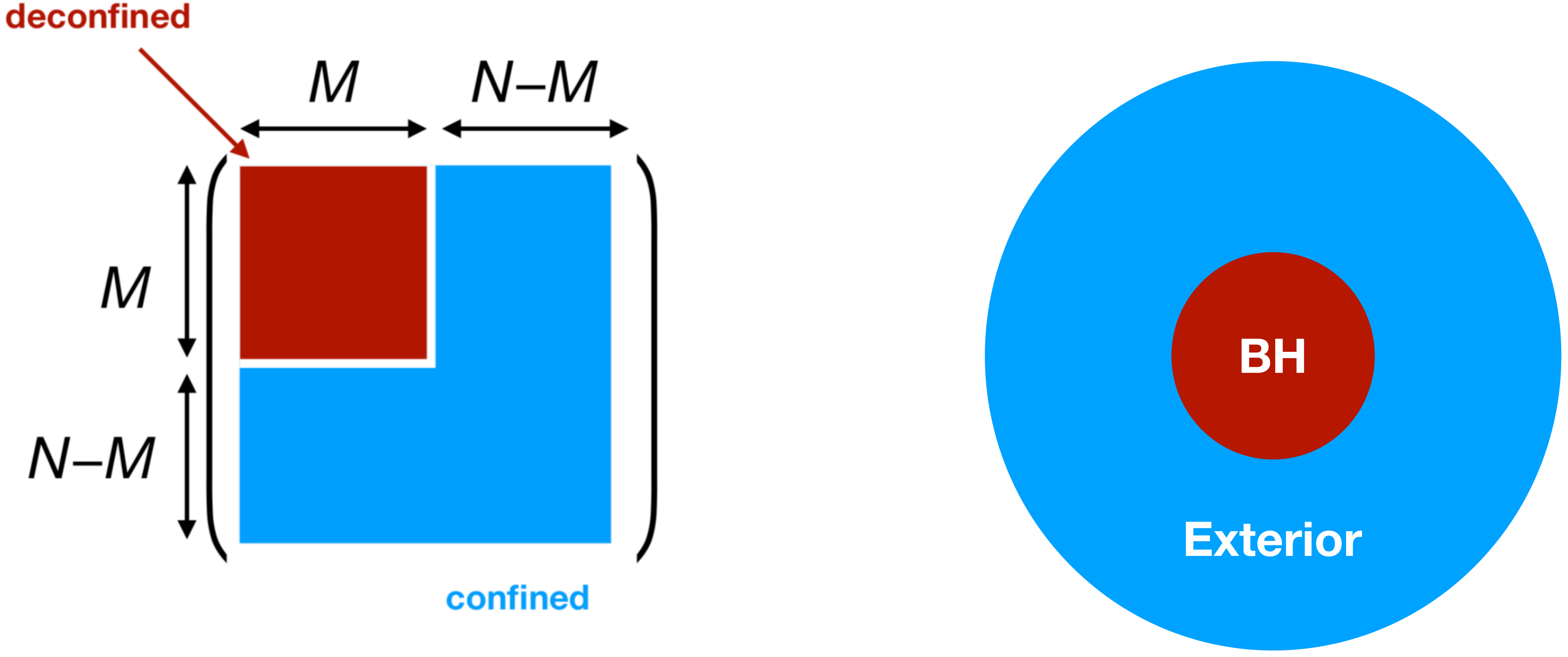
What if $E \sim N^2/100$?



<u>'partially' deconfine</u>

(MH-Maltz, 2016)

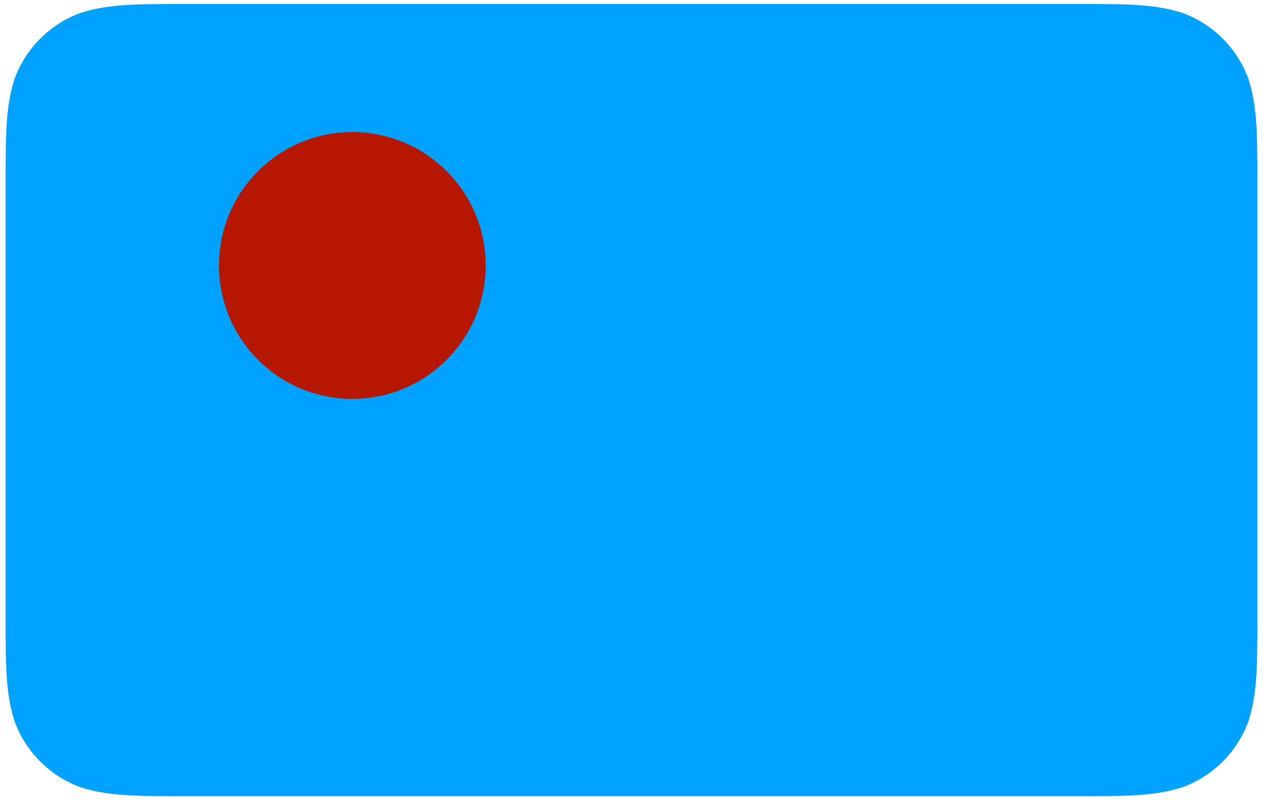



deconfined
M
N−M
M
N−M
confined
BH
Exterior

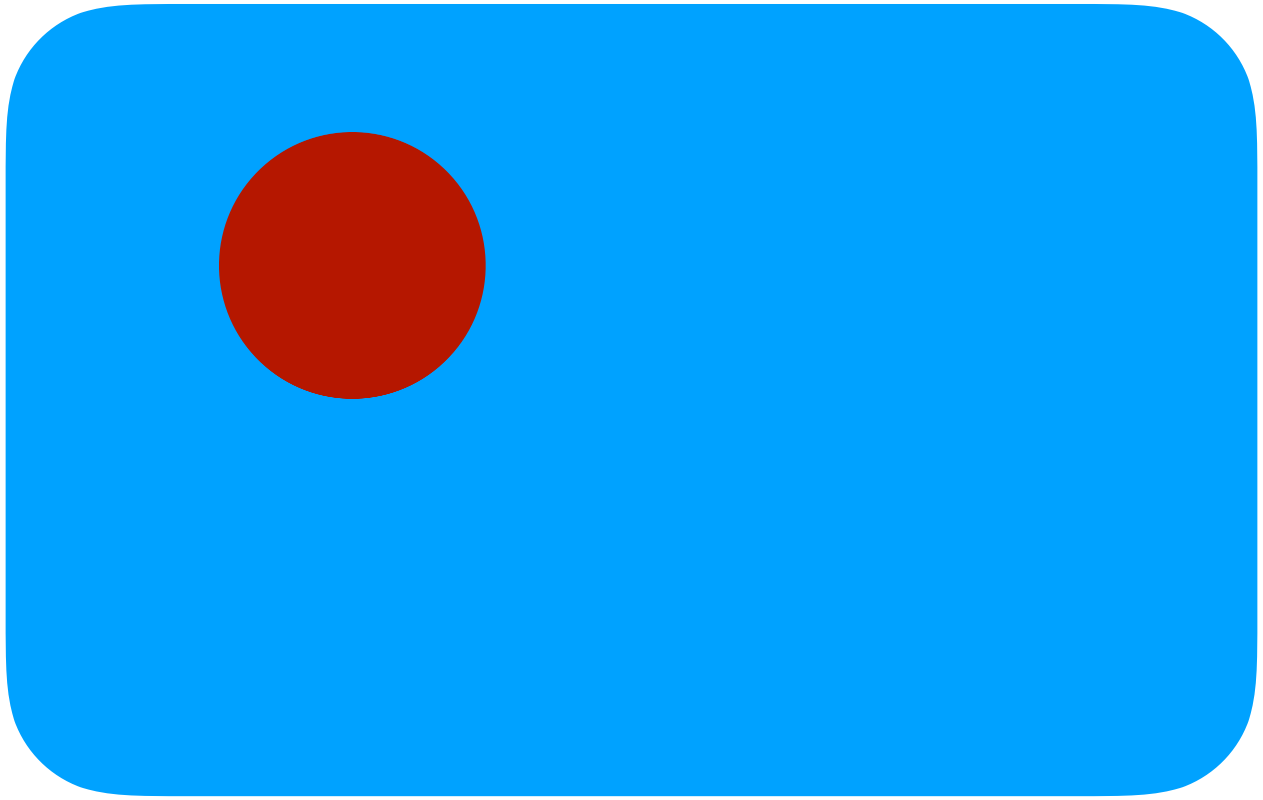
Heuristic justification

(more precise argument is given later)

Why doesn't a part of the volume deconfine?



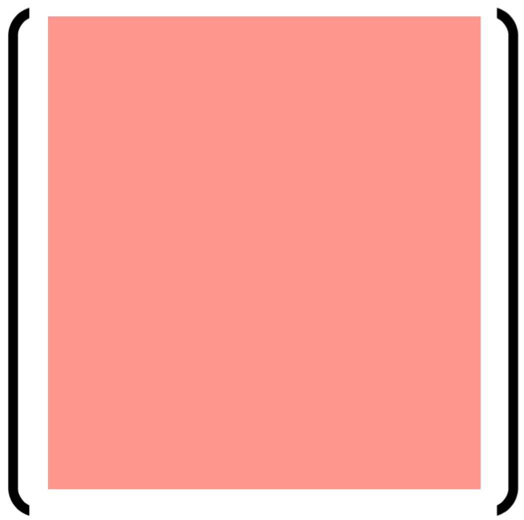
Why doesn't a part of the volume deconfine?



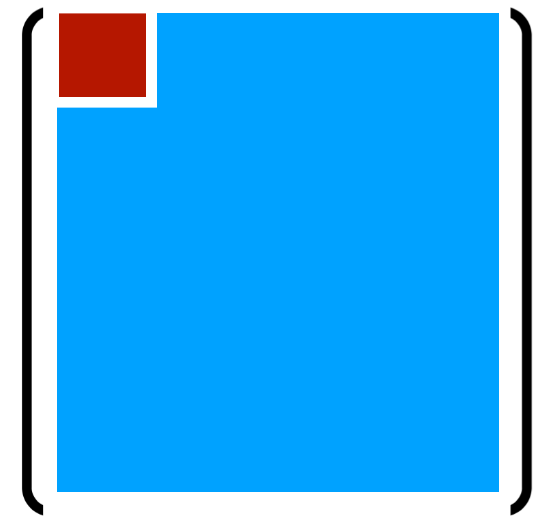
Deconfinement takes place even in matrix model, which has no spatial dimensions.

(Exception: first order transition, large volume)

Why don't all N^2 d.o.f. gently excited?

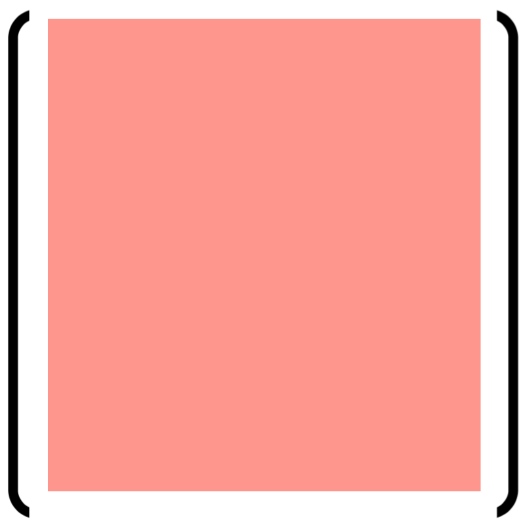


instead of

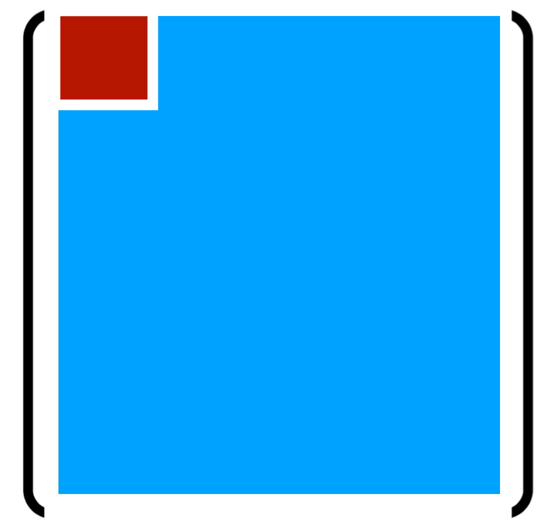


?

Why don’t all N^2 d.o.f. gently excited?



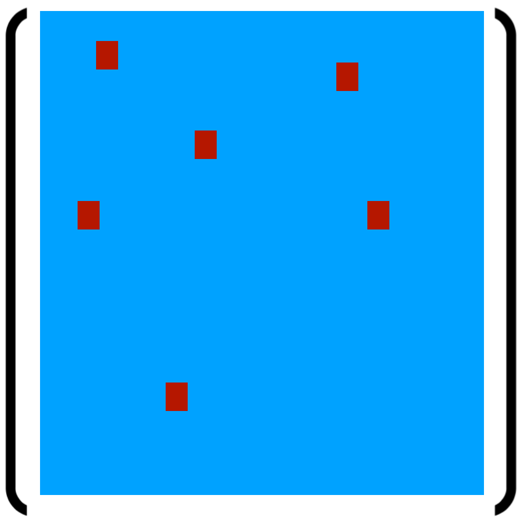
instead of



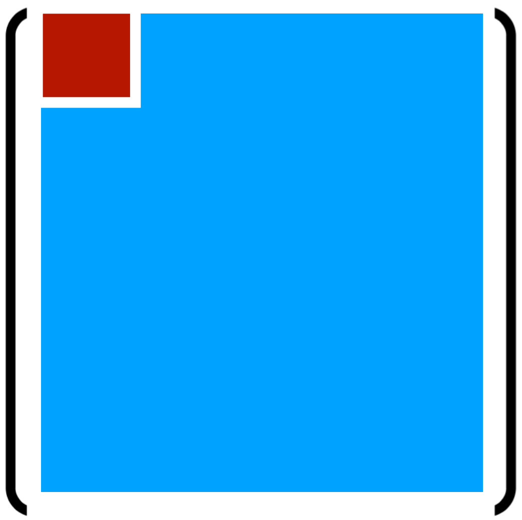
?

In quantum mechanics, parametrically small excitation is impossible.

Why should symmetry preserved partly?

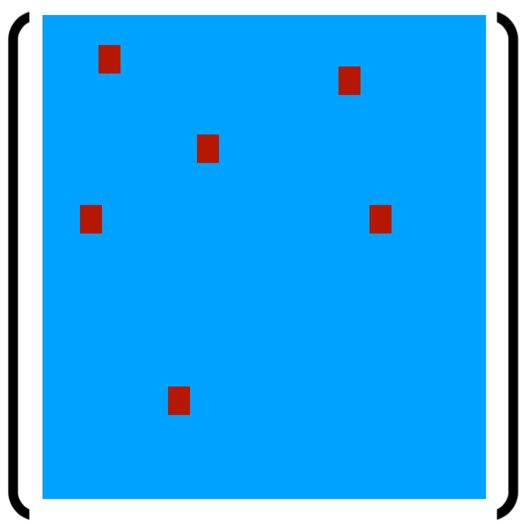


instead of

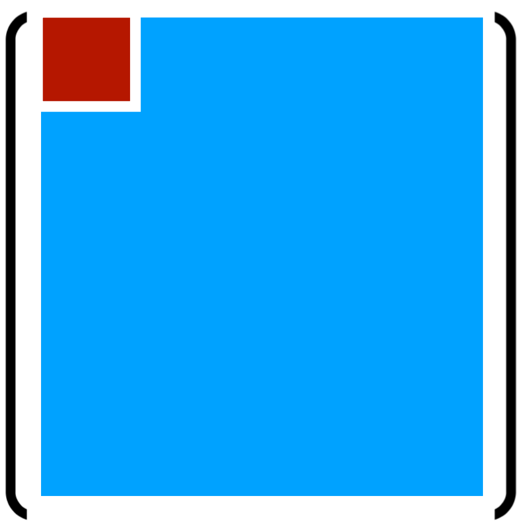


?

Why should symmetry preserved partly?



instead of



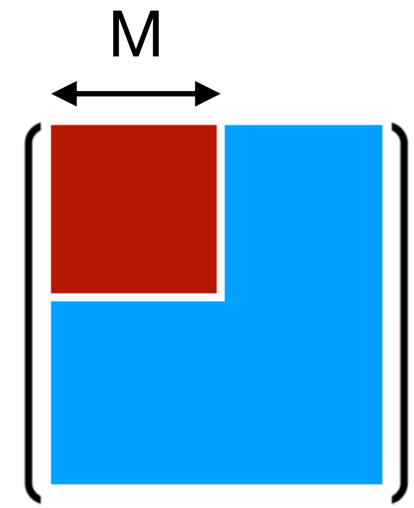
?

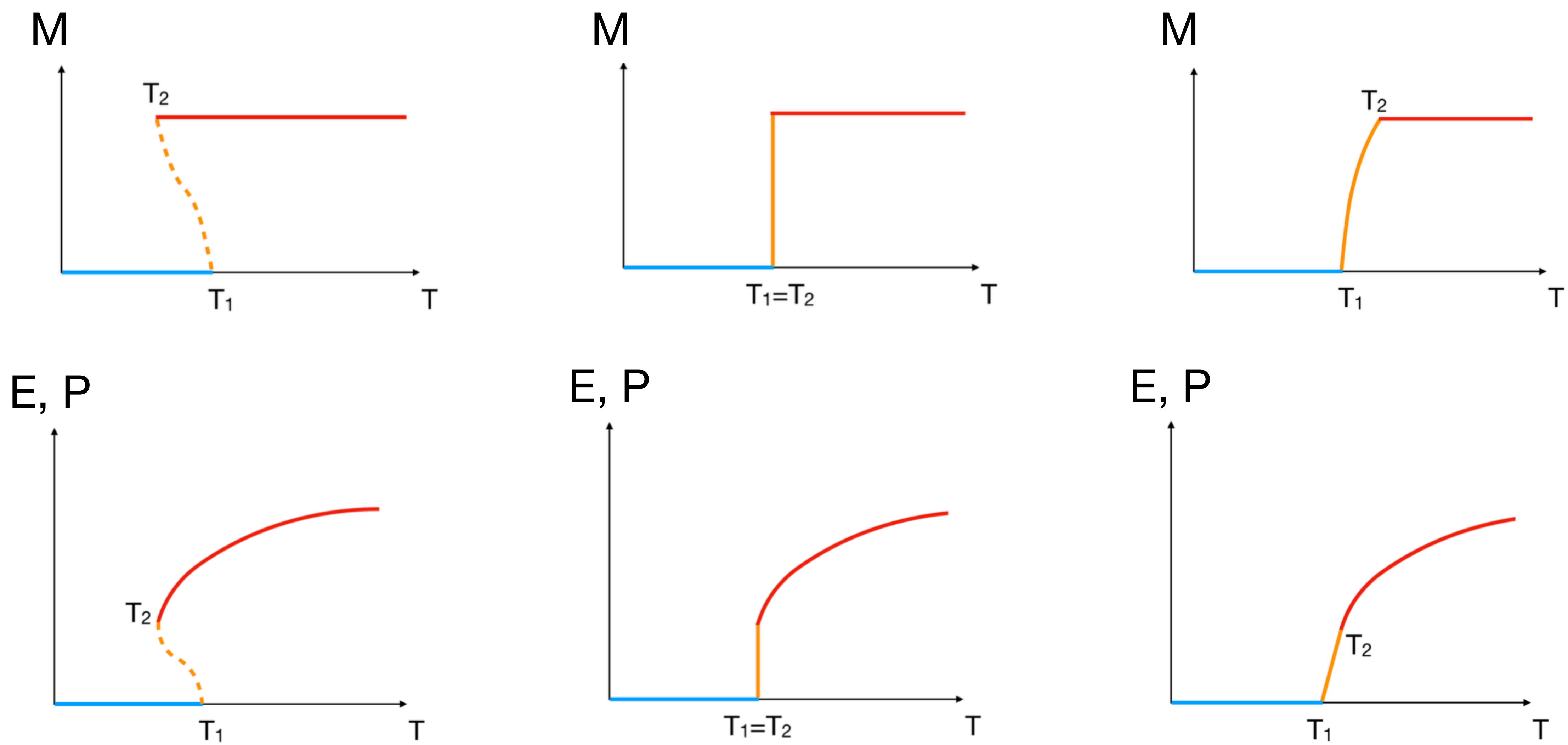
It is natural to expect a large symmetry
at saddle point.

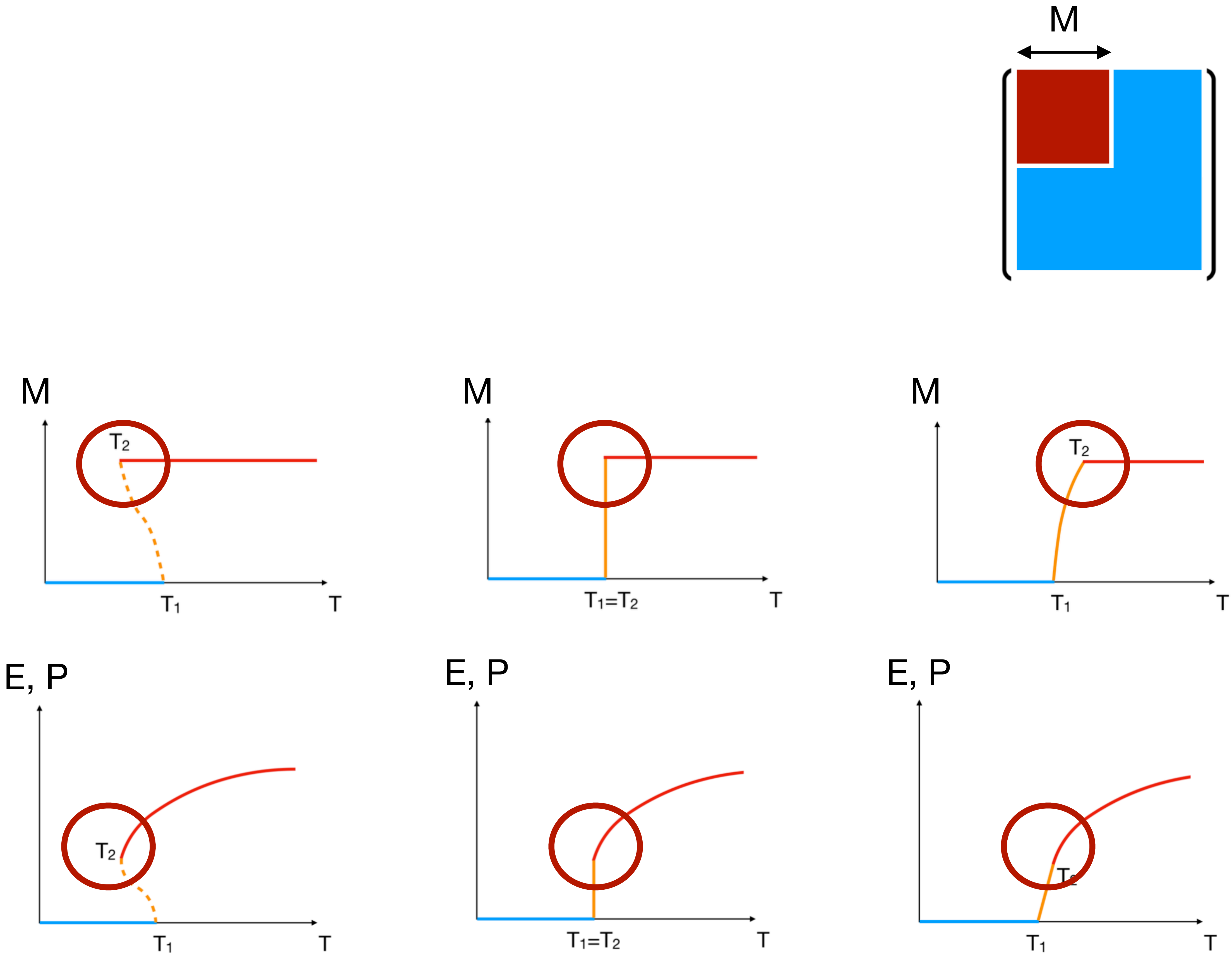
Phase Diagrams

Hagedorn transition
Gross-Witten-Wadia transition
Gauge symmetry breaking

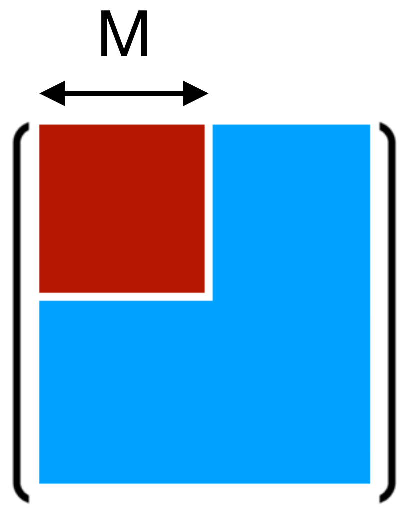
(more precise argument is given later)


M

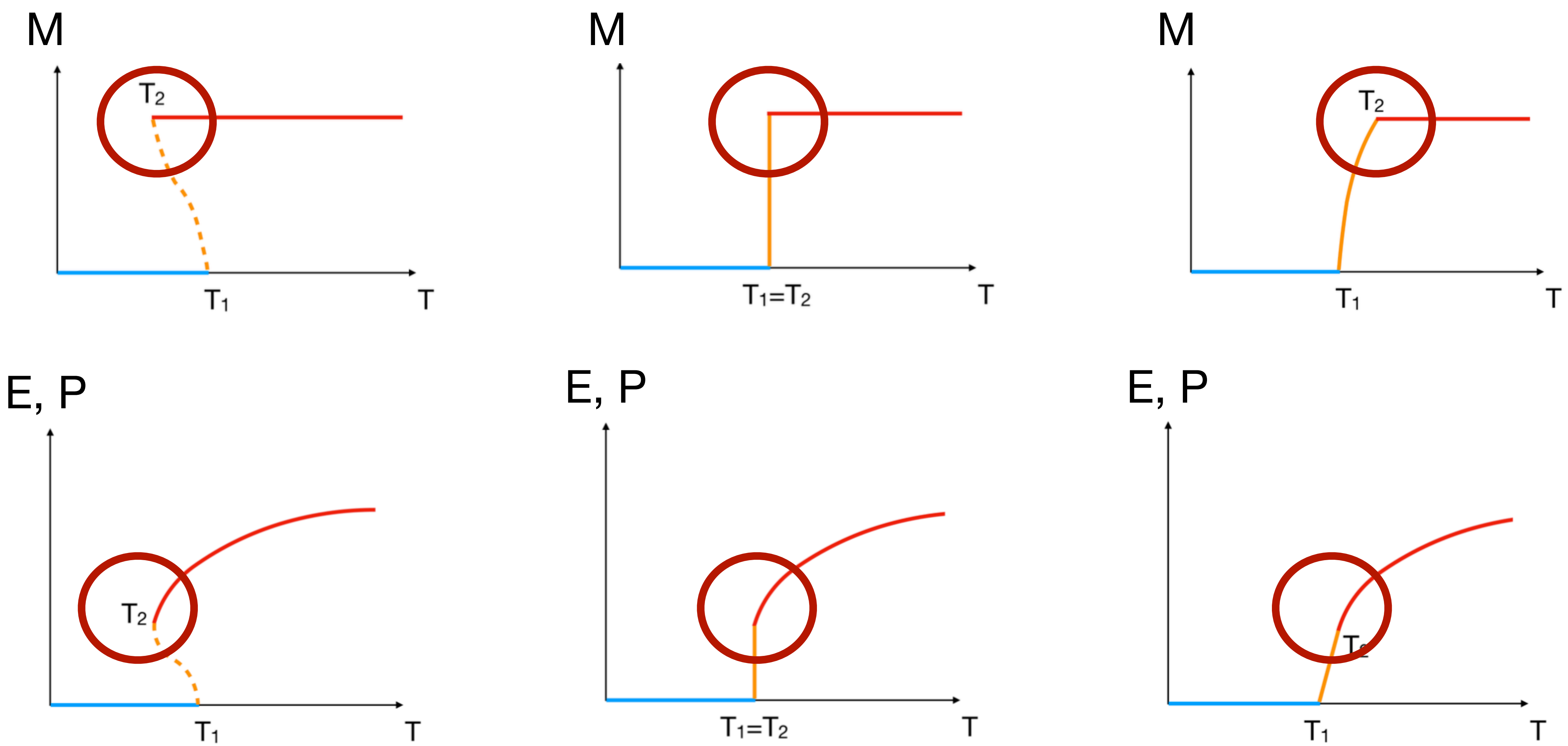

M
T2
T1
T
M
T1=T2
T
M
T2
T1
T
E, P
T2
T1
T
E, P
T1=T2
T
E, P
T2
T1
T



M
M
T2
T1
T
M
T1=T2
T
M
T2
T1
T
E, P
T2
T1
T
E, P
T1=T2
T
E, P
T1
T



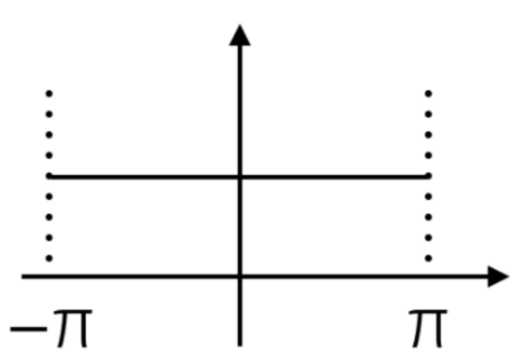
Gross-Witten-Wadia transition



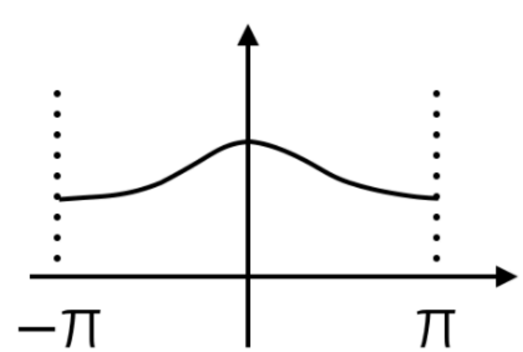
- Polyakov loop $$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:

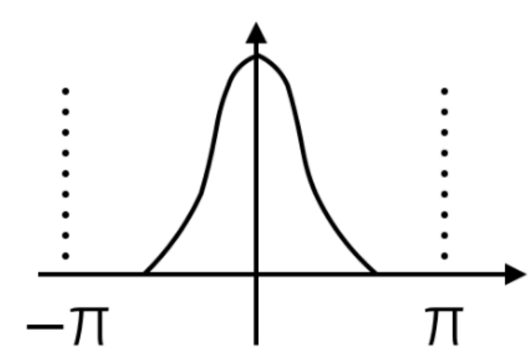
confined phase
P=0



deconfined phase
P ≠ 0



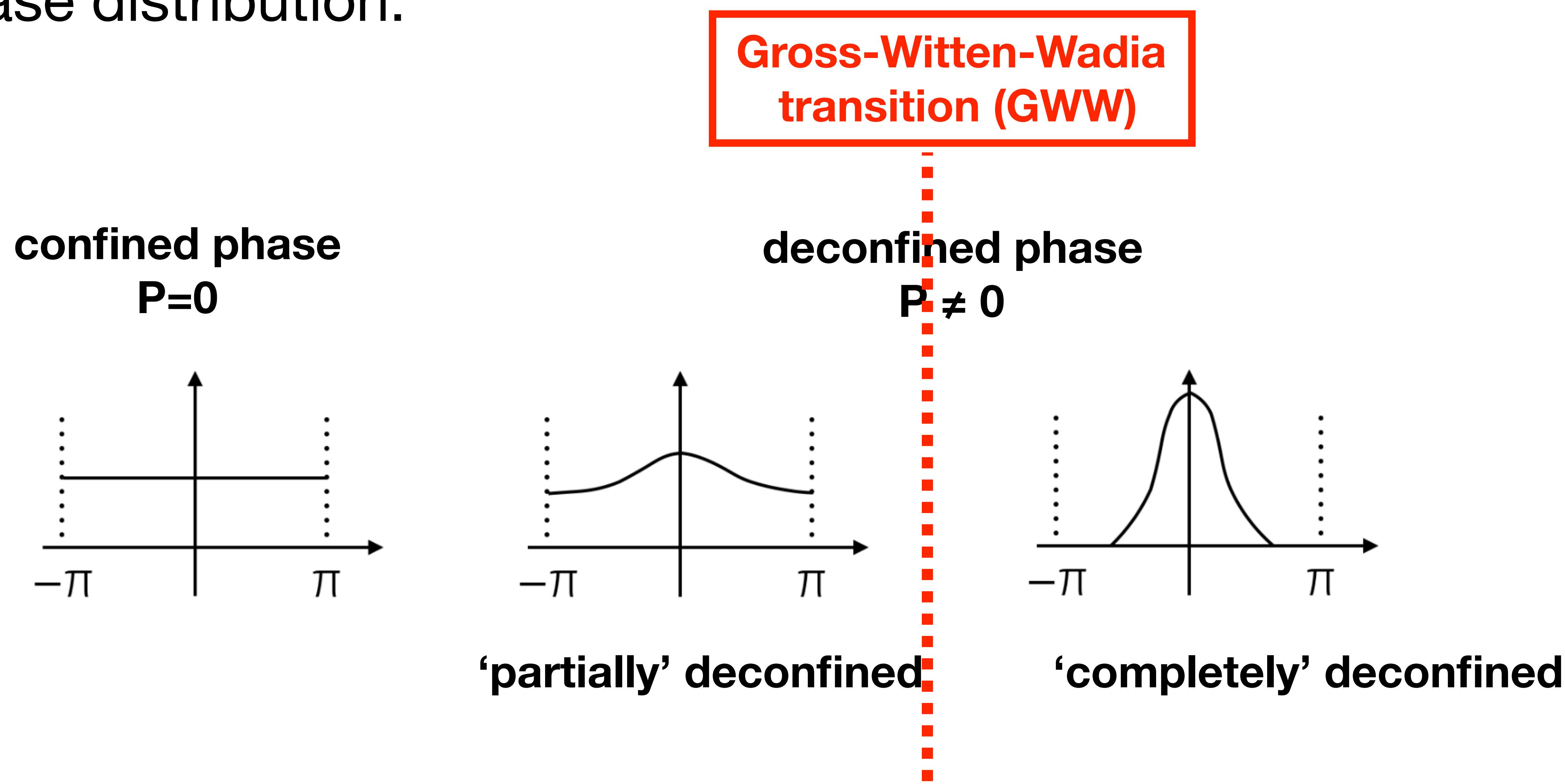
‘partially’ deconfined



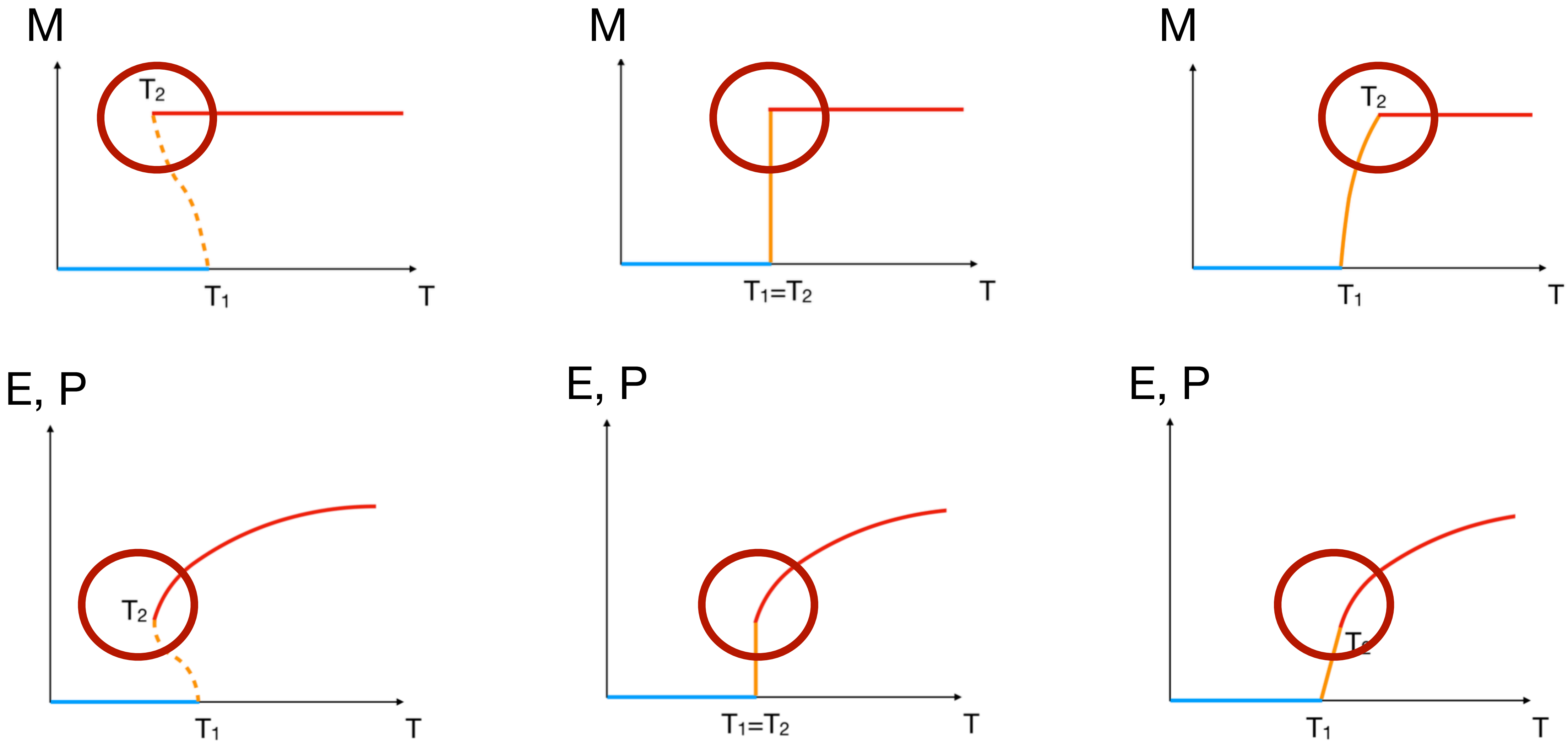
‘completely’ deconfined

- Polyakov loop $$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:



Gross-Witten-Wadia transition = "partial deconfinement → complete deconfinement" transition



- Polyakov loop $$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

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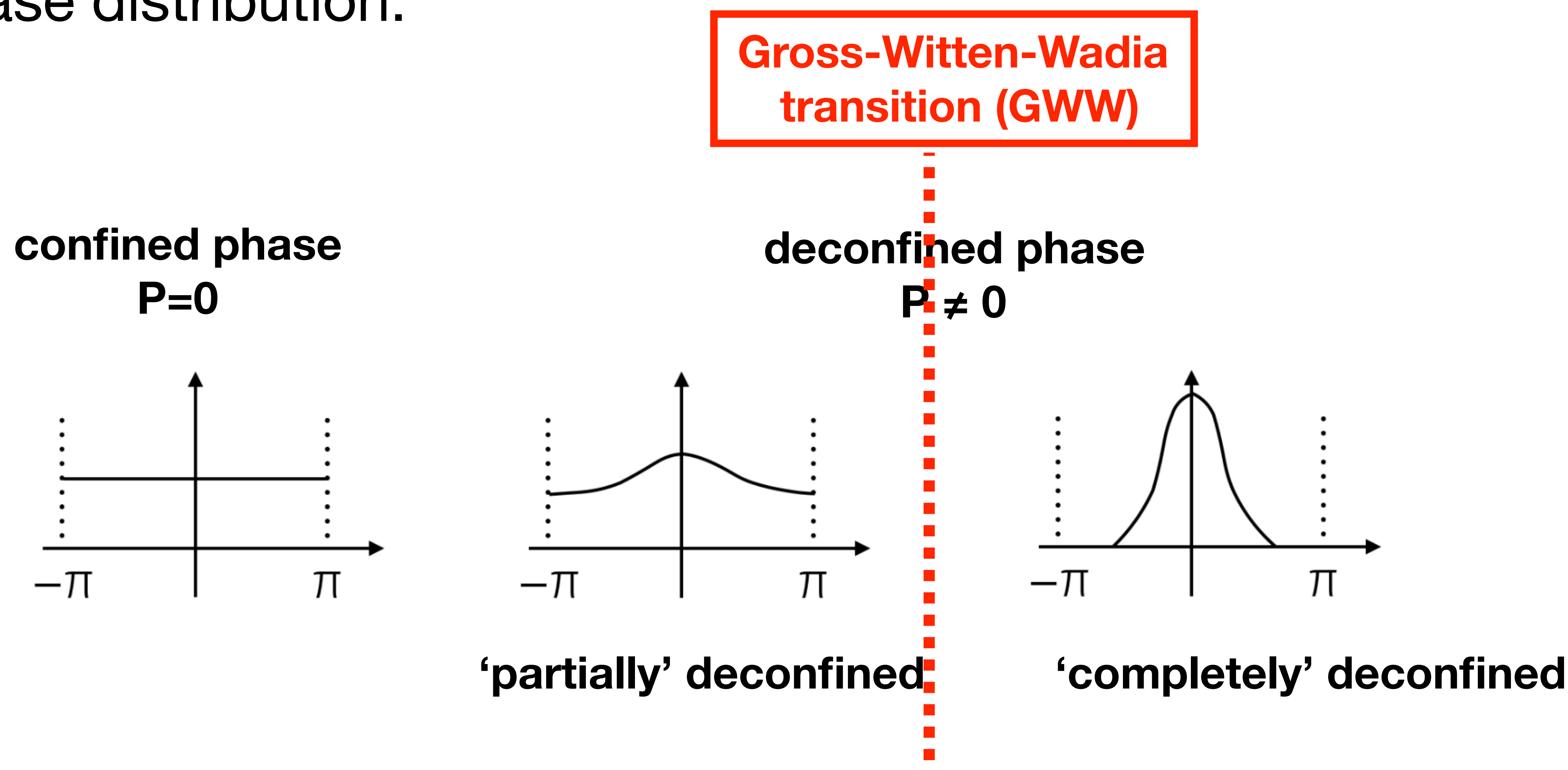

- Polyakov loop $$P = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j}$$

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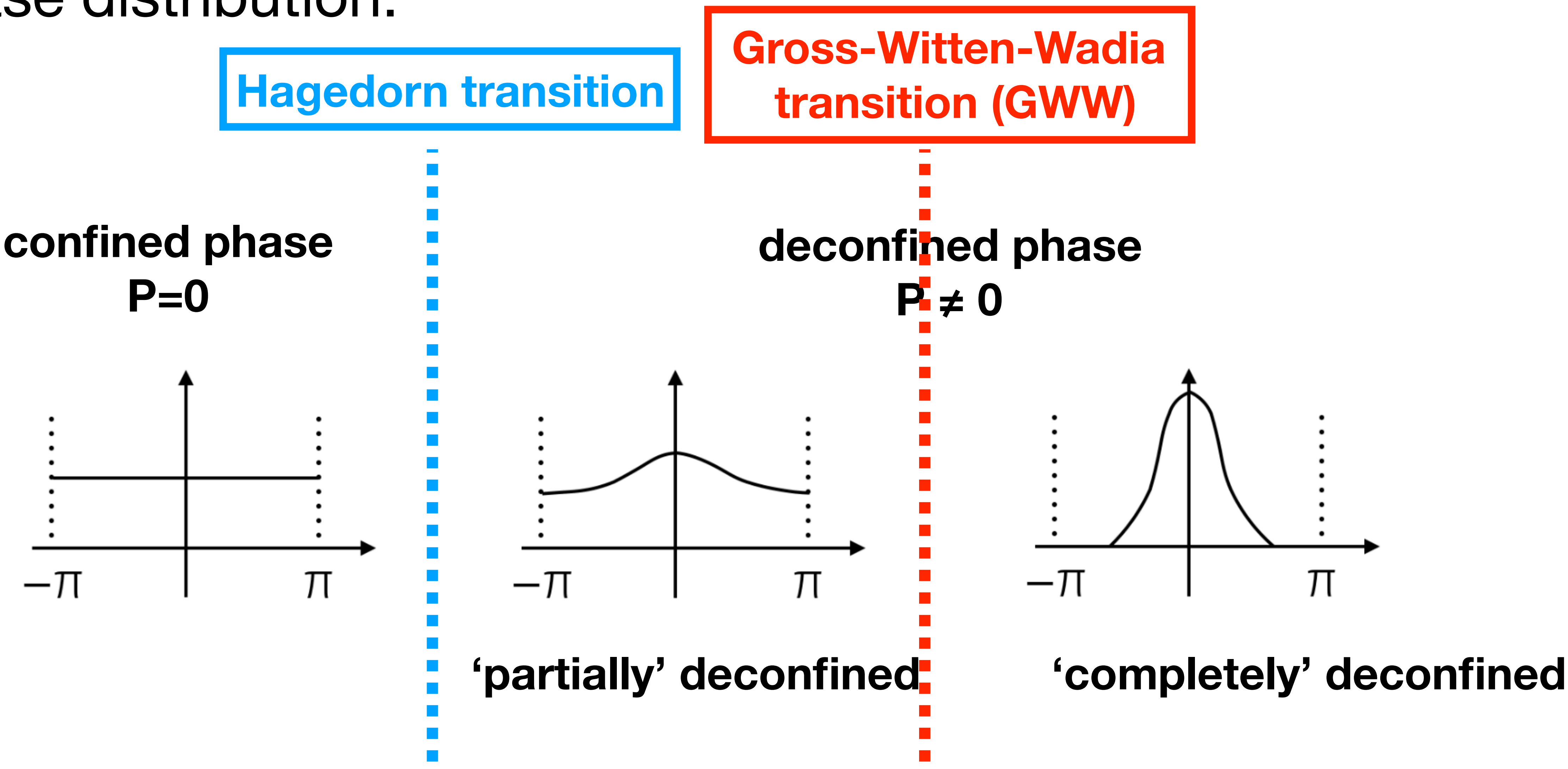
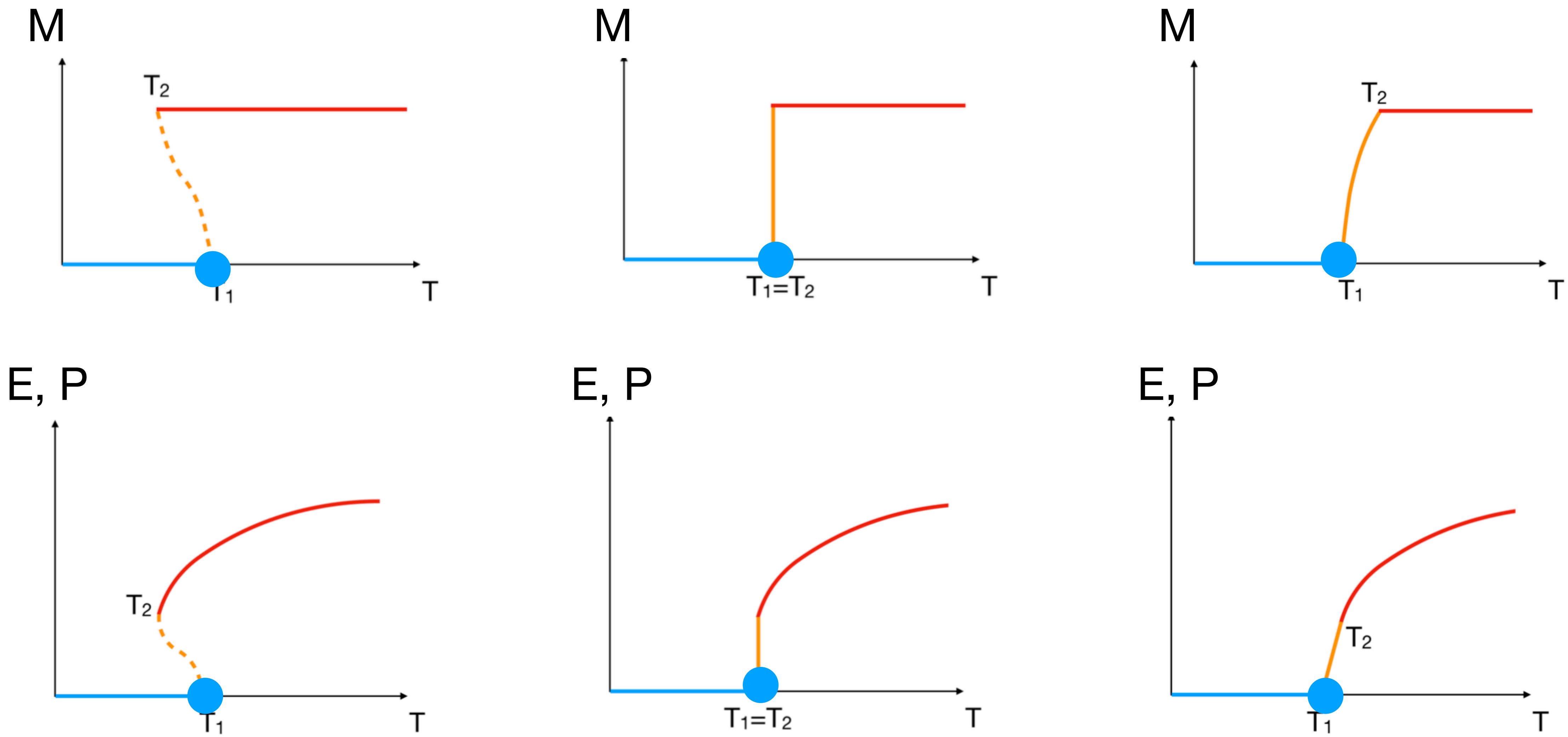
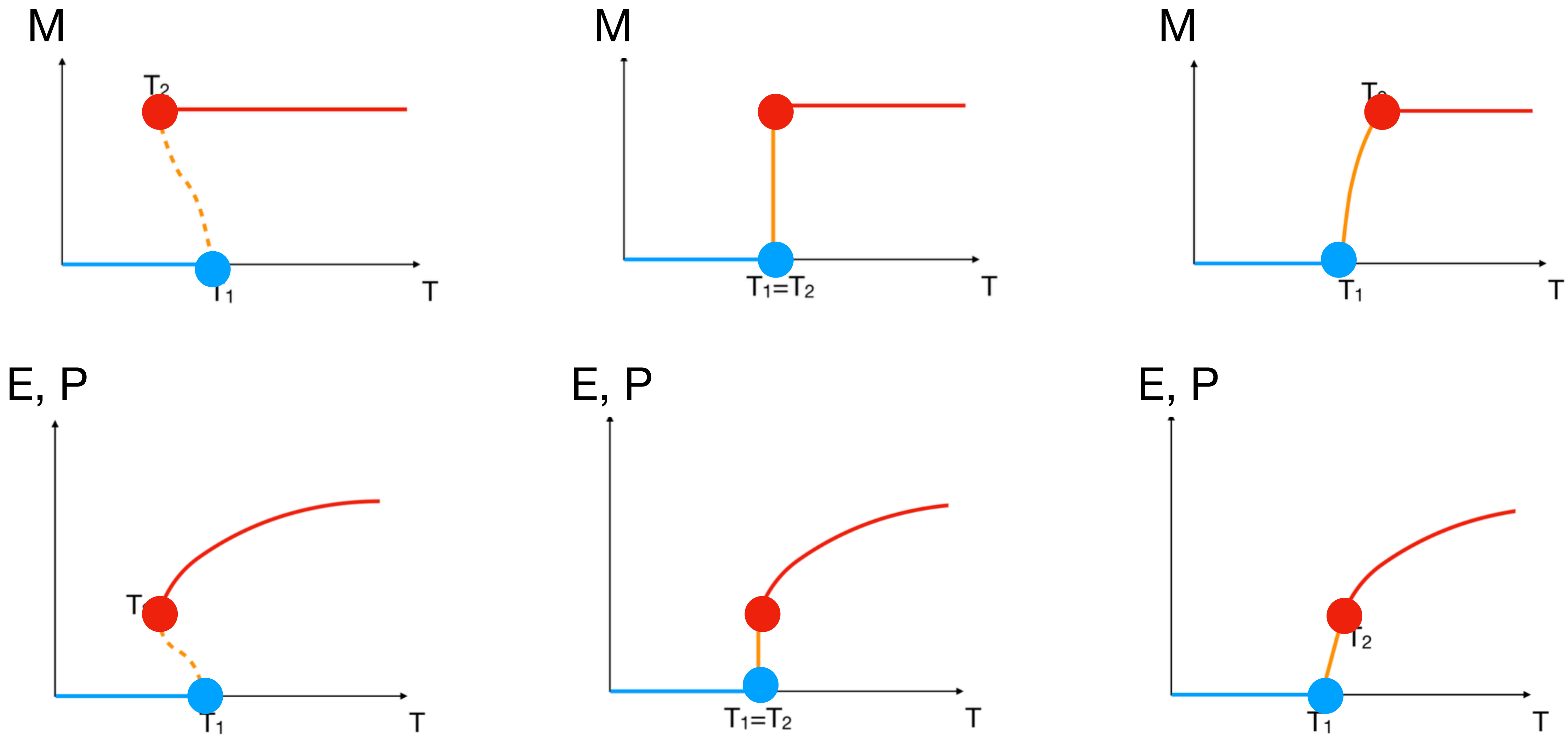


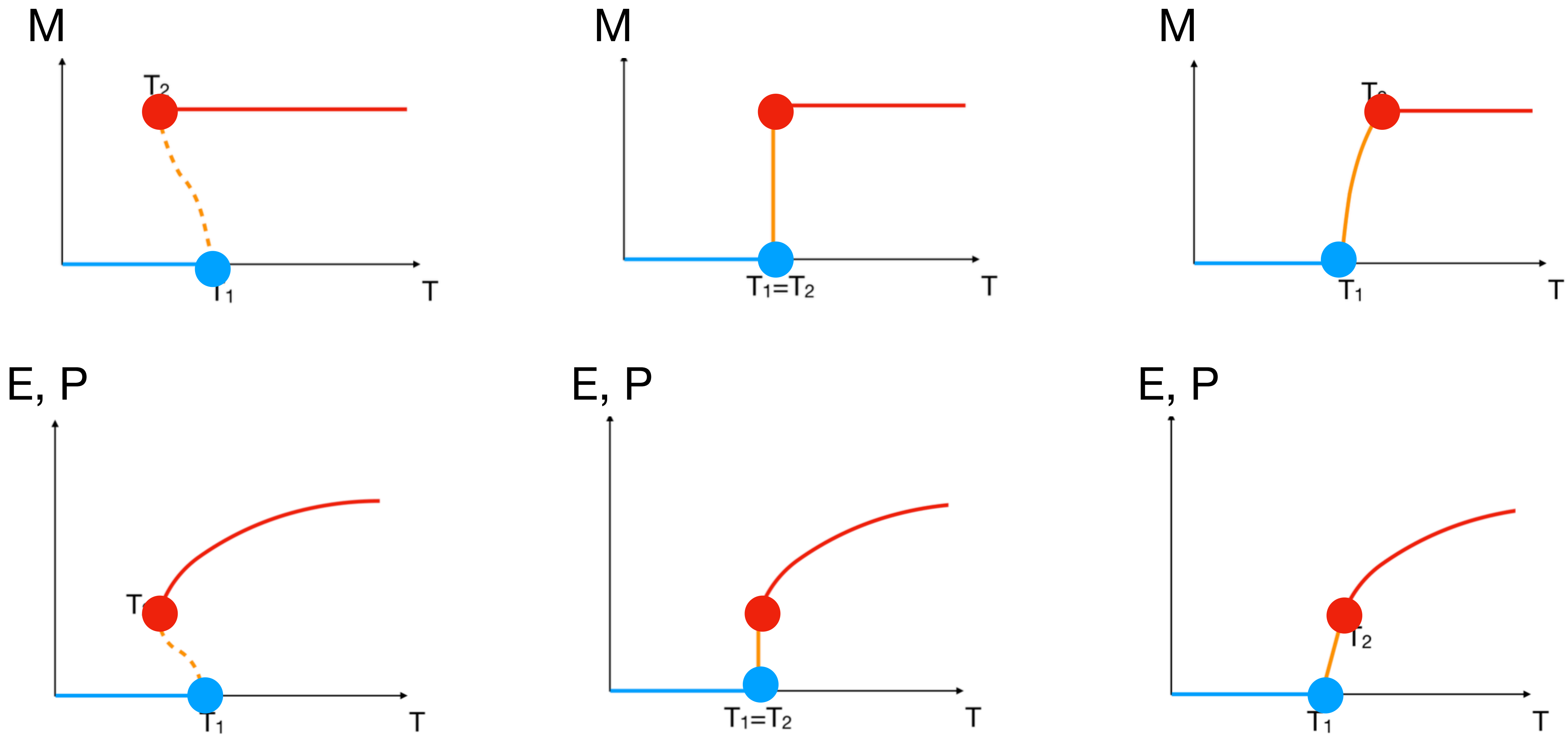


transition 1: confinement to partial deconfinement
(black hole formation begins)



transition 1: confinement to partial deconfinement
(black hole formation begins)

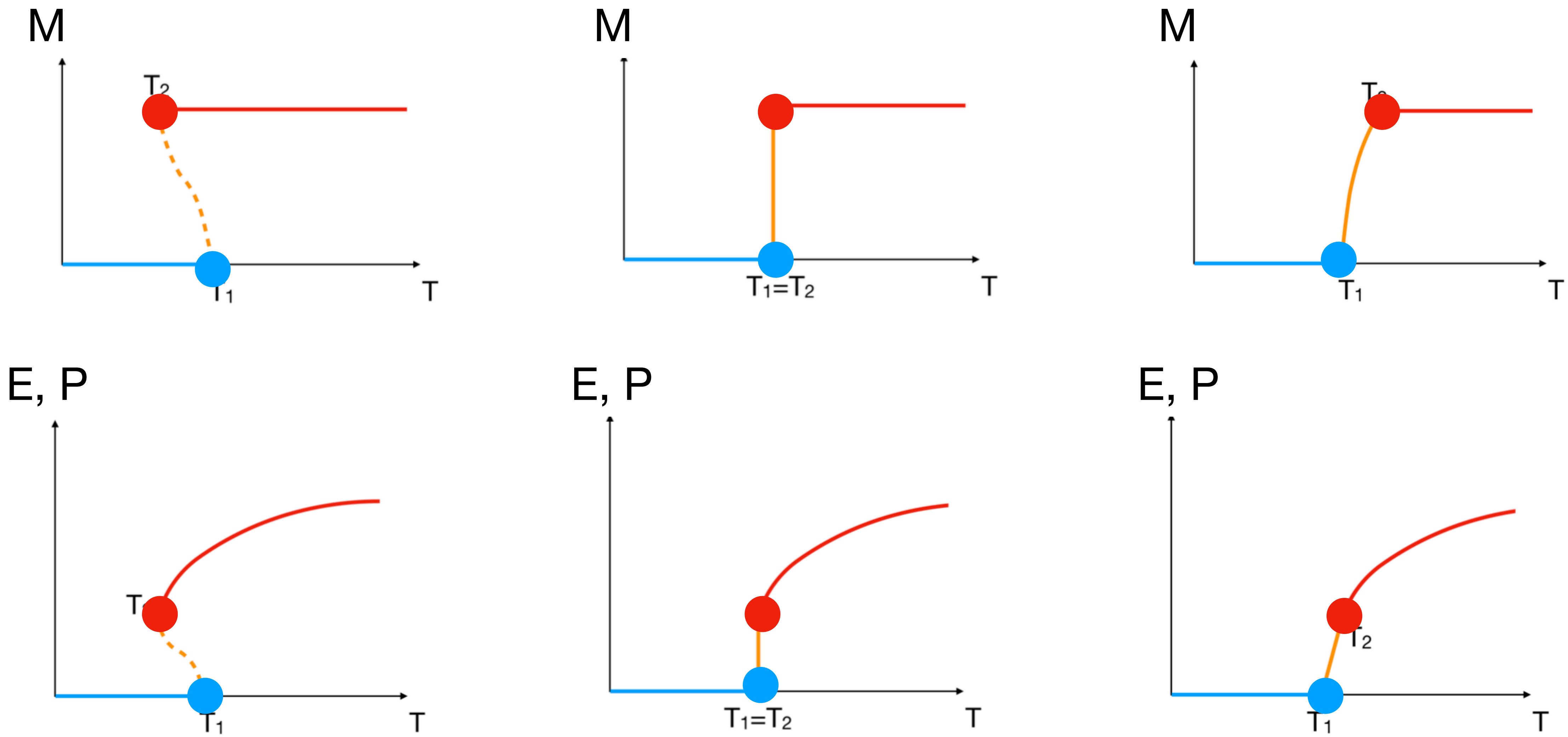
transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$SU(N) \rightarrow SU(M)\times SU(N-M)\times U(1) \rightarrow SU(N)$$



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$SU(N) \rightarrow SU(M)\times SU(N-M)\times U(1) \rightarrow SU(N)$$

No need for center symmetry. No need for chiral symmetry. Applies to QCD.

Where did it come from?

J. Maltz

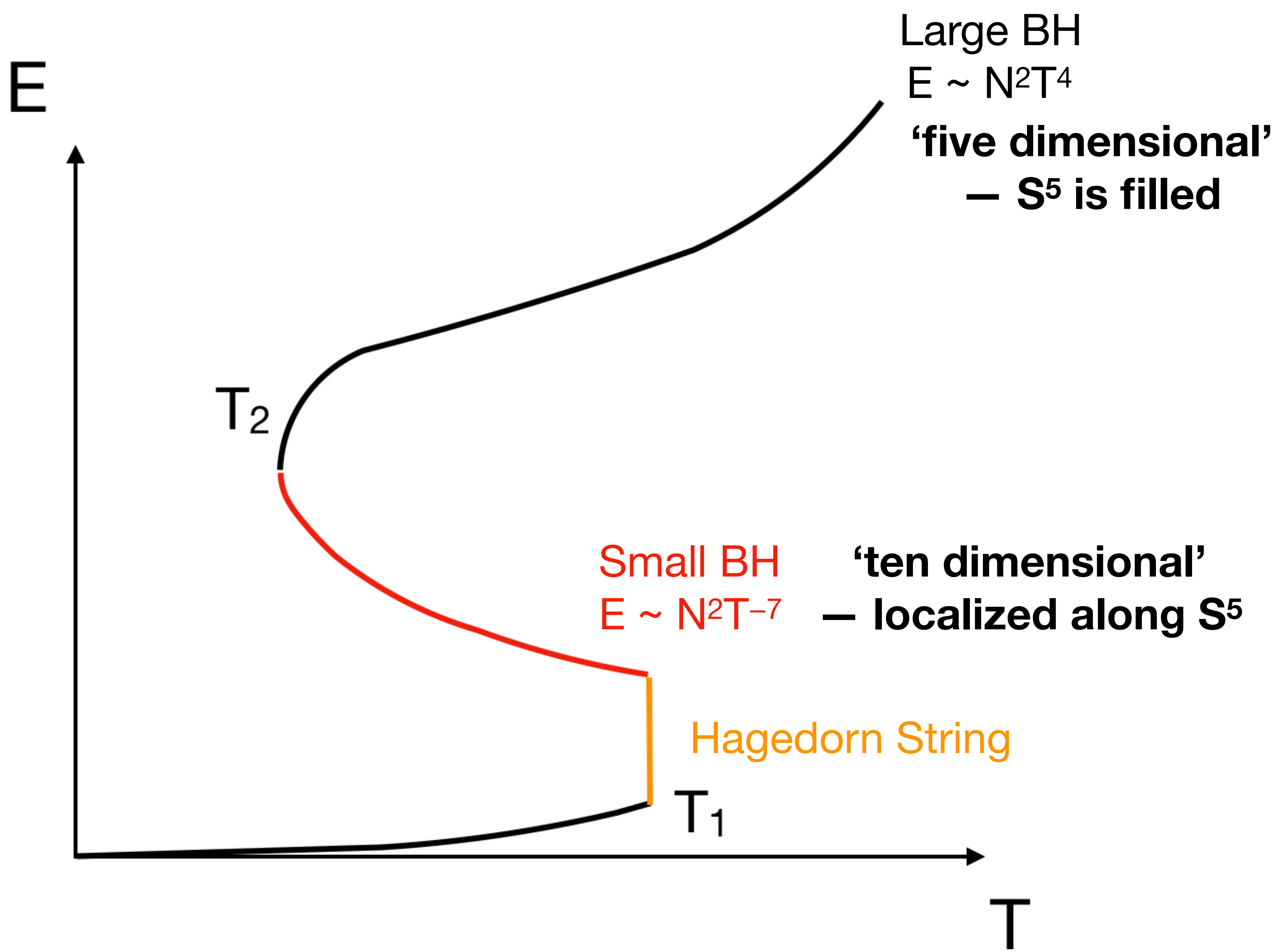
E. Berkowitz

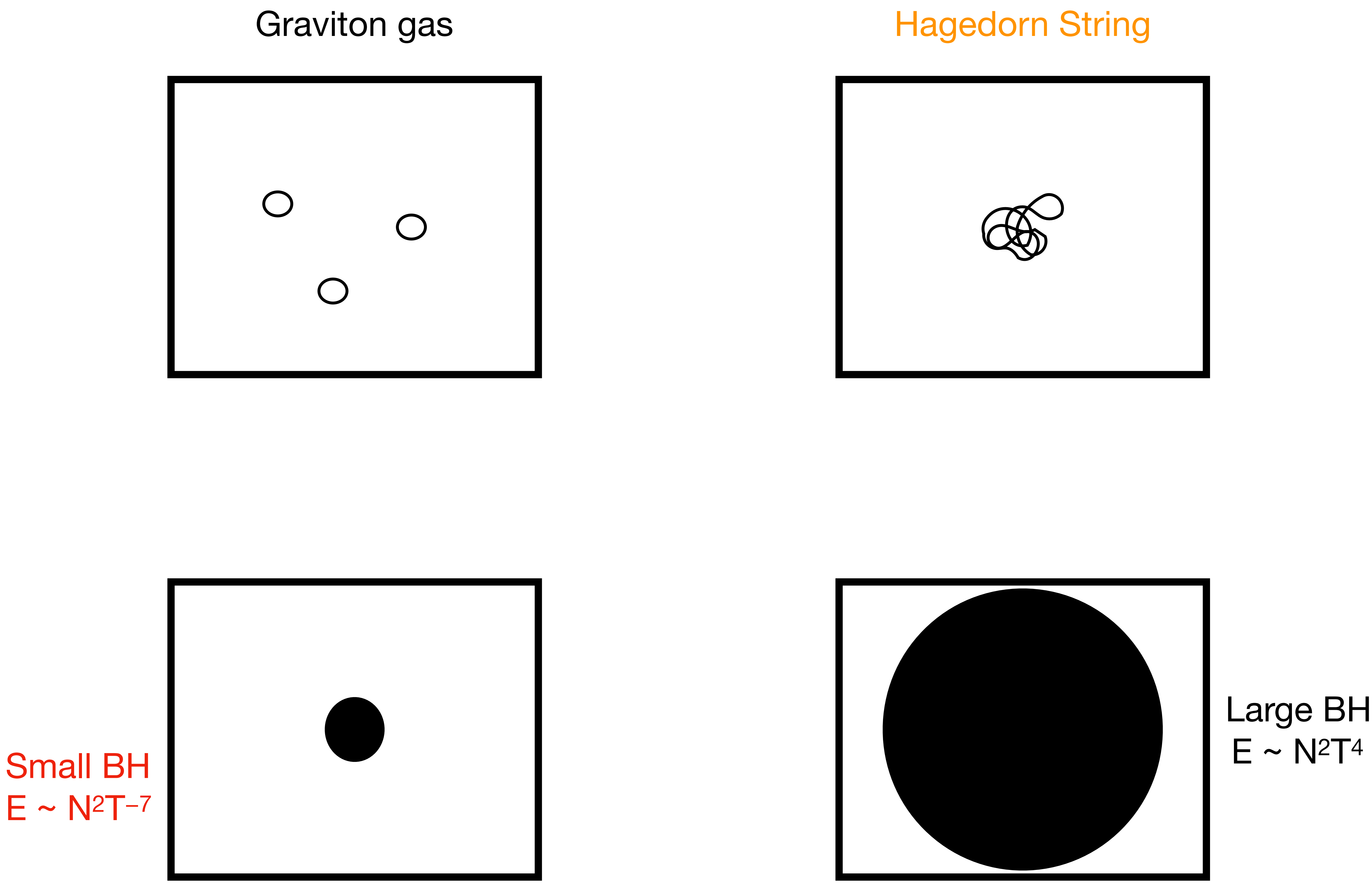
Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

M.H.-Maltz, 2016, JHEP; Susskind, unpublished

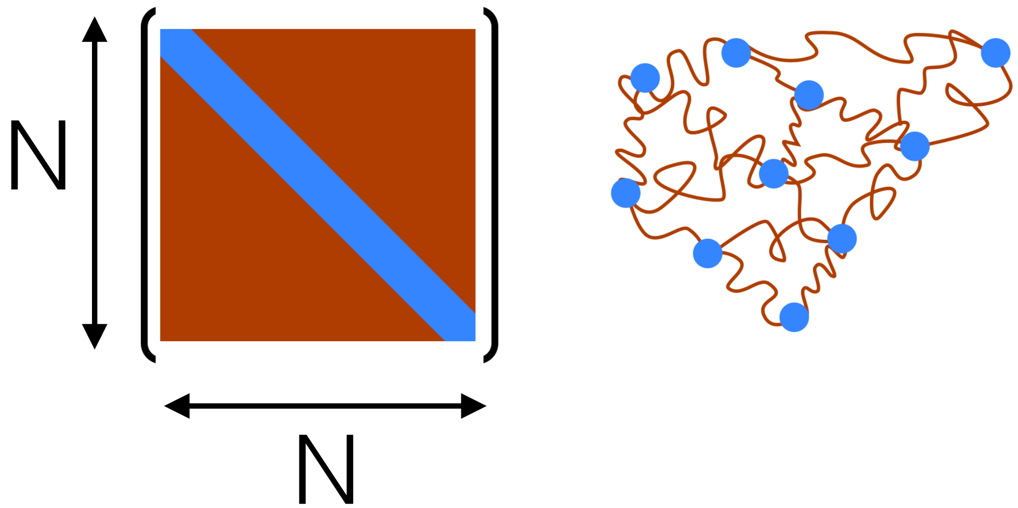
Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

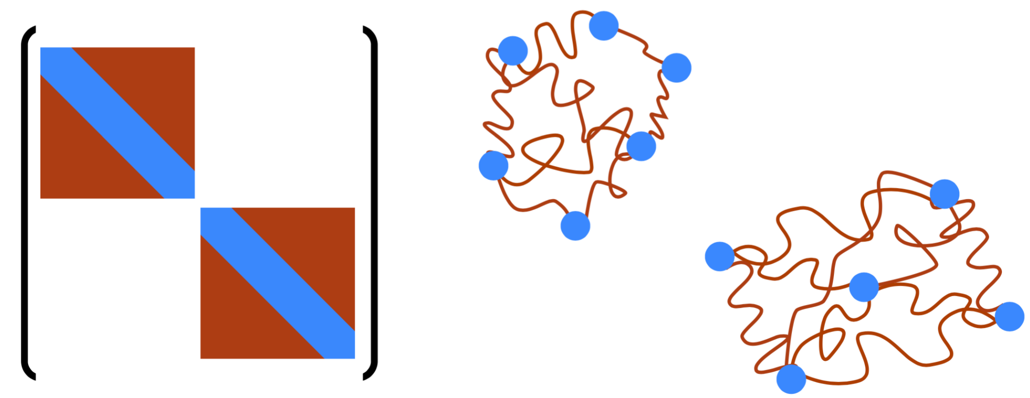
microcanonical
ensemble
(E fix)

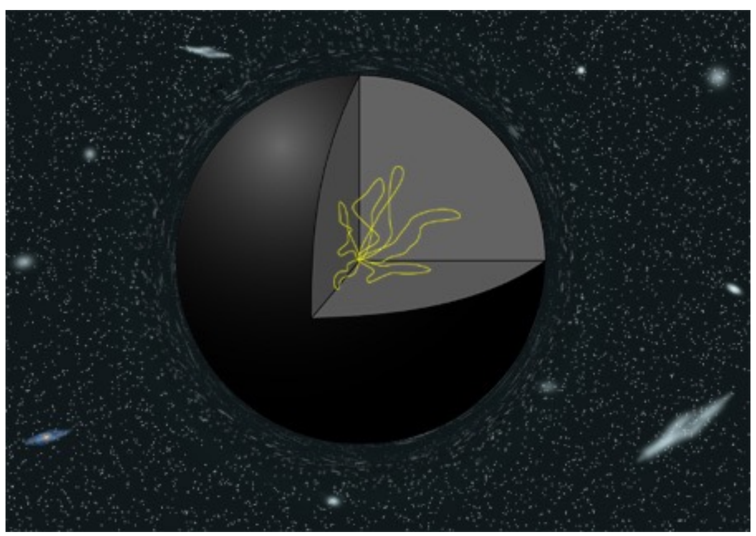



Graviton gas
Hagedorn String
Small BH
E ~ N^2T^{-7}
Large BH
E ~ N^2T^4

Higgsing Picture of Yang-Mills and String







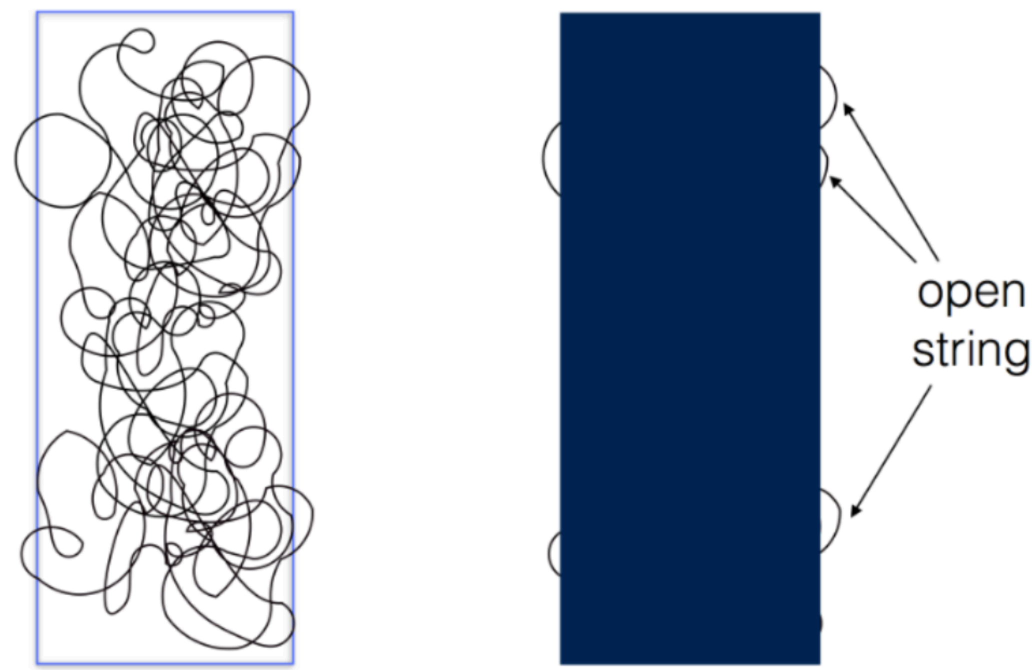
diagonal elements = particles (D-branes)
off-diagonal elements = open strings

(Witten, 1994)

black hole = bound state of D-branes and strings

‘SU(N) theory describes N D-branes + strings’

Confinement/Deconfinement Picture of Yang-Mills and String

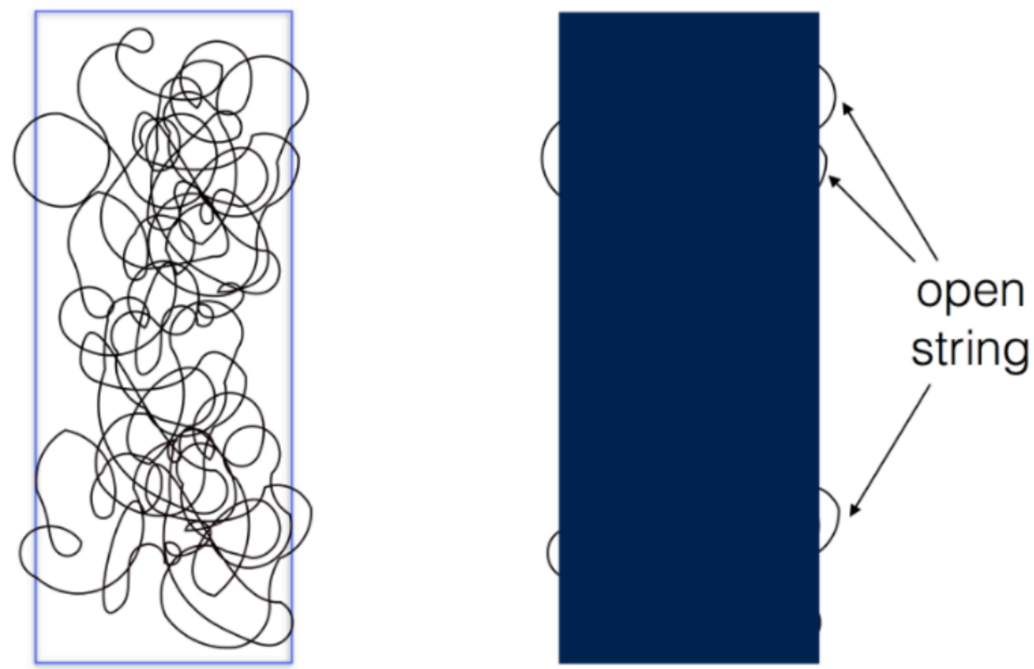


D-brane = condensation of string = deconfinement phase

Confinement phase → no D-brane

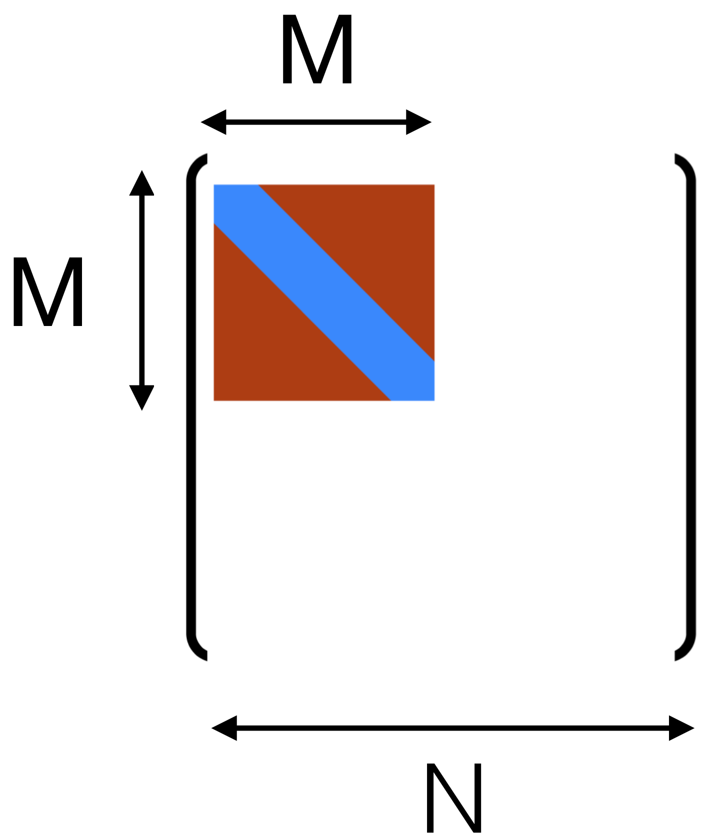
Deconfinement phase → N D-branes

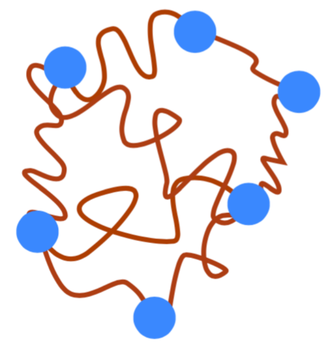
Partial Deconfinement Picture of Yang-Mills and String



D-brane = condensation of string
= deconfined sector

'SU(N) theory describes <u>N or less</u> D-branes + strings'





Bound state of M D-branes

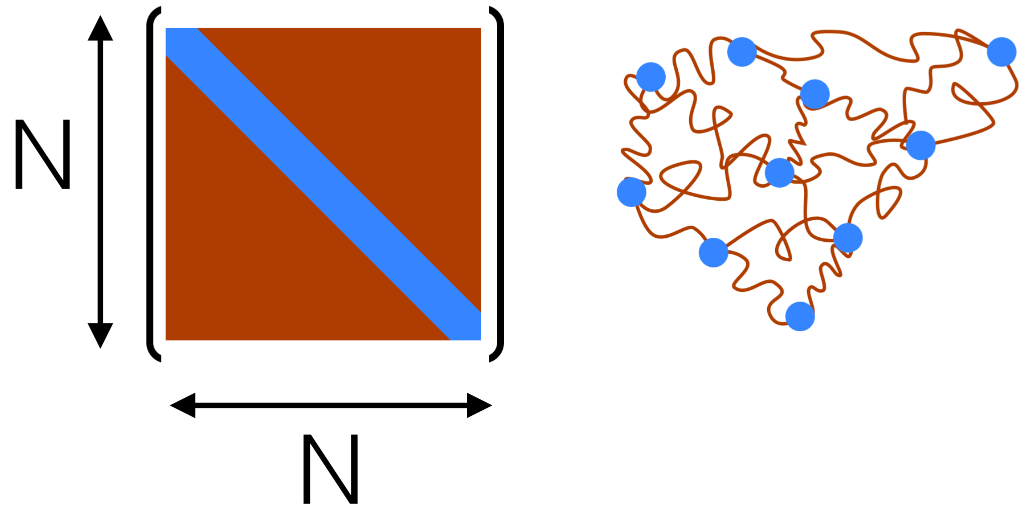
U(M) is deconfined — ‘partial deconfinement’

It can explain $E \sim N^2 T^{-7}$ for 4d SYM, $N^{3/2} T^{-8}$ for ABJM

(String Theory → 10d) (M-Theory → 11d)

(MH-Maltz, 2016)

Why is positive specific heat natural?

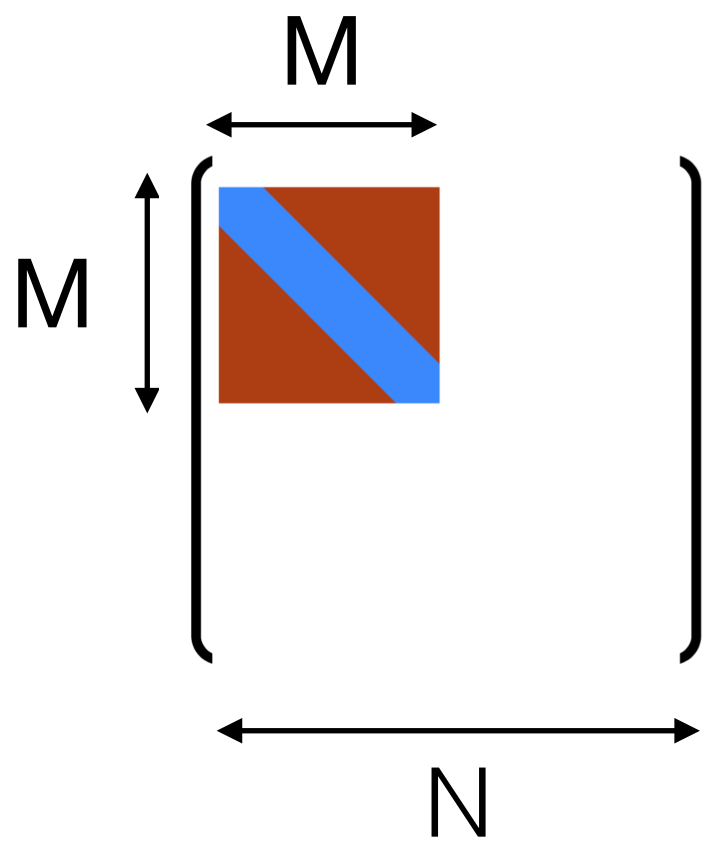


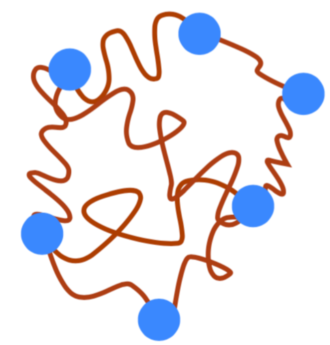
$T \sim E/N^2$

$T' \sim E'/N^2$

N^2 is fixed → $T' > T$ if $E' > E$

Why can negative specific heat appear?





$T \sim E/M^2$

M is a function of E

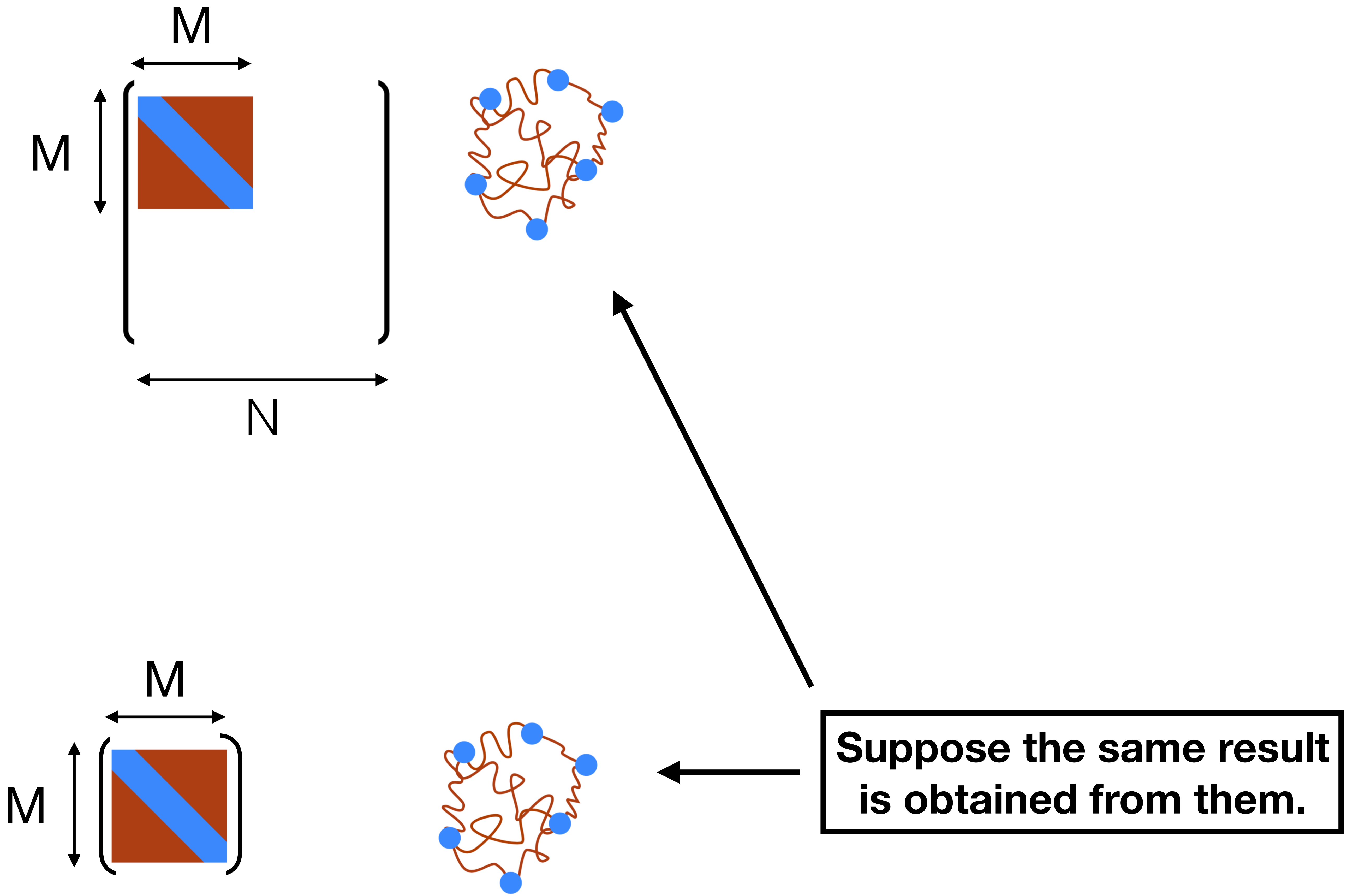
Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

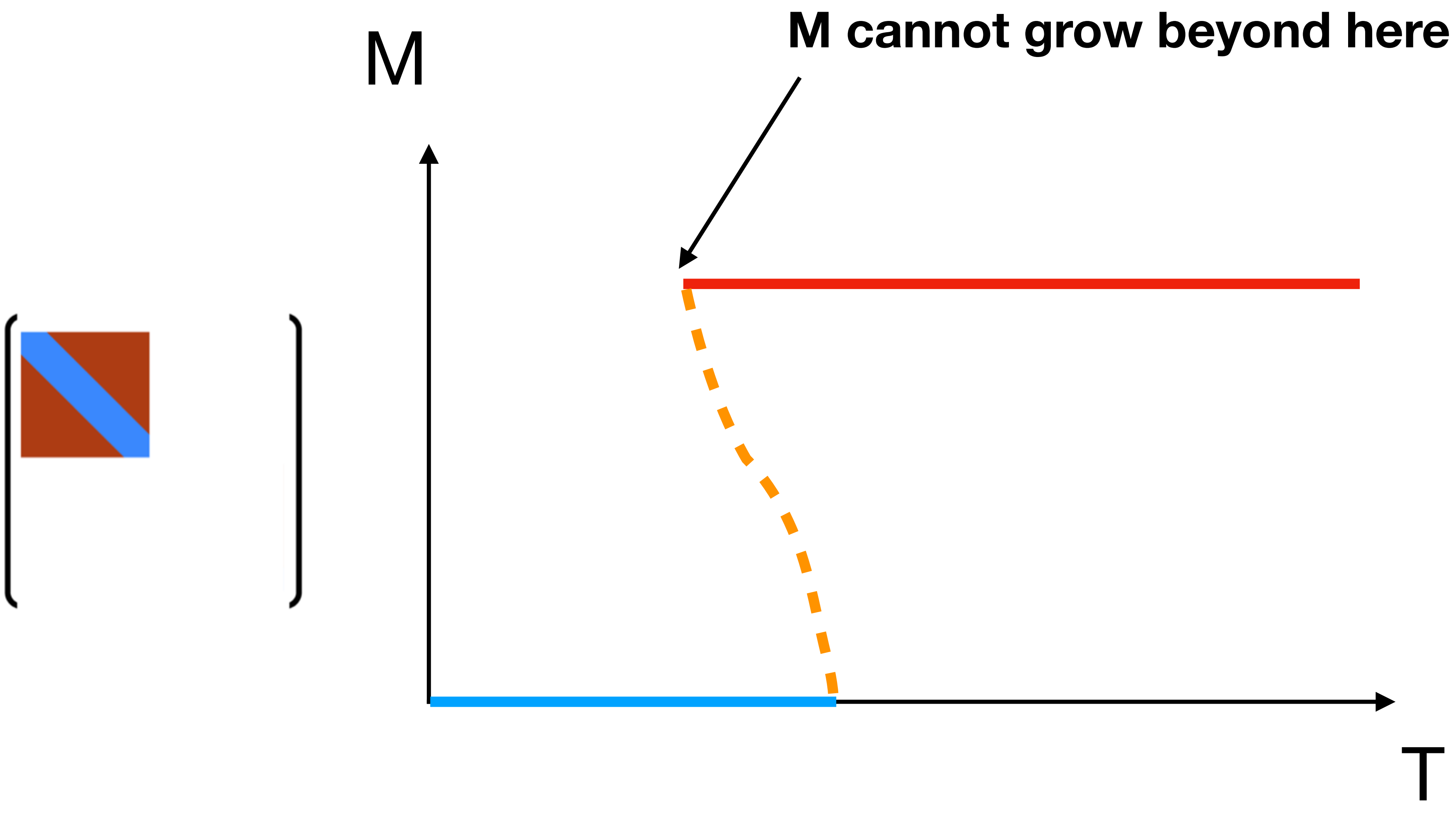
M.H.-Maltz, 2016, JHEP

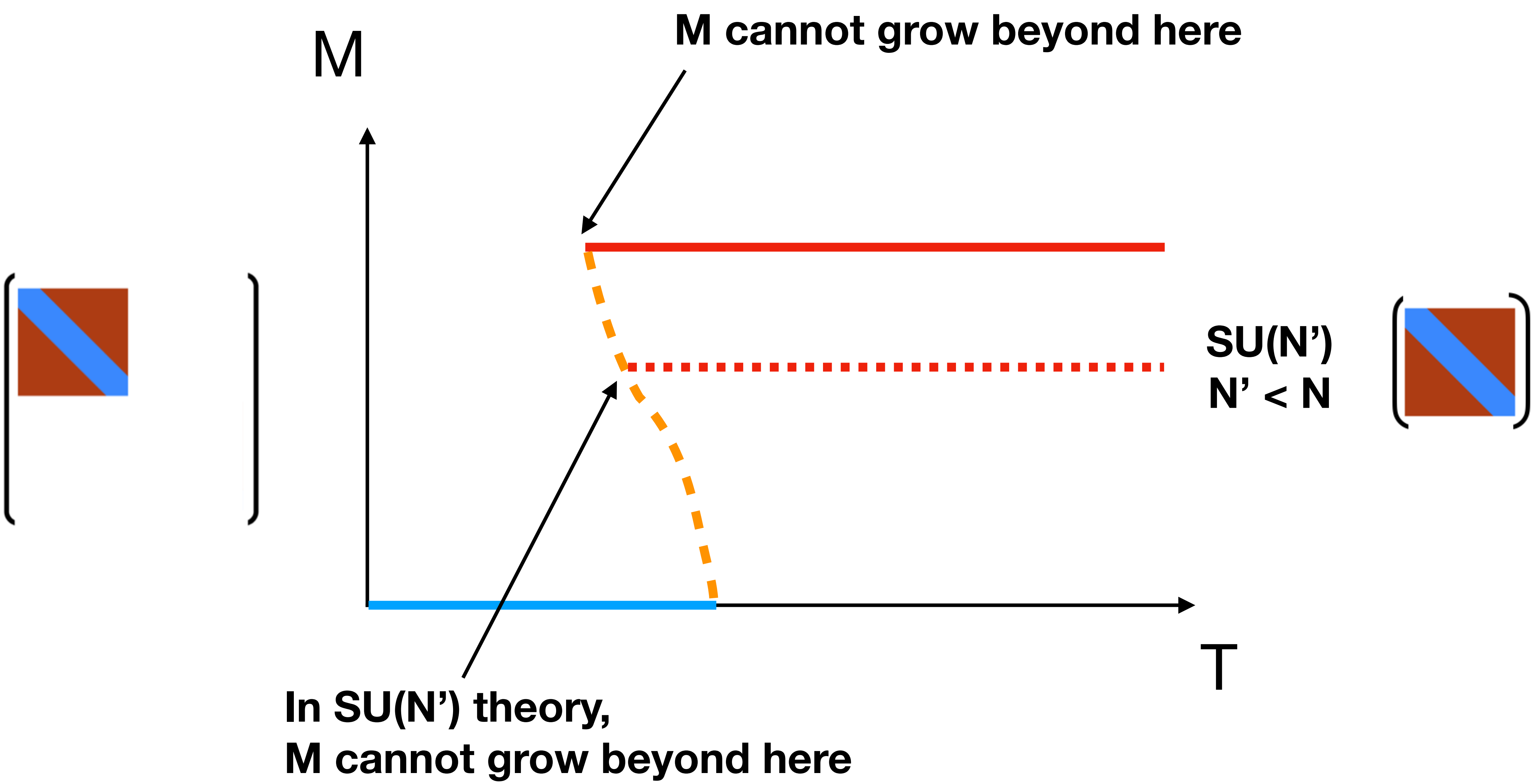
Explicit demonstration

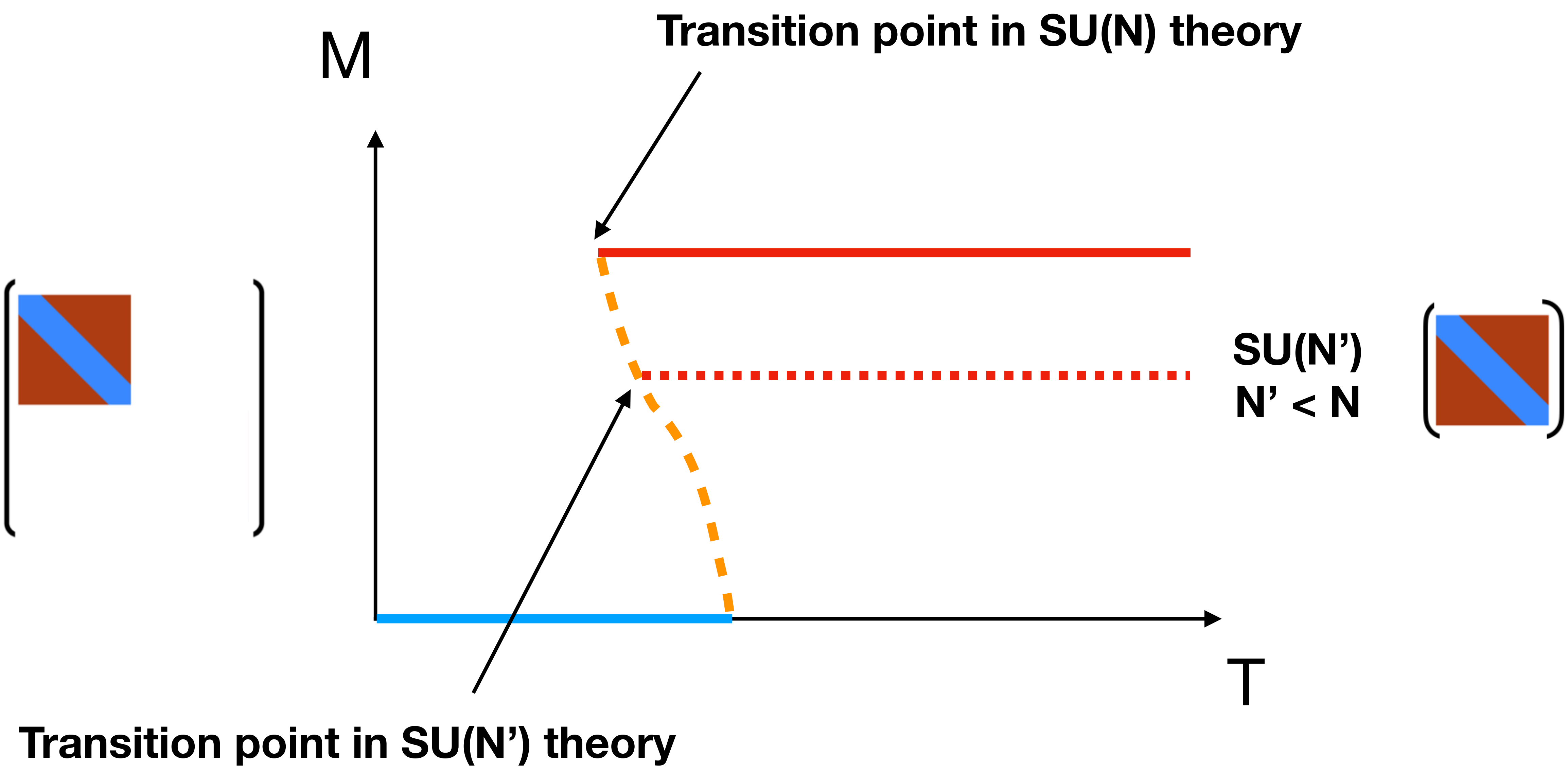
in simple theories

M.H., Jevicki, Peng, Wintergerst, 1909.09118 [hep-th]

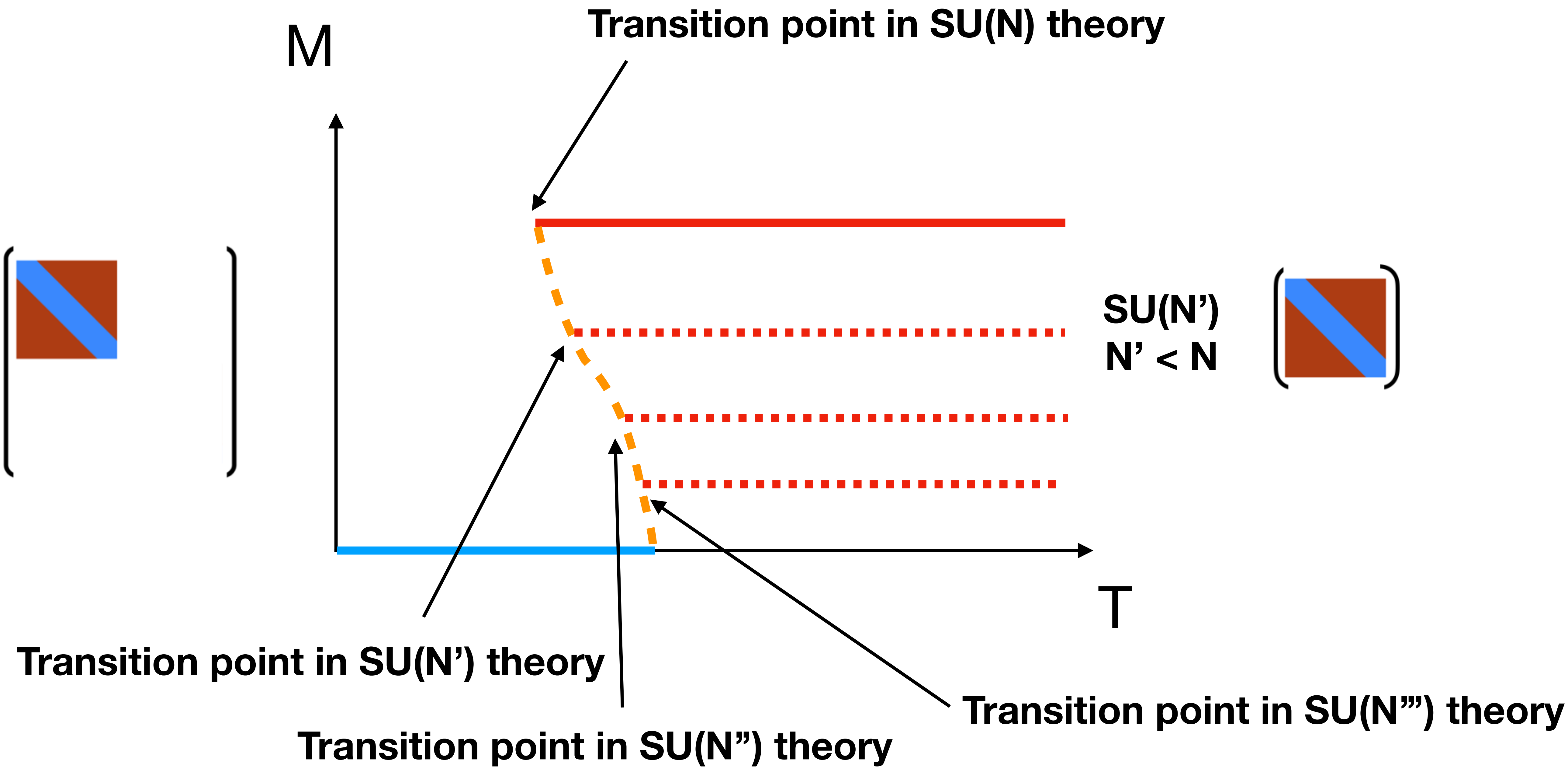

M
M
N
M
M
Suppose the same result is obtained from them.


M cannot grow beyond here
M
T


M cannot grow beyond here
M
SU(N')
N' < N
T
In SU(N') theory,
M cannot grow beyond here



M.H., Maltz, 2016



$$N'''' < N'' < N' < N$$

M.H., Maltz, 2016

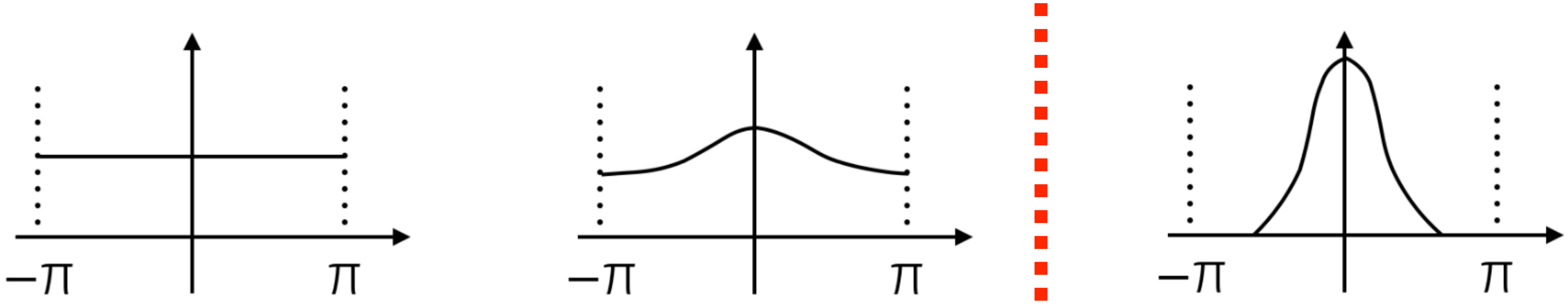
- Polyakov loop $$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:

Gross-Witten-Wadia transition (GWW)

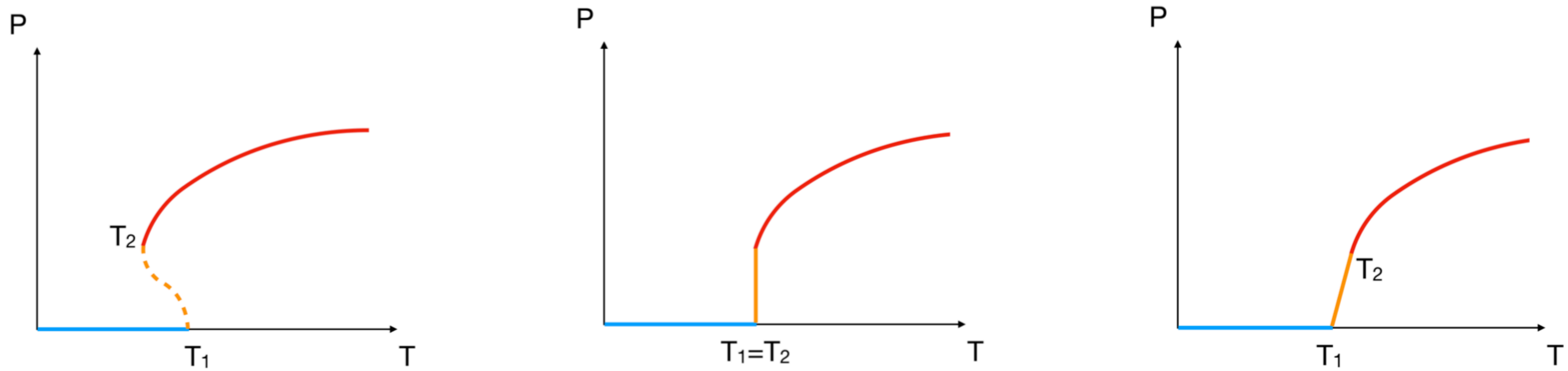
confined phase
P=0

deconfined phase
$P \neq 0$

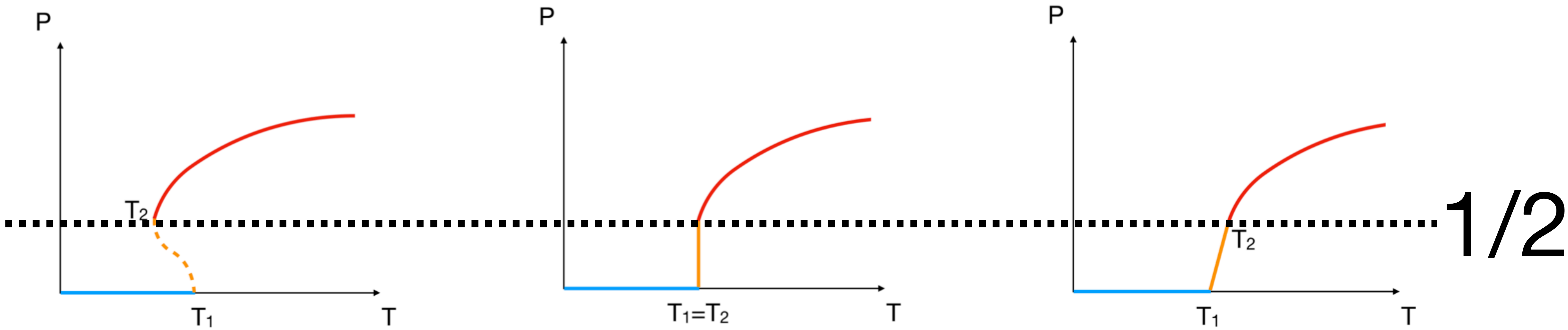


‘partially’ deconfined

‘completely’ deconfined



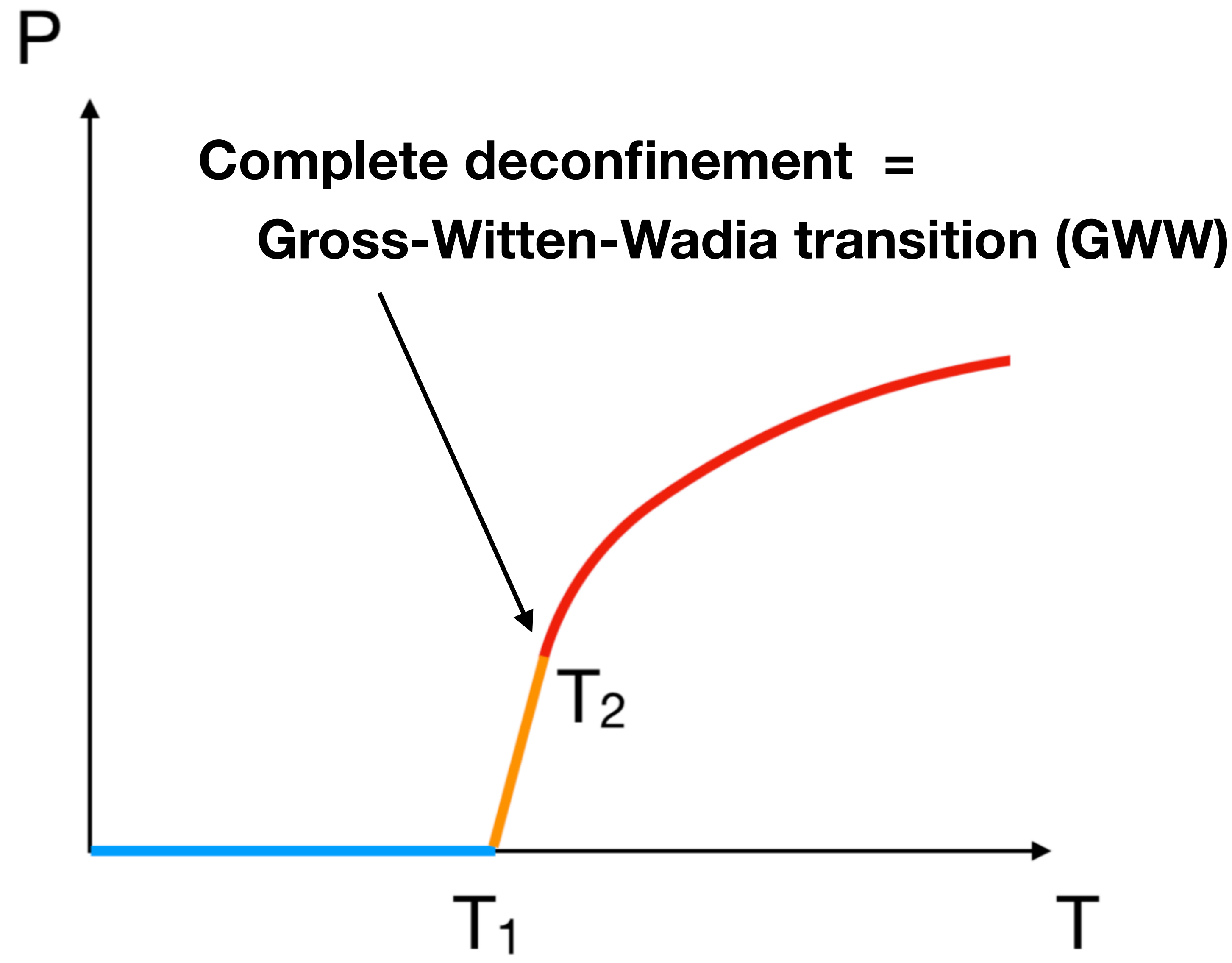
P
T
T_2
T_1
P
T
T_1=T_2
P
T
T_2
T_1

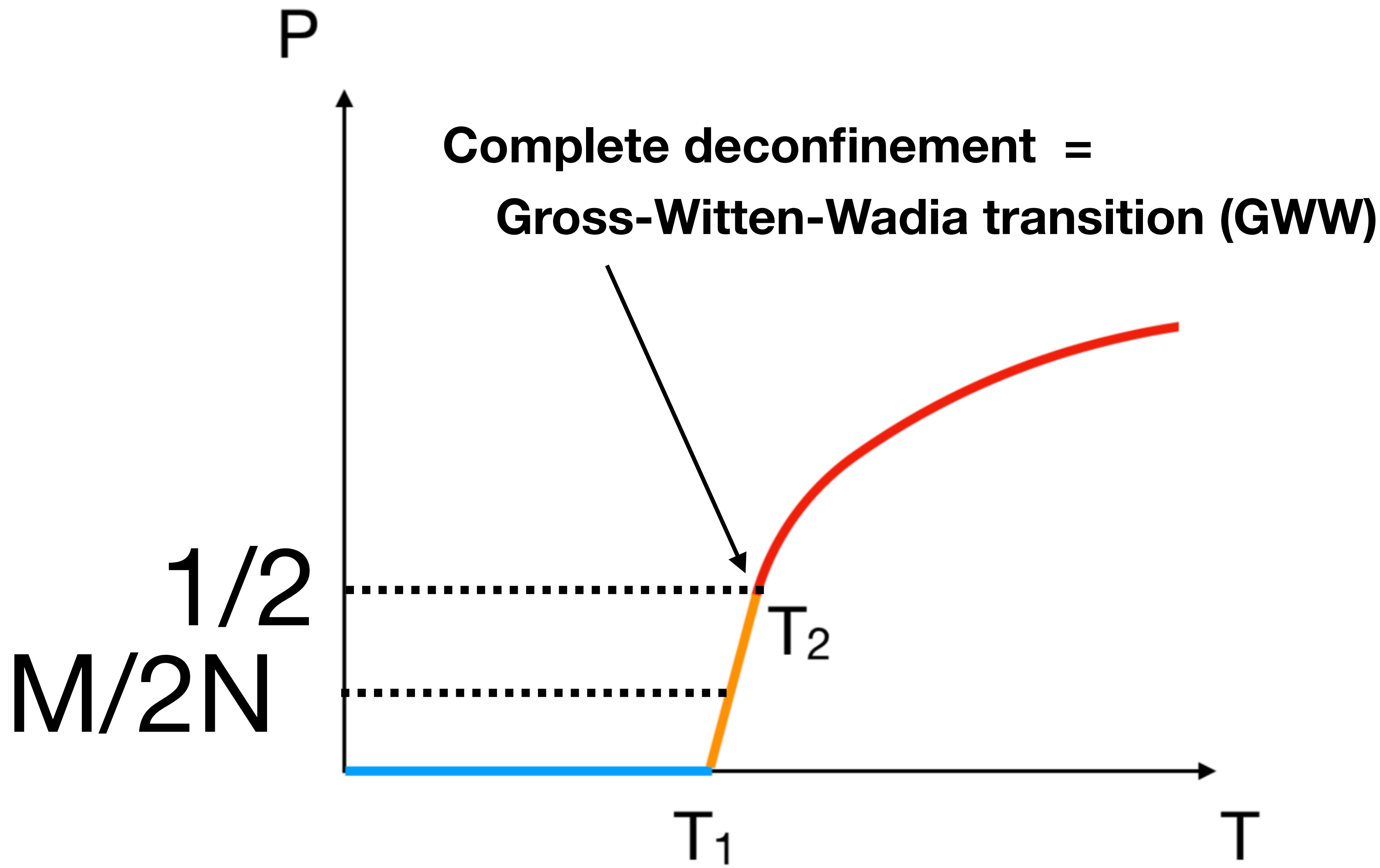


$$
\begin{aligned}
\rho(\theta) &= \left(1-\frac{M}{N}\right)\rho_{\rm confine}(\theta)+\frac{M}{N}\cdot\rho_{\rm GWW}(\theta;M)\\
&= \frac{1}{2\pi}\left(1-\frac{M}{N}\right)+\frac{M}{N}\cdot\rho_{\rm GWW}(\theta;M).
\end{aligned}
$$

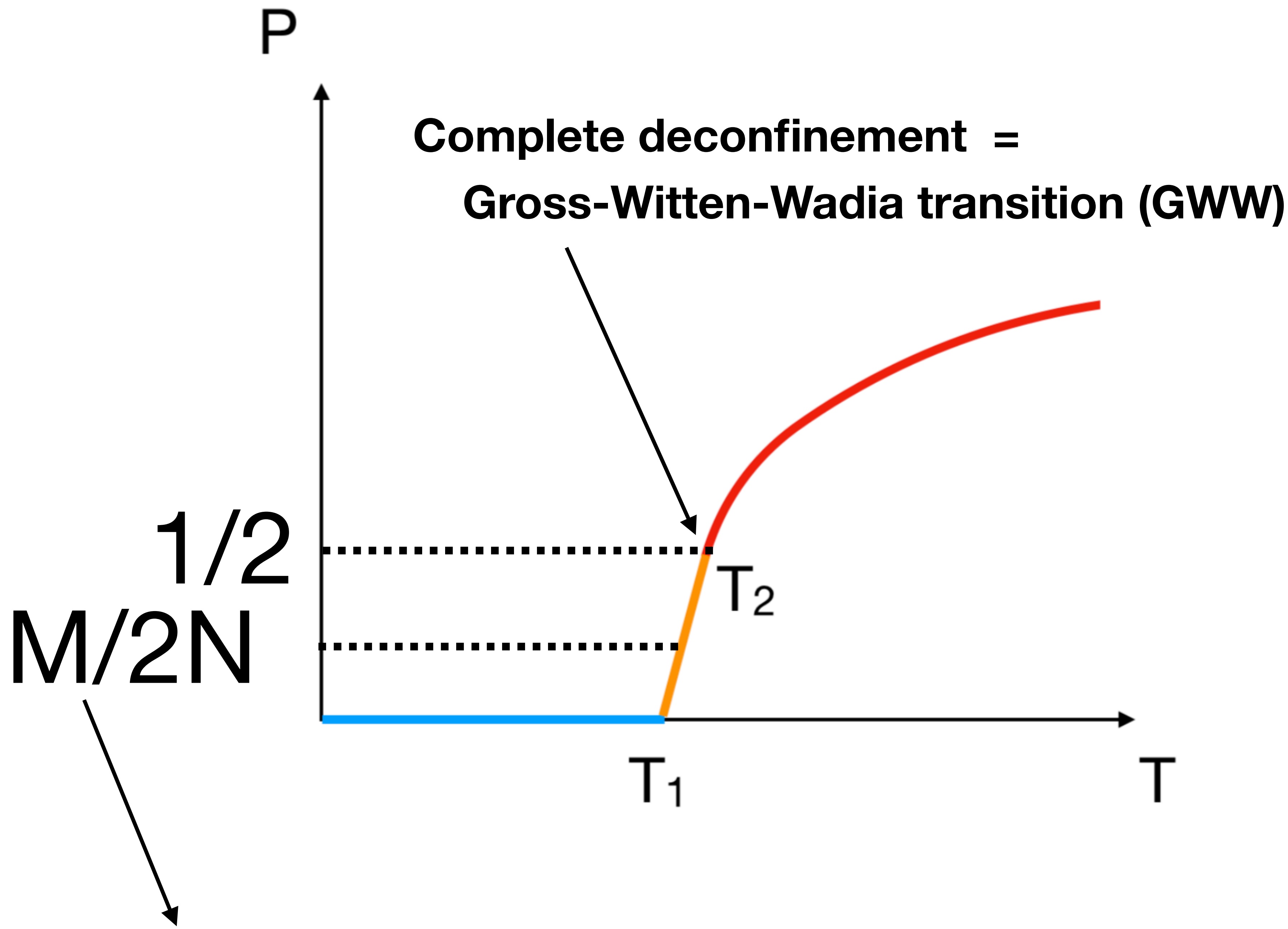
Holds in all examples we have studied.

M.H.-Ishiki-Watanabe, 2018


P
Complete deconfinement =
Gross-Witten-Wadia transition (GWW)
T2
T1
T



P
Complete deconfinement =
Gross-Witten-Wadia transition (GWW)
1/2
M/2N
T2
T1
T



$$E = E_{\mathrm{GWW}}(M)$$

$$S = S_{\mathrm{GWW}}(M)$$

Simplest Example:

Gauged Gaussian Two Matrix Model

$$\hat{H} = \frac{1}{2}\mathrm{Tr}\left(\hat{P}_X^2 + \hat{X}^2 + \hat{P}_Y^2 + \hat{Y}^2\right)$$

(Other cases are very similar)

$$\hat{H} = \frac{1}{2}\mathrm{Tr}\left(\underbrace{\hat{P}_X^2 + \hat{X}^2}_{\hat{A},\,\hat{A}^\dagger} + \underbrace{\hat{P}_Y^2 + \hat{Y}^2}_{\hat{B},\,\hat{B}^\dagger}\right)$$

$$\hat{P}_X^2 + \hat{X}^2 \rightarrow \hat{A}, \hat{A}^\dagger \qquad \hat{P}_Y^2 + \hat{Y}^2 \rightarrow \hat{B}, \hat{B}^\dagger$$

$$\mathrm{Tr}\underbrace{\left(\hat{A}^\dagger \hat{A}^\dagger \hat{B}^\dagger \hat{A}^\dagger \cdots\right)}_{L}|0\rangle$$

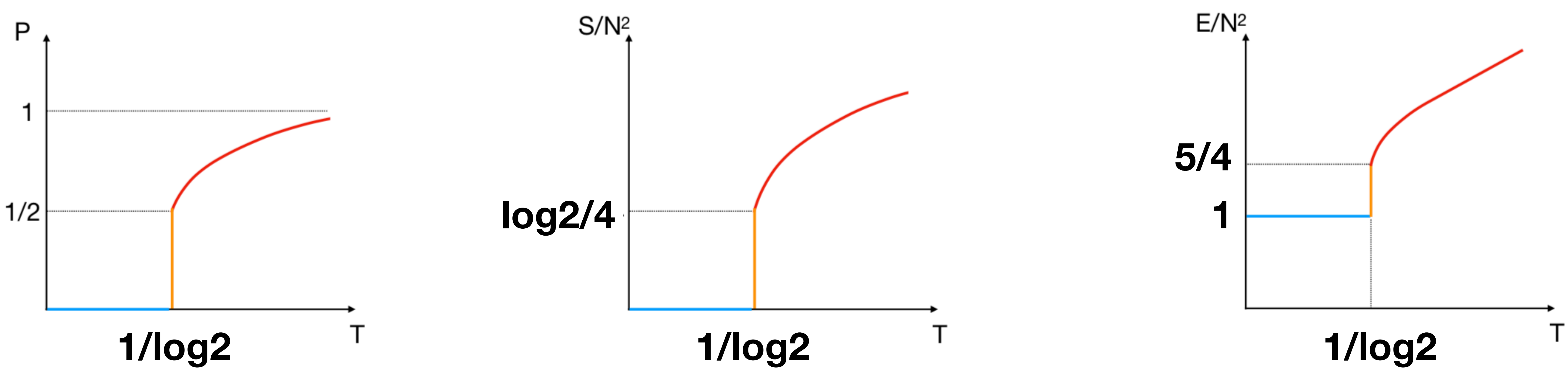
$E = L$ (up to zero-pt energy)

$S = L\log 2$ (# of states ~ 2^L)

$$F = E - TS = L(1 - T\log 2)$$

(up to zero-pt energy)

$$F = 0 \quad @ \quad T = \frac{1}{\log 2}$$



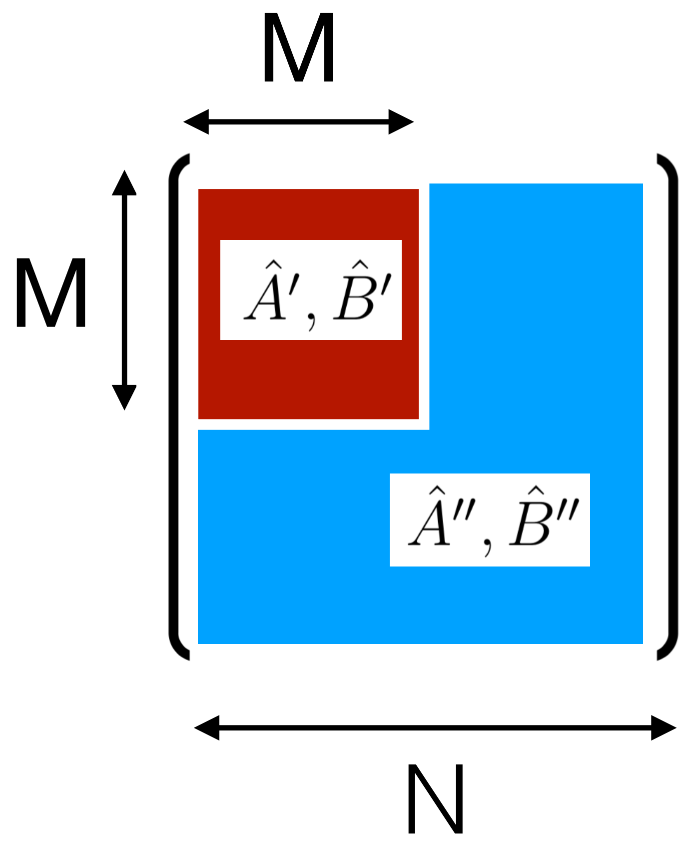
$$E(T=T_c,P,N)=N^2+N^2P^2=N^2+\frac{M^2}{4}$$

$$S(T=T_c,P,N)=\frac{M^2}{4}\log 2$$

$$\begin{aligned}\rho(\theta)=\frac{1}{2\pi}\left(1+2P\cos\theta\right) &= (1-2P)\cdot\frac{1}{2\pi}+2P\cdot\frac{1}{2\pi}\left(1+\cos\theta\right)\\ &= \left(1-\frac{M}{N}\right)\cdot\frac{1}{2\pi}+\frac{M}{N}\cdot\frac{1}{2\pi}\left(1+\cos\theta\right)\end{aligned}$$

$$\boxed{\begin{aligned}E&=E_{\rm GWW}(M)\\ S&=S_{\rm GWW}(M)\end{aligned}}$$

We will construct the states explicitly,
and demonstrate the partial deconfinement.



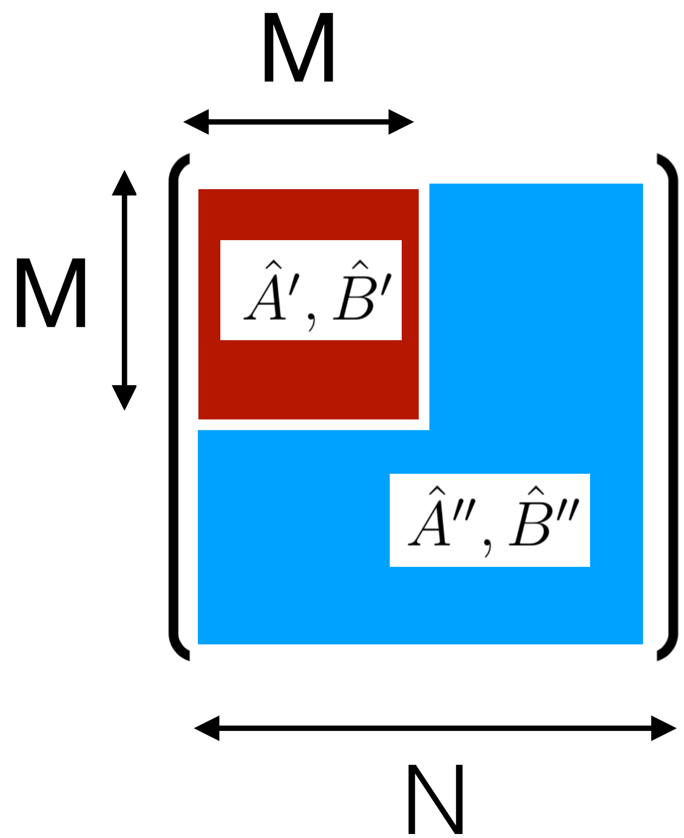
not SU(N)-invariant

$$|E; \mathrm{SU}(M)\rangle = \mathrm{Tr}\left(\hat{A}'^{\dagger}\hat{A}'^{\dagger}\hat{B}'^{\dagger}\hat{A}'^{\dagger}\cdots\right)|0\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\mathrm{GWW}}(M)$$

M.H.-Jevicki-Peng-Wintergerts, 2019



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$$S = S_{\mathrm{GWW}}(M)$$

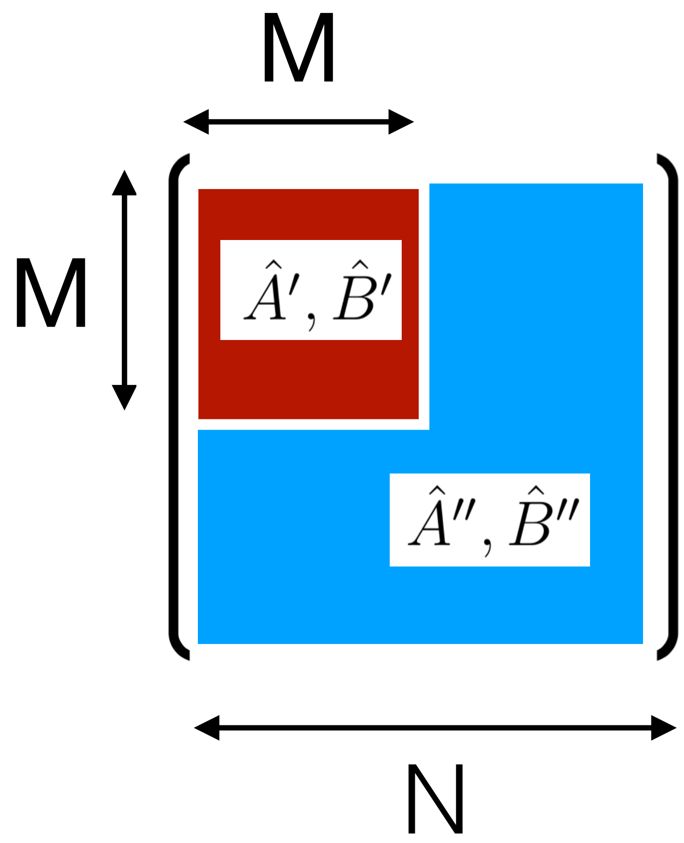
one-to-one correspondence

SU(N)-invariant

This is also an energy eigenstate.

$$|E\rangle_{\mathrm{inv}} \equiv \mathcal{N}^{-1/2}\int dU\, \mathcal{U}\left(|E; \mathrm{SU}(M)\rangle\right)$$

M.H.-Jevicki-Peng-Wintergerts, 2019



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This is also an energy eigenstate.

$$|E\rangle_{\mathrm{inv}} \equiv \mathcal{N}^{-1/2}\int dU\, \mathcal{U}\left(|E; \mathrm{SU}(M)\rangle\right)$$

<u>These states explain the entropy precisely.</u>

M.H.-Jevicki-Peng-Wintergerts, 2019

‘Spontaneous gauge symmetry breaking’

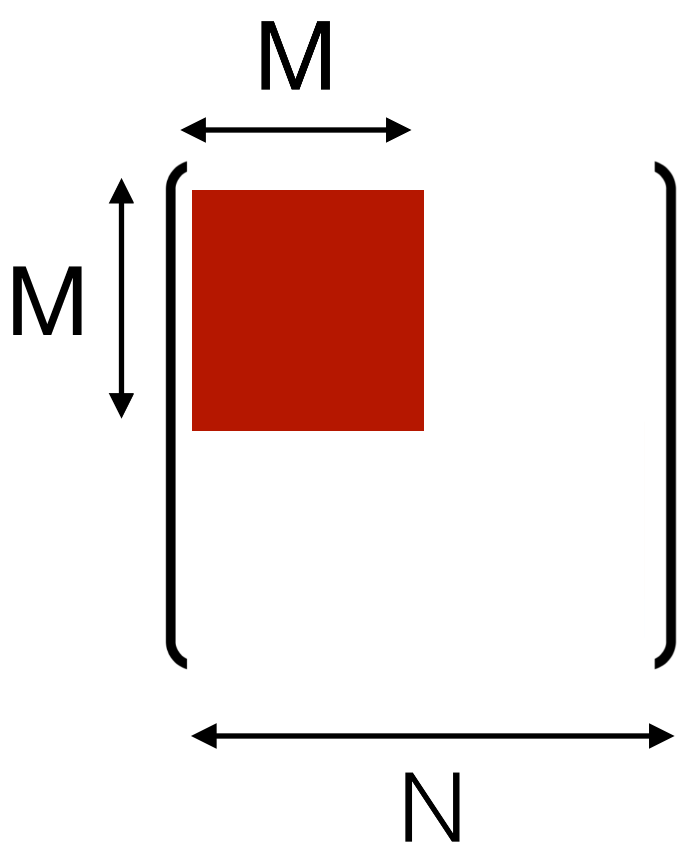
M.H.-Jevicki-Peng-Wintergerts, 2019

SU(N)-invariant

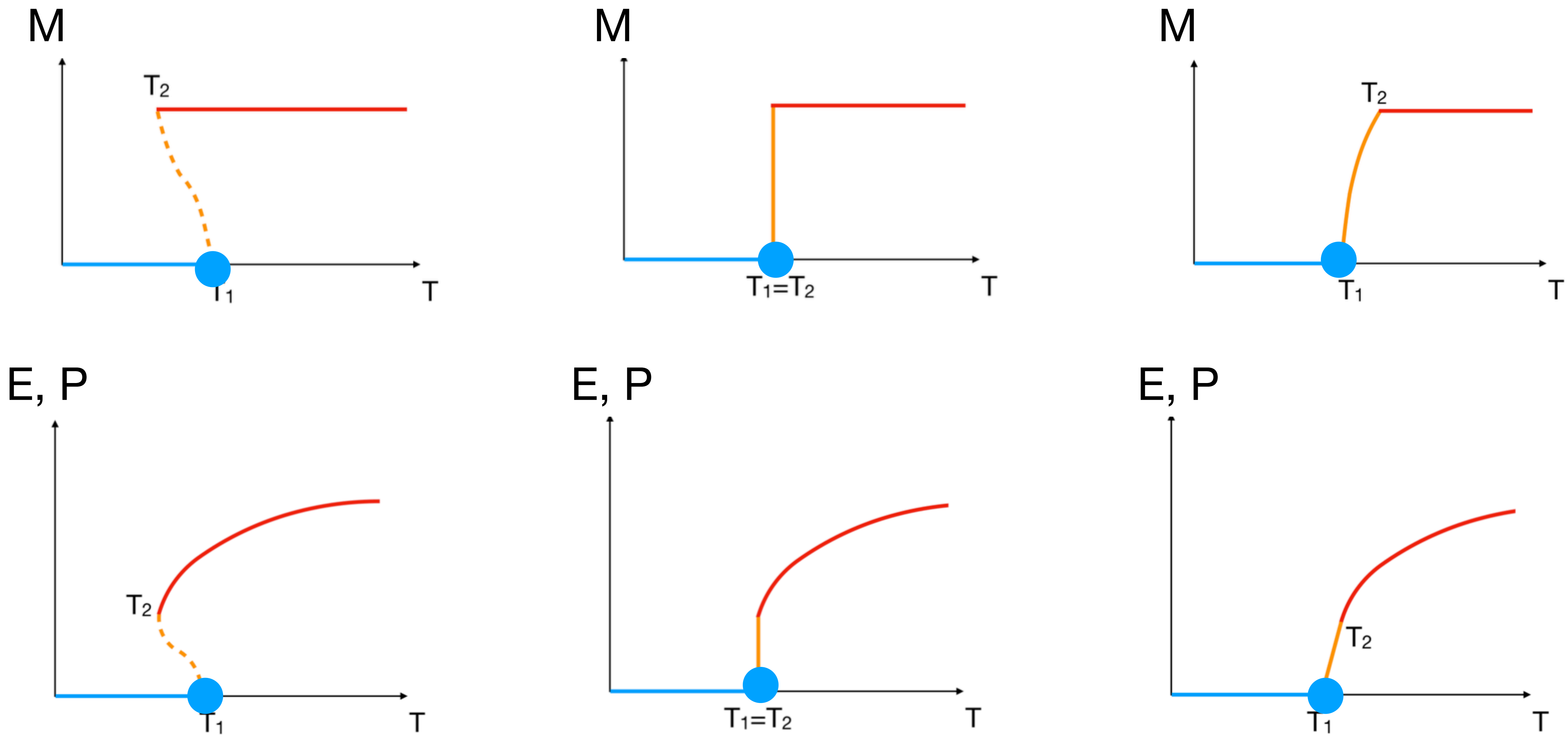
$$|E\rangle_{\rm inv} \equiv \mathcal{N}^{-1/2} \int dU\, \mathcal{U}\left(|E; {\rm SU}(M)\rangle\right)$$

↓ gauge fixing

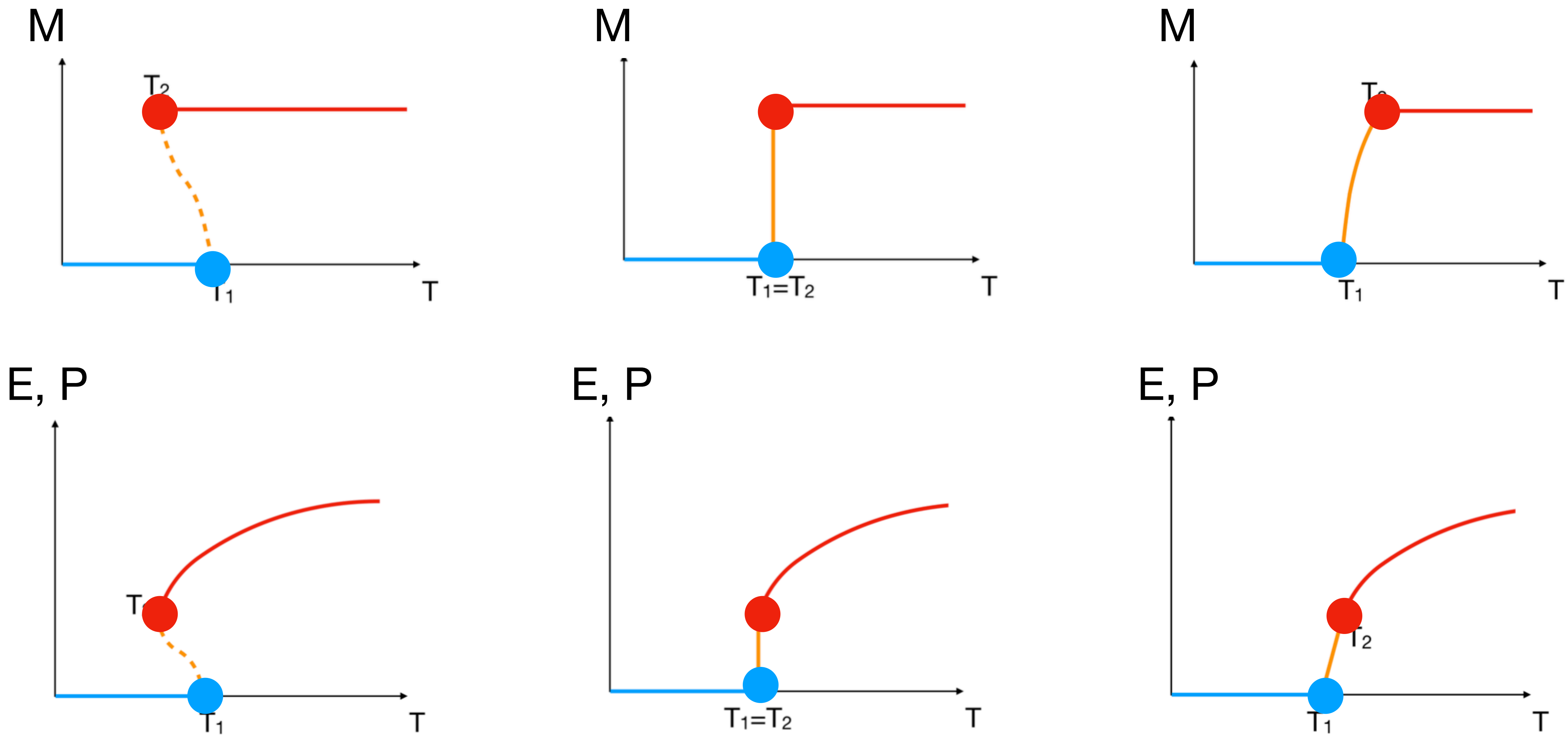
not SU(N)-invariant $|E; {\rm SU}(M)\rangle$



- Global part of gauge symmetry breaks spontaneously.
- It is convenient to fix the local part, like usual Higgsing.
- ‘Gauge symmetry breaking’ provides us with a ‘useful fiction’ which makes physics understandable.



transition 1: confinement to partial deconfinement
(black hole formation begins)



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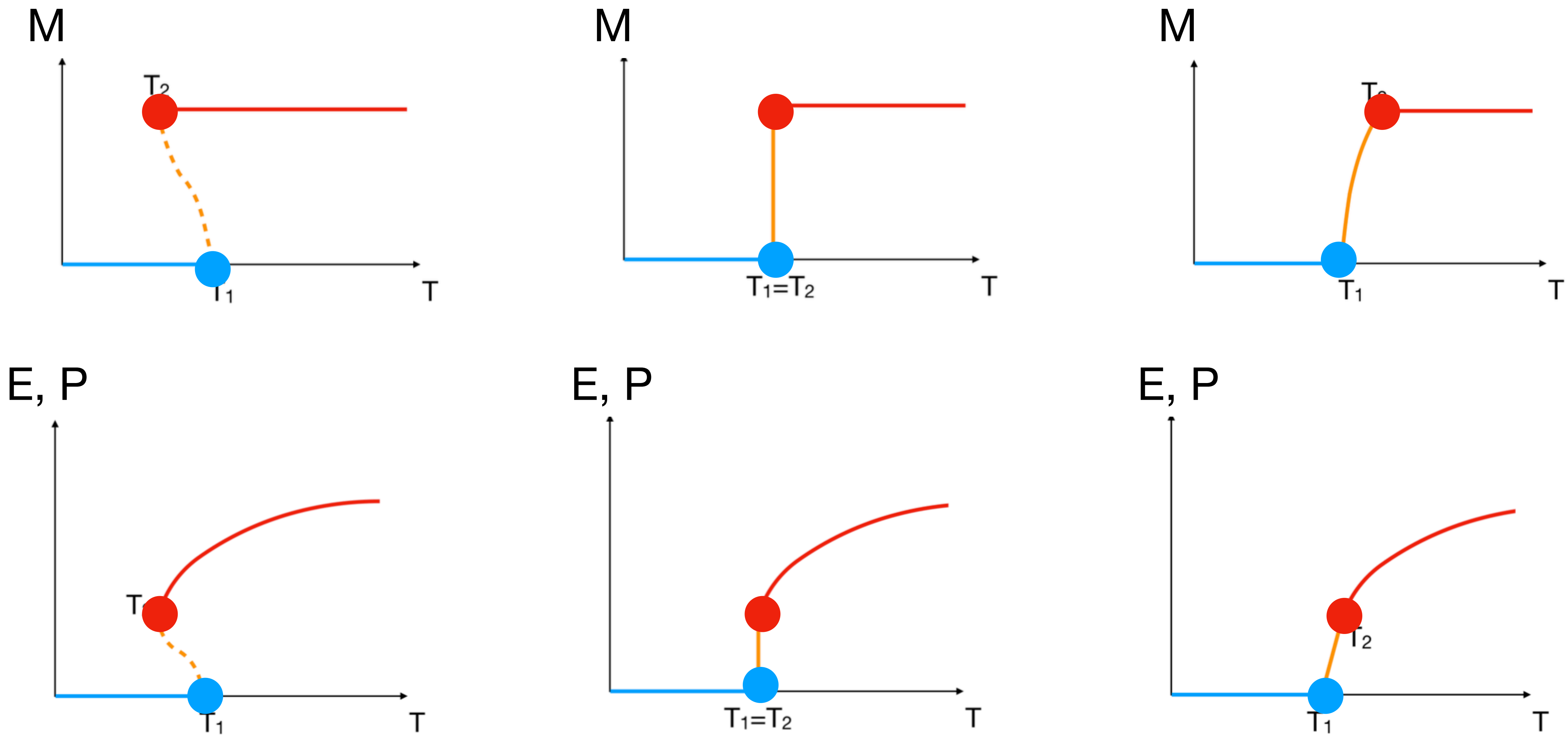

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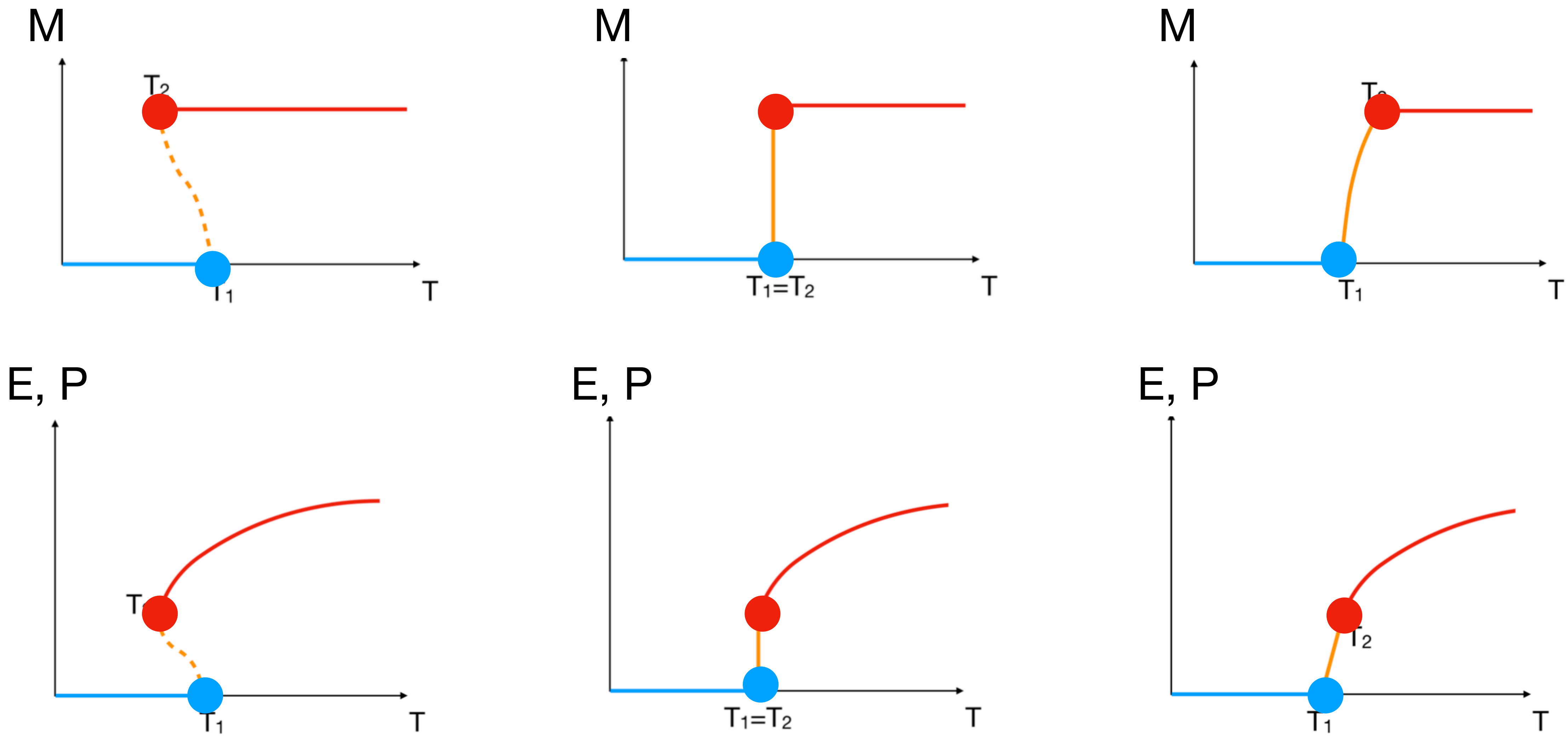
$$SU(N) \to SU(M)\times SU(N-M)\times U(1) \to SU(N)$$



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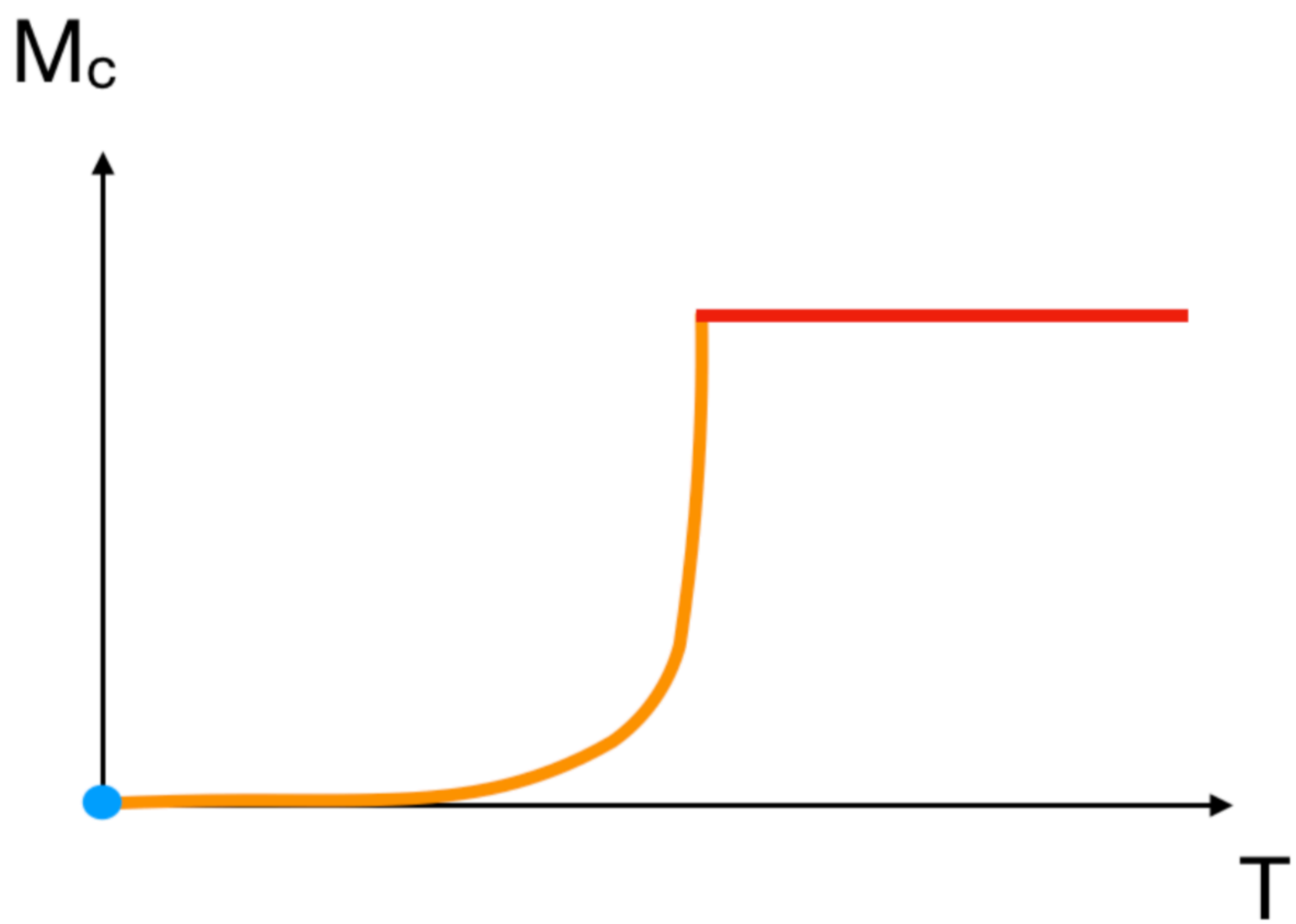
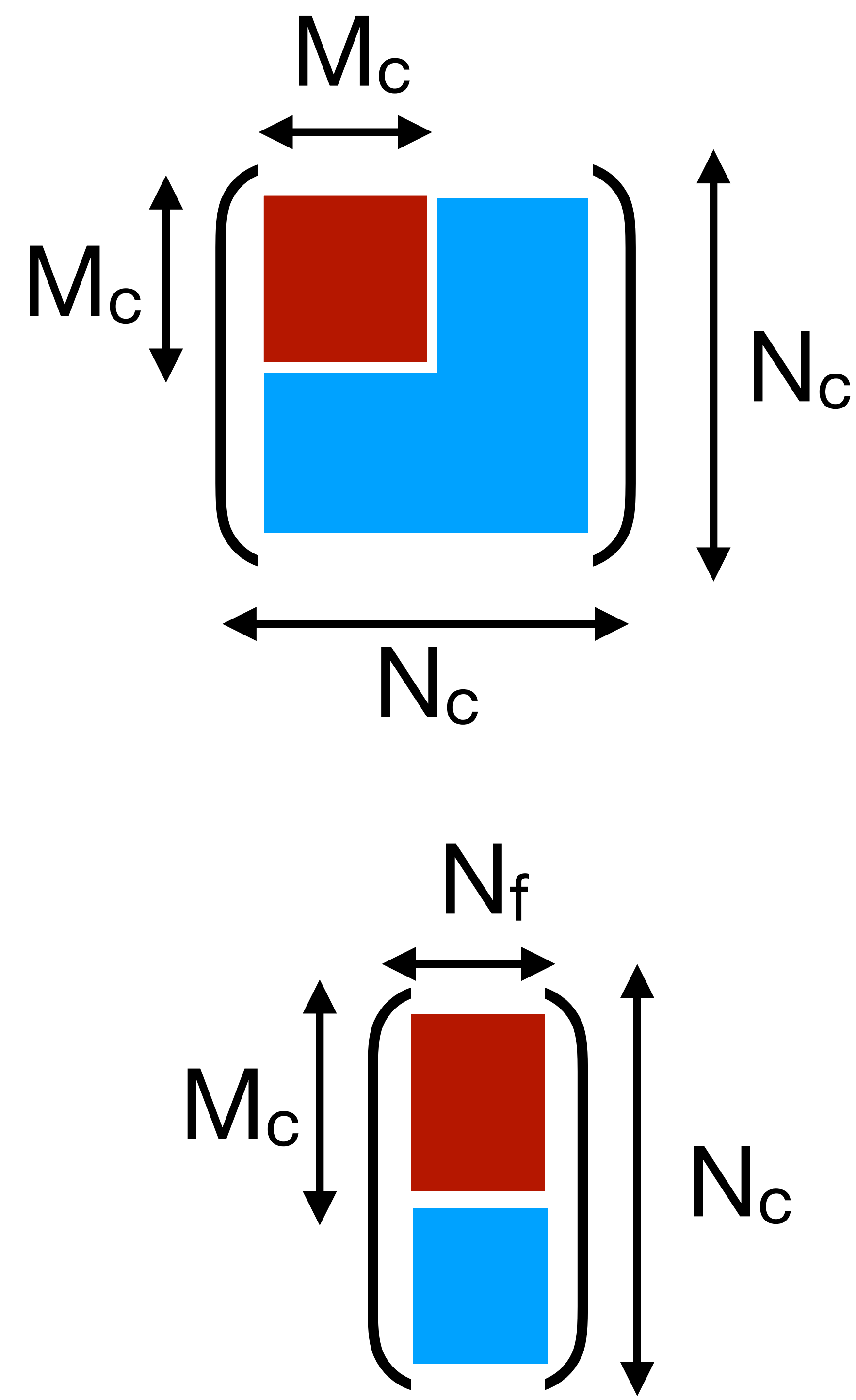
No need for center symmetry. No need for chiral symmetry. Applies to QCD.

Weakly-coupled QCD on S^3





Quantum Entanglement

between color d.o.f.

- Typically, ground state of interacting system is highly entangled.
- Thermal excitations can destroy the entanglement.

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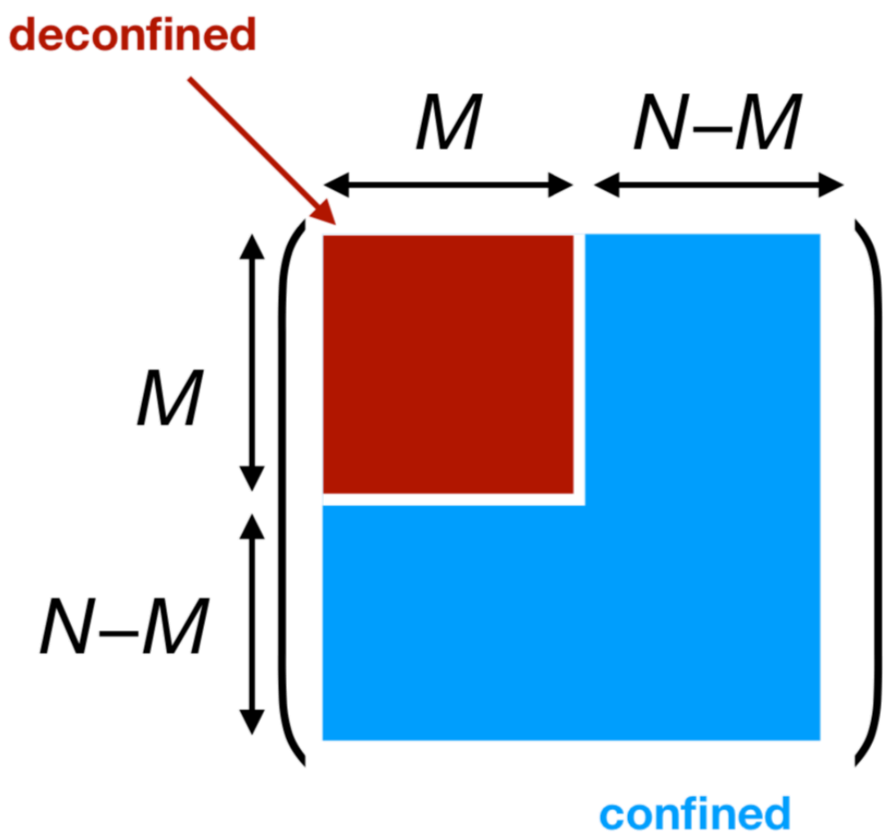

Confined → ground state up to 1/N corrections

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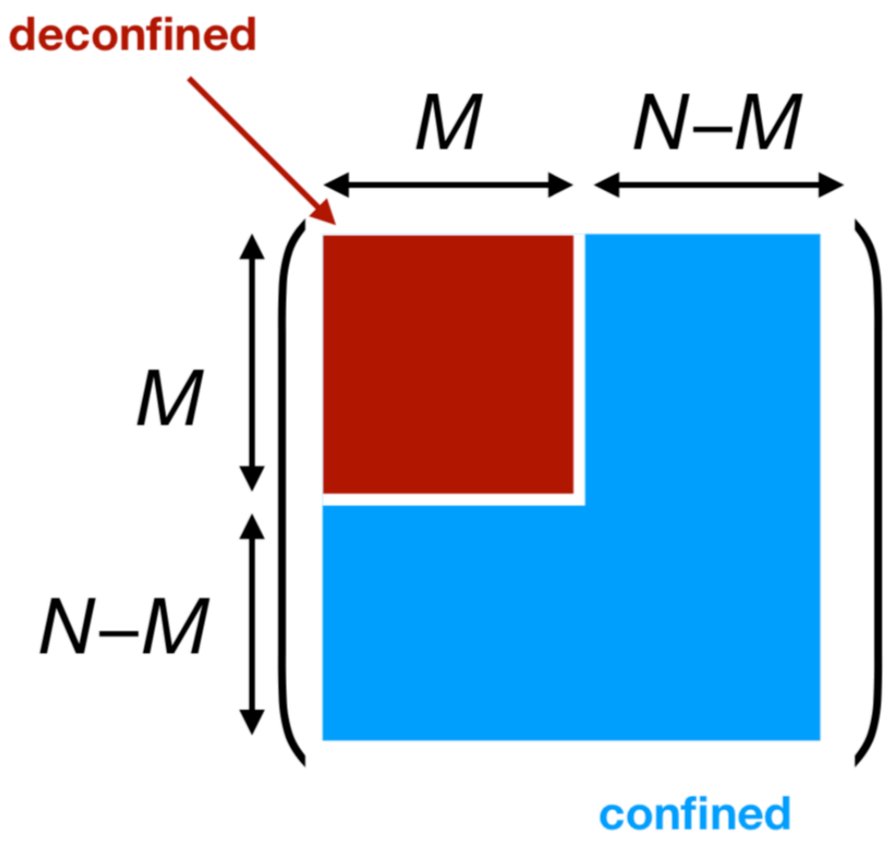
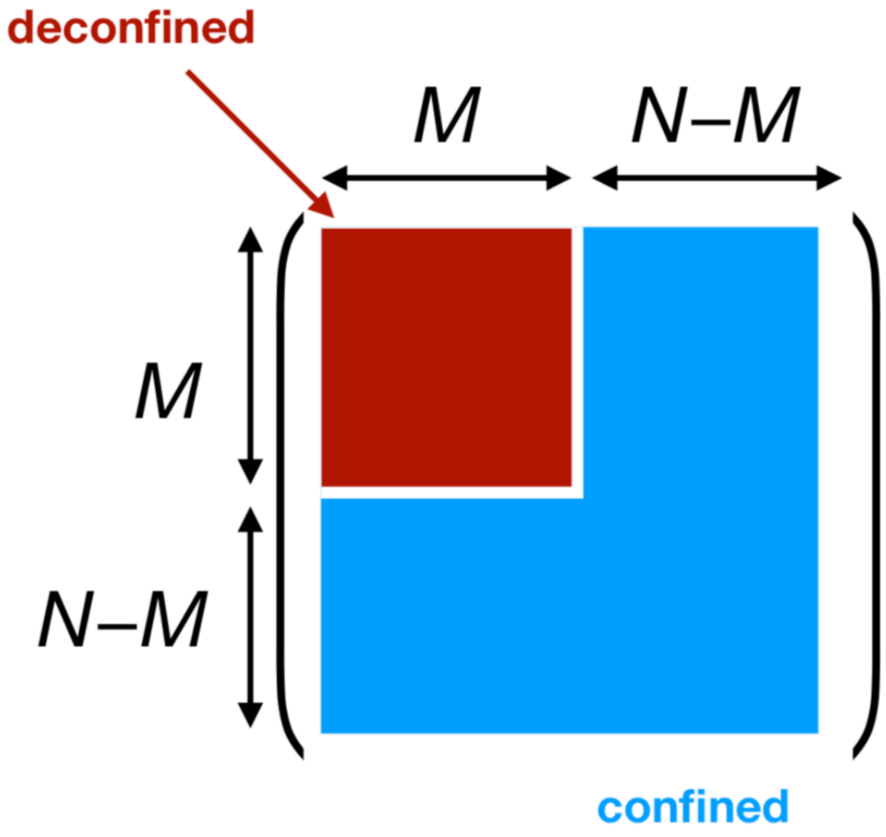
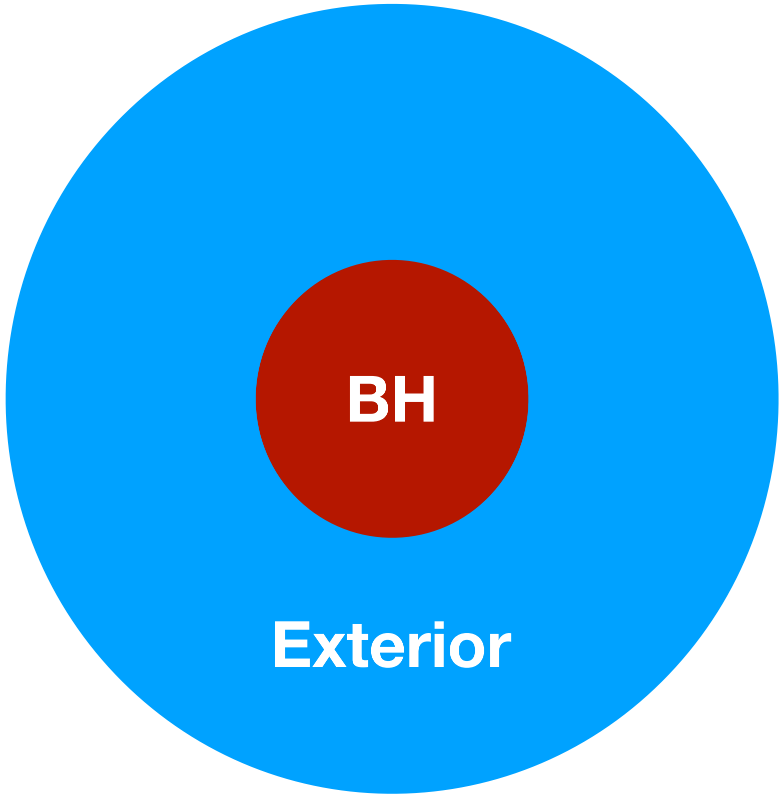


Confined → ground state up to 1/N corrections

Large entanglement can survive even at finite temperature.

gauge/gravity duality





Entanglement between color d.o.f.
→ geometry outside the horizon?

Future Directions

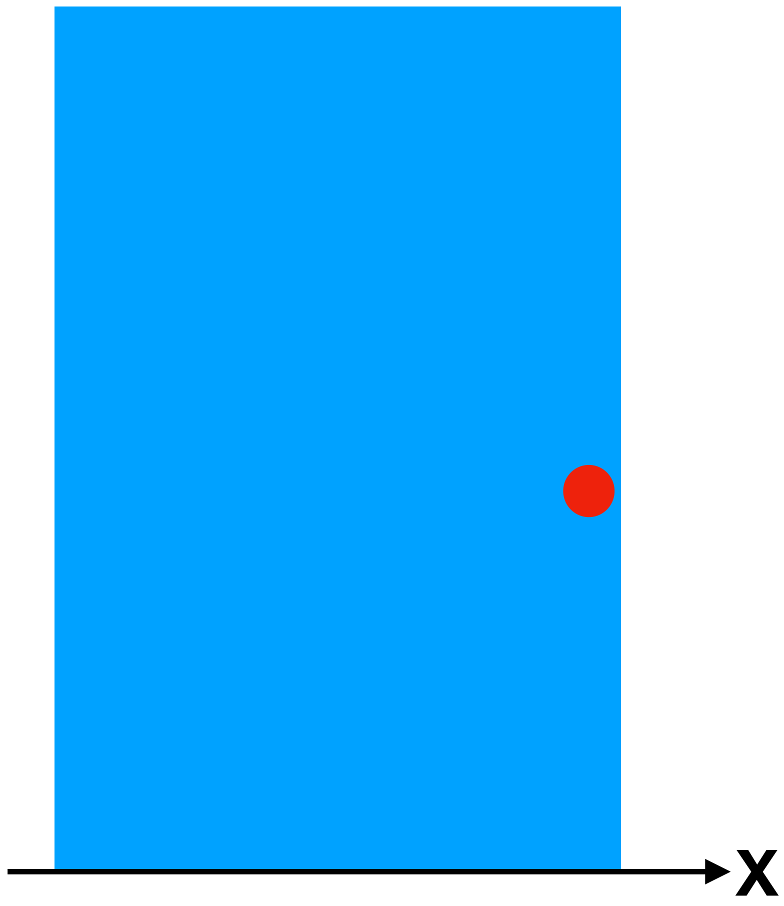
Hiromasa Watanabe is visiting us from Univ. Tsukuba, until Oct 31.

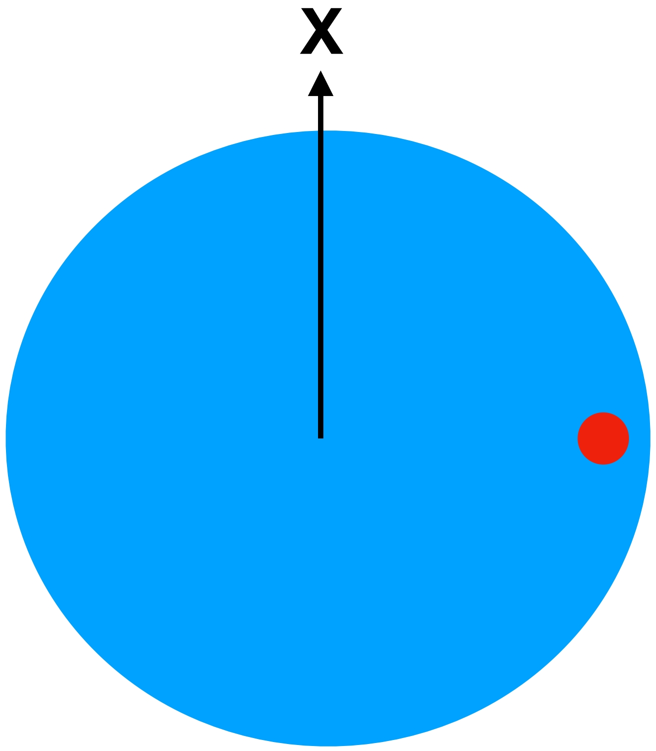
- We can use lattice simulation for nonperturbative study.
- We can get actual matrices as lattice configurations.
- It should be possible to separate color degrees of freedom to “black hole” and “exterior” by fixing gauge.

Get Gauge Fixing Done

More on
‘Bulk from Matrices’?

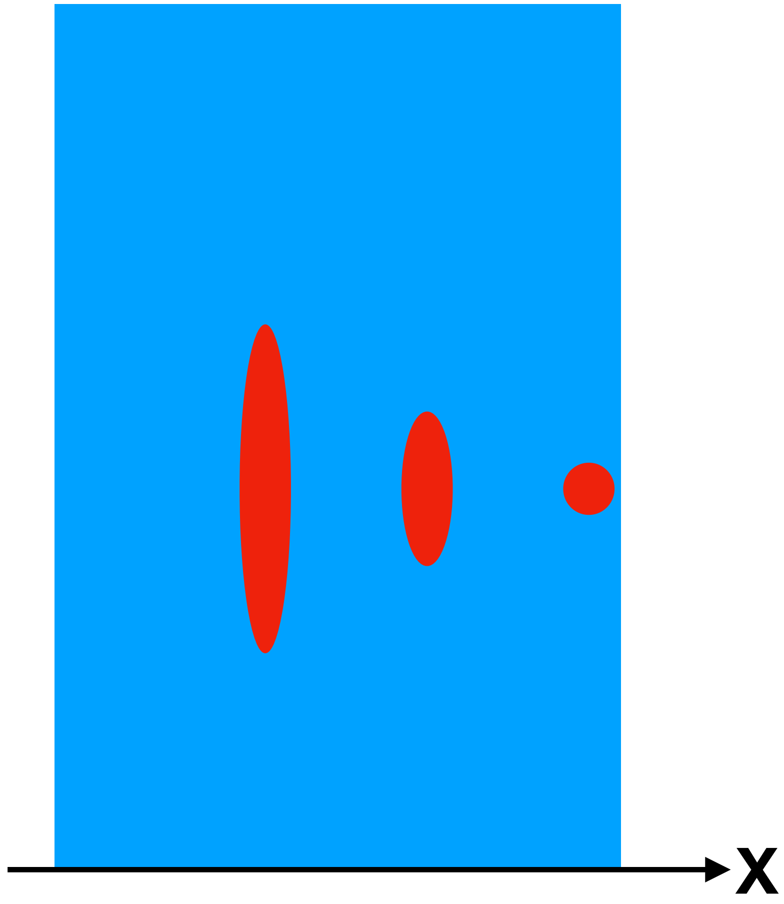
(popped up in my mind last night)

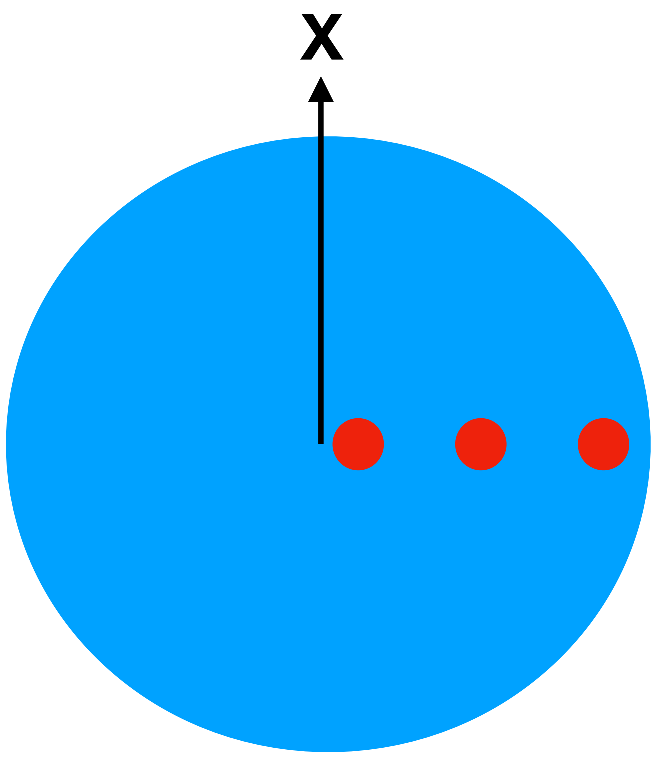




Local operator adds energy to the state and creates small deconfined block

‘Boundary’ = large X = high energy





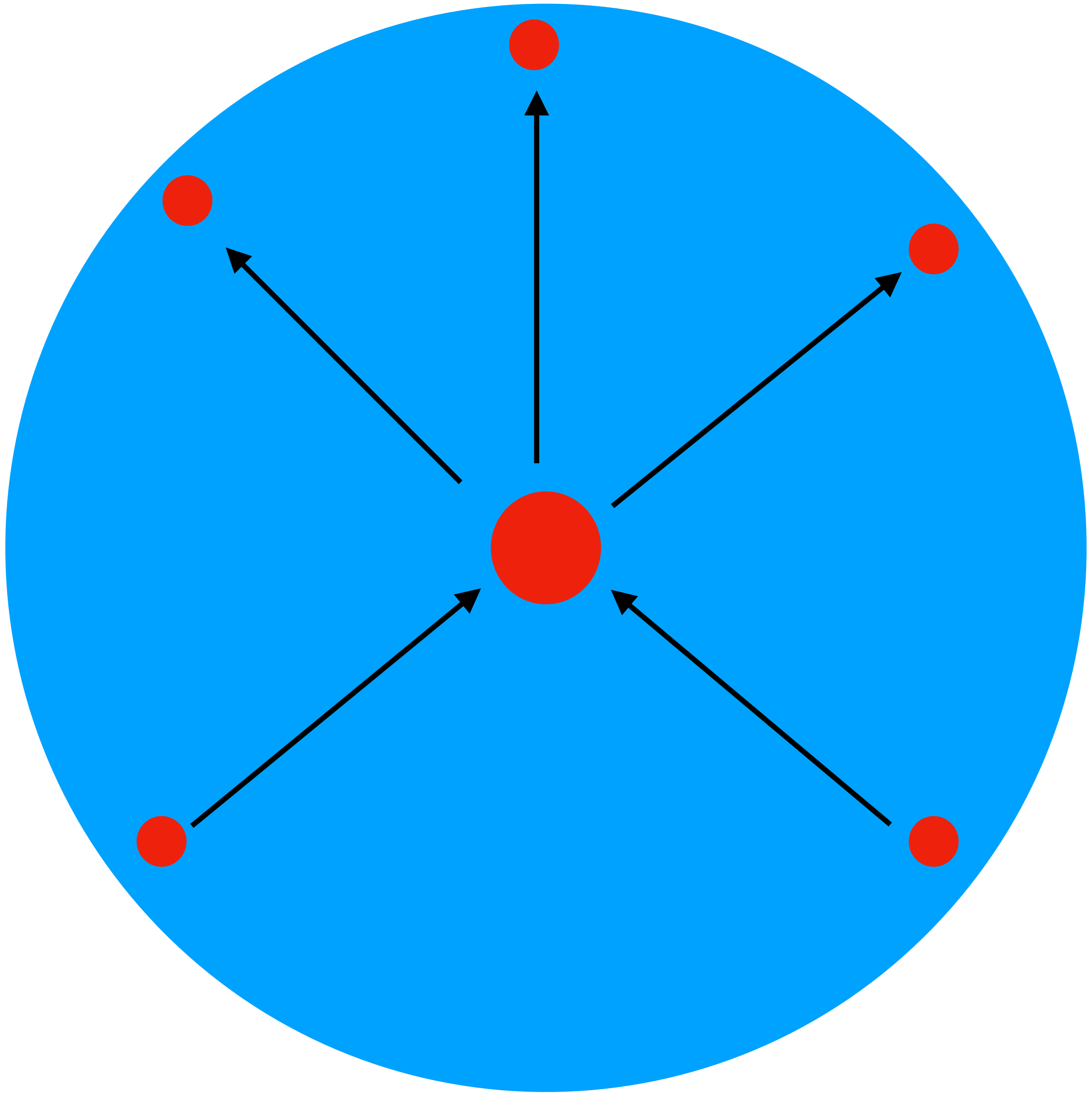
Local operator adds energy to the state and creates small deconfined block

‘Boundary’ = large X = high energy

‘Bulk’ = small X = low energy

Volume of the deconfined block increases (total energy fixed)

May lead to a better understanding about “space from colors”?



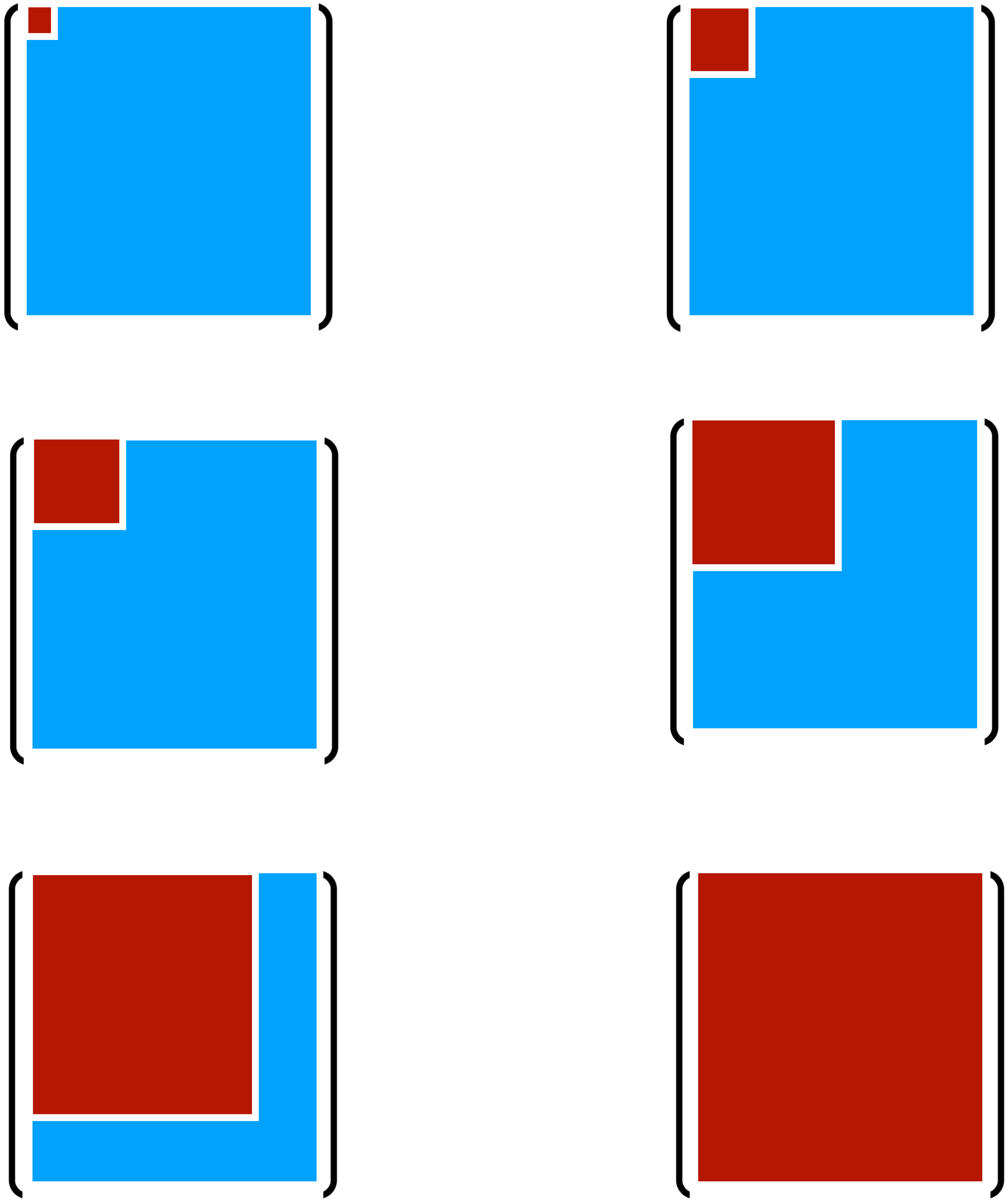
Bulk dynamics looks like Banks-Fischler-Shekner-Susskind picture

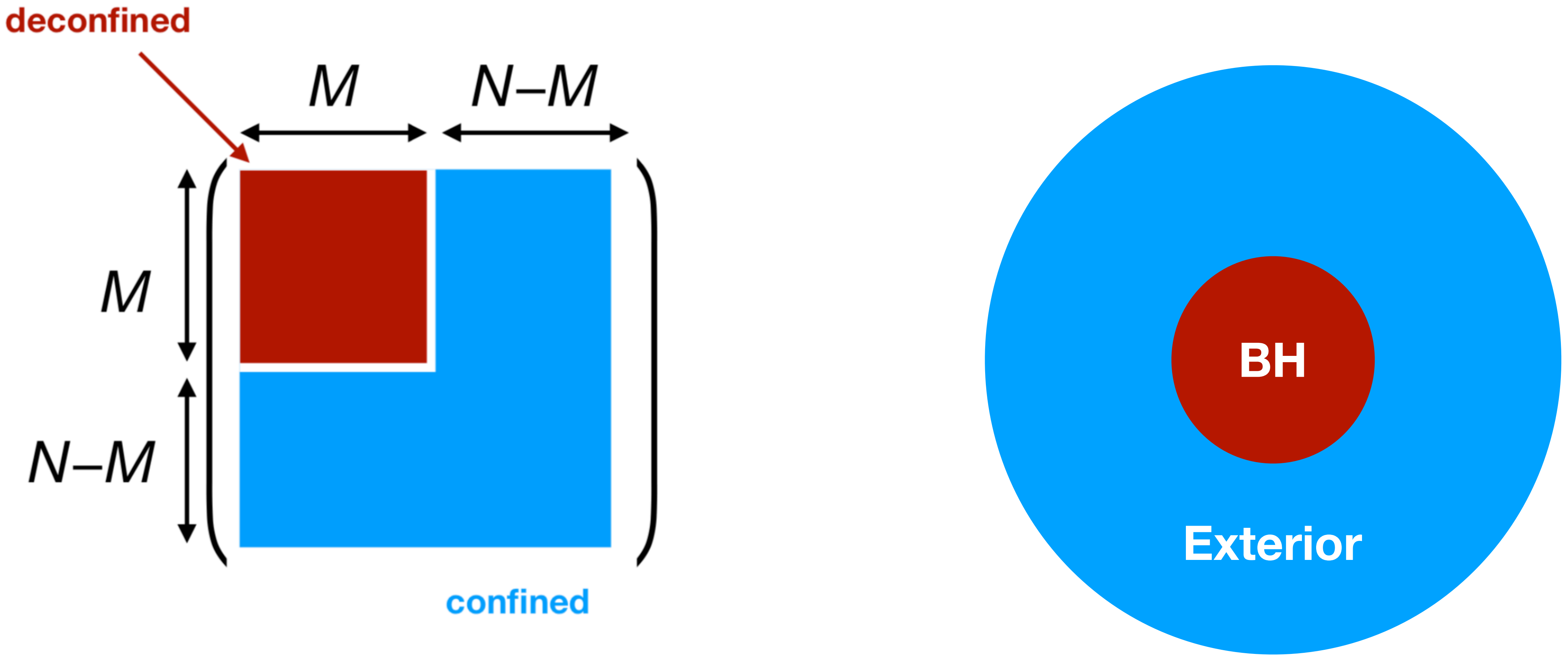
‘Matrix model as second quantization’

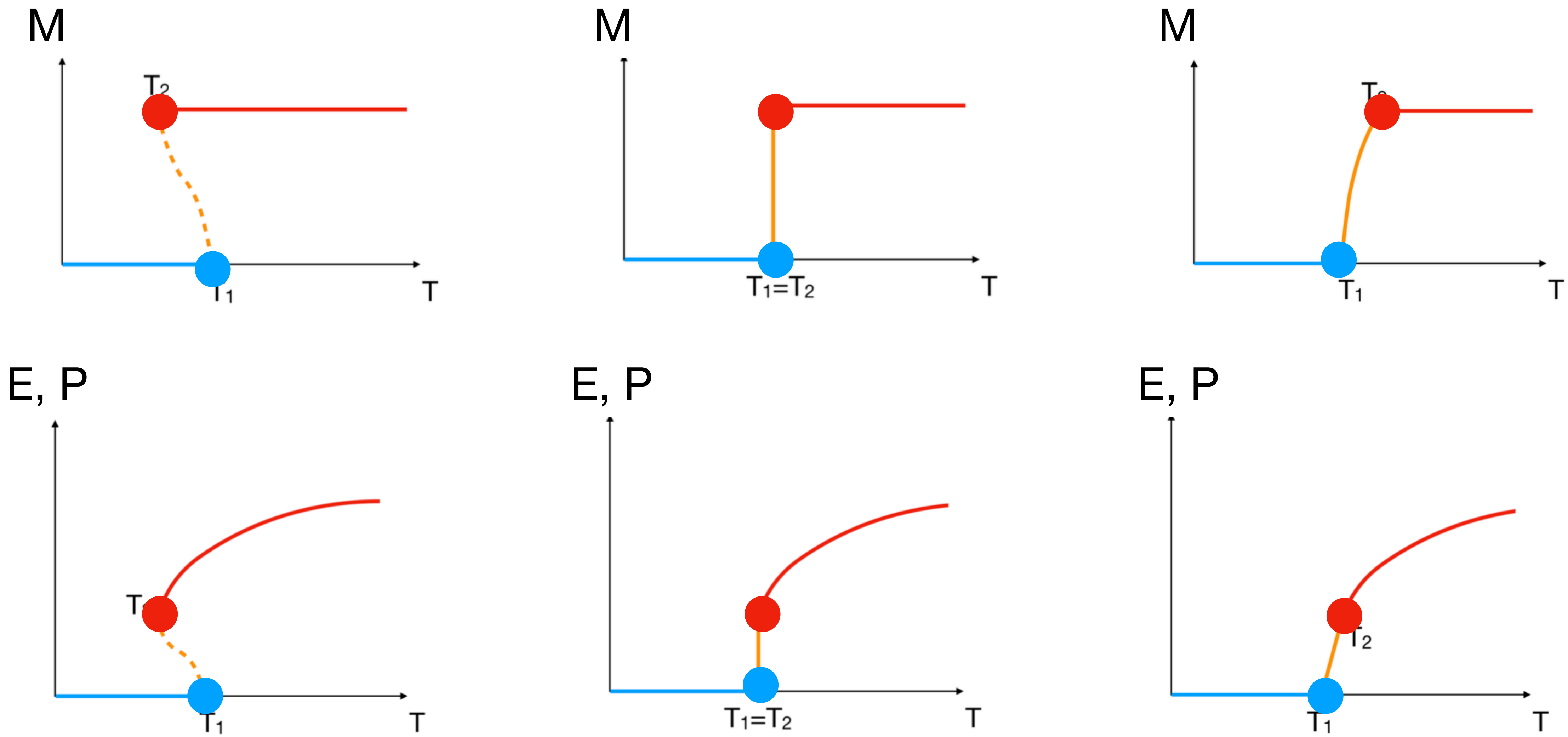
Partial deconfinement, instead of Higgsing

Number of ‘D-branes’ = N or less

Summary




deconfined
M
N−M
M
N−M
confined
BH
Exterior



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