Parental Transfers, Incomplete Credit Markets and Economic Growth

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Abstract

We present cross-country evidence suggesting that there is a statistically robust association between the intensity of household expenditures on education and economic growth: positive at relatively high levels of credit market development and negative at relatively low levels. The evidence also shows that 86% of parents pay for their children's education, while the intensity of parental transfers towards the education of their children varies significantly across countries. Using an overlapping generations framework, we investigate the impact of the intensity of parental financing of children's education on economic growth, under complete and incomplete credit markets for education loans. Among others, we justify theoretically the empirical finding of the negative relationship between growth and the intensity of household spending on education under missing financial markets. We also demonstrate conditions under which incomplete financial markets, in the presence of the parental altruism motive, increase economic growth and do not preclude dynamic efficiency.

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1 Introduction

In most real-world economies, well-developed credit markets for financing educational investments do not exist. This raises the question of how private transfer arrangements between

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parents and children influence human capital accumulation and economic growth, especially when public education is not easily accessible. Galor and Zeira (1993) show that income distribution can have long run effects on economic development when credit market imperfections hinder investment in human capital. As demonstrated by Boldrin and Montes (2005), if young individuals are unable to borrow to finance educational investment then the competitive equilibrium of the economy is dynamically inefficient and may involve stagnation: human capital will not be accumulated and hence growth comes to a standstill. To alleviate dynamic inefficiency, they propose government intervention, accompanied by a pension scheme. In this paper, we argue that the latter is not needed, as long as the parental altruism motive is sufficiently high. Specifically, this paper aims to analyze the extent to which household funding of education – parental transfers for children's education in particular – impacts economic growth and dynamic efficiency.

We begin our analysis by presenting empirical evidence which shows that the intensity of household expenditures on education varies significantly both across countries and across broader regions, with particularly high levels in East Asia and the Pacific, and relatively low levels in the western world. Using an extended sample of 92 countries, we estimate the relationship between intensity of household expenditures on education and economic growth, controlling for both credit market development and public education. The latter is proxied by the share of government expenditures on education in GDP, while the former is measured by a newly constructed credit index. Our results suggest that there is a positive relationship between the intensity of household expenditures on education and economic growth at relative high levels of credit market development, but a negative relationship at relatively low levels of credit market development.

An important component of household expenditures on education are parental transfers toward children's education. Cross-country data limitations on the latter constrain our analysis to a small but indicative sample from HSBC's Value of Education study (2017). Using data from the HSBC survey we show that (i) cross-country differences in intensity of parental financing of children's education are even larger than differences in the intensity of household expenditure on education, and (ii) the intensity of parental transfers for education is highly correlated with the share of government expenditures on education. We argue that public education can be considered as an alternative to a credit market for education loans where individuals privately borrow in order to finance their education. Rather than having individuals borrowing to pay for their education in exchange of future interest payments, the government borrows on their behalf and then taxes them to pay off the debt. In what follows, our analysis focuses on the credit markets for education loans, but the results may be generalized to cover a government that acts as an intermediary between individual borrowers/lenders and financial institutions.

To investigate these issues, we consider an overlapping generations (OLG) framework where growth is determined endogenously and parents are altruistic toward their children. Parental transfers to children play an important role because the young would like to invest in human capital, but they face incomplete credit markets for educational loans; hence children may be reliant on parents for their educational investment.[1] Parents derive utility from

[1] Mukherjee (2018) provides evidence of parent's altruistic behavior towards their children using US data. Specifically, it is shown that parents provide sources to their children without expectations for reciprocal caregiving.

the amount transferred to their children. If parents choose optimal allocations of consumption and transfers, then the intensity of parental transfers towards the children is directly linked to parental altruism.[2] We show that economies where markets for education loans are absent may have higher growth rates than economies with complete credit markets and a dynamically efficient balanced growth path.[3] Low growth rates and dynamic inefficiency are linked to high ratios of physical to human capital which occur either due to over-saving or under-borrowing relative to the growth/efficiency optimal levels. In particular, we show that economies with missing credit markets exhibit higher growth rates than economies with complete credit markets, either when both the parental altruism motive and intergenerational correlation of human capital are relatively low than or when they are relatively high. For instance, when markets are complete and the levels of parental altruism and intergenerational correlation of human capital are high, parental transfers are also high and the young individuals save part of the transfers in financial intermediaries rather than accumulating additional human capital that drives growth. We also consider the case where financial markets exist but education loans are restricted to a certain limit. We demonstrate conditions that relate to the degree of tightness of the borrowing constraint, the degree of intergenerational correlation in human capital and the level of parental altruism, under which an economy where education loans are limited, exhibits higher growth than otherwise identical

[2] These joy-of-giving preferences differ from bequests in that middle-aged parents make transfers to their young children, who in turn use the transfer primarily to finance education. Examples of the bequest version of joy-of-giving (or 'warm glow') preferences include Yaari (1964) and Galor and Zeira (1993), among many others. This intuitive assumption makes altruistic models more tractable than under the dynastic approach of Barro (1974) and thus allows us to provide a full characterization of the impact of the parental altruism motive on growth at different levels of credit market development.

[3] In the empirical literature there is no clear consensus on the impact of financial development on economic growth (see Levine, 2005). Law and Singh (2014) provide evidence that financial development exerts a positive impact on growth below a certain threshold, before turning negative.

economies with complete markets.

Boldrin and Montes (2005) argue that when markets for education loans are missing the BGP is dynamically inefficient due to a high ratio of physical to human capital, and that the inefficiency can be alleviated by government intervention, accompanied by a pension scheme. We argue that parental altruism can substitute missing credit markets for education loans, enabling dynamic efficiency along the BGP. Government intervention and pensions are not needed to achieve dynamic efficiency when credit markets for education loans are missing, as long as the level parental altruism is sufficiently high, as it implies a level of parental transfers that prevents under-accumulation of human capital relative to physical capital. Furthermore, we demonstrate that as long as the level of parental altruism is sufficiently high, economies with missing credit markets are dynamically efficient while identical economies with complete markets are not. Under complete markets, high levels of parental altruism imply parental transfers in excess of what the young would like to invest in education, which then leads to an over-investment in the financial market and to an inefficiently high ratio of physical to human capital. Missing financial markets on the other hand, prevent the channelling of excessive funds in investment in physical capital, restricting the funds into investment in human capital that promotes growth. In short, not only we find that the absence of well-developed credit markets may not hinder economic growth but also dynamic efficiency. We further show that when financial markets exist but education loans are restricted to a certain limit, the BGP is always dynamically inefficient whenever the borrowing constraint is binding. In general, as the level of parental altruism decreases, it becomes less likely that the laissez-faire BGP will be dynamically efficient if markets are incomplete.

These results may provide a possible explanation for the high growth rates of several emerging East Asian economies with high levels of parental investment but relatively undeveloped credit markets. For instance, Seth (2002) discusses the cultural roots of high parental investment in education in South Korea, known as 'education fever'. The high levels of parental investment in education in East Asian economies have also received mainstream media attention (BBC, 2013; The Economist, 2013). Such differences in altruistic motives across countries could be a factor behind the mixed findings in the empirical literature on borrowing constraints and growth: Japelli and Pagano (1994) show that borrowing constraints are associated with higher growth, whereas De Gregorio (1996) finds borrowing constraints negatively affect human capital accumulation and growth. Our results suggest that at certain levels of parental altruism, over-investment in physical capital reduces growth relative to the growth of an otherwise identical but credit constrained economy. Our results are also consistent with the observation, highlighted in Coeurdacier et al. (2015), that savings in emerging Asian markets with high growth rates are higher than savings in advanced economies with lower growth rates. Thus, the mechanism we highlight may be a factor behind cross-country differences in saving and growth that the literature has hitherto had difficulty explaining.

Our model is related to a large literature on credit market development and economic growth. Galor and Zeira (1993) show that the combination of credit market imperfections and initial wealth differentials can drive persistent differences in economic development through the impact on human capital accumulation. Much of the literature has focused specifically on borrowing constraints and growth. Using an endogenous growth model, Japelli and Pagano (1994) show that borrowing constraints raise economic growth due to higher accumulation of physical capital that drives productivity growth. This result depends on the absence of human capital investment. If there are borrowing constraints which hinder investment in human capital, the positive relationship between credit constraints and growth may be reversed (De Gregorio, 1996), though this need not be the case (de la Croix and Michel, 2007; Kitaura, 2012). Here, we show parental altruism has important implications for this debate: its effects on growth and dynamic efficiency are not monotonic, but depend crucially on the level of credit market development.

The remainder of the paper proceeds as follows. Section 2 provides cross-country empirical evidence on private financing of education and its association with growth at different levels of credit market development. It also provides separate evidence on parental transfers towards children's education and its association with government expenditures. Section 3 introduces the economic environment and derives a number of results relating to growth and dynamic efficiency. Finally, section 4 concludes.

2 Private Financing of Education, Credit Constraints and Growth

In this section, we present cross-country empirical evidence on private financing of education, and then demonstrate the relationship of the intensity of the latter with economic growth in the presence of credit constraints.[4] Finally, we present empirical evidence on parental

[4]The intensity of private financing of education is defined as the percentage of the latter in consumption or income. Here we focus on consumption, though income will give similar results due the high correlation

transfers toward children's education, one of the main components of private financing in education. Although an extended cross-country empirical analysis for the latter is not feasible due to data limitations, cross-country evidence from HSBC's Value of Education survey (2017) is quite informative and indicative.

Data from Eurostat (2015) and Global Consumption Database (2010) demonstrate that household expenditures on education, as a percentage of household consumption, vary significantly both across countries and across broader regions.[5] The share of household expenditure on education is relatively high in East Asia and the Pacific (6%) and relatively low in the western world such as the European Union (1.1%) and the USA (2.4%). In China the share of household expenditures on education is about 6 times higher than the corresponding EU share, while the share of education expenditures in South Korea is almost triple the corresponding USA share. High levels of household expenditure on education are not confined only to the East Asia and the Pacific region. The share of education expenditures in Latin America and the Caribbean is 4.1% and more than three times the corresponding share in the EU.[6]

To investigate the relationship between the intensity of private financing of education and economic growth, we consider an extended sample of 92 economies, most of which are

with former.

[5]Household consumption expenditure on goods and services include indirect taxes such as VAT and excise duties.

[6]Brazil has an expenditure share of 3.5%, which is more than triple the share in the EU. Japan as well as the broader regions such as the Middle East & North Africa and Sub-Saharan Africa exhibit shares which are, at least, twice as high as the share in the EU. Within the EU, the share of expenditure on education is lowest in Sweden, Finland and Belgium (0.3-0.4%) and is highest in Ireland, Cyprus and Greece (3.5%, 2.6% and 2.4%, respectively).

developing countries. The cross-country growth regression that we estimate is the following:

$$Growth_i = \alpha_0 + \alpha_1 HExp_i + \alpha_2 GExp_i + \alpha_3 HExp_i \times Credit_i + \epsilon_i$$

where $Growth_i$ is the growth rate of real GDP of country i, $HExp_i$ is the ratio of household expenditure on education to household consumption in country i, $GExp_i$ is the corresponding public expenditure on education as a % of the country's Gross Domestic Product. $Credit_i$ is a discrete-valued credit index that measures the extent of credit provision, with low values reflecting developed credit markets and high values reflecting credit constrained economies.[7] The credit index may take on five different values, $\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$, depending on whether certain thresholds for credit indicators are met. More information about the construction of the credit market index along with descriptive statistics of the variables used in the regression can be found in the Appendix.[8] Table 1 displays estimates for the full sample, a sample which includes data only from GCD (GCD only: i.e. excluding countries reported only by Eurostat) and a sample which excludes countries with moderate growth rates (Mod. Growth). Our estimates in table 1, indicate that the share of household expenditures on education has statistically significant explanatory power for cross-country growth rates, both directly and through its interaction with credit market development. This result holds

[7]Contrary to specifications where credit is included as a separate regressor, either linearly or in high powers, our current specification rejects misspecification, using the Ramsey RESET test, in all three samples (full, GCD-only, without outliers). Therefore, we only include credit in the interaction term. *Growth* corresponds to the average growth rate of real GDP per capita for the period 1970-2010 (in % p.a.) computed from Penn World Table 9. Due to data availability for 18 economies the period 1990-2010 was used. *HExp* was computed using data from the Global Consumption Database (GCD, 2010), Eurostat (code: nama_10_co3_p3, 2010) and Eurostat: EU(2013, 2015). *GExp* was computed using data from the World Bank Development Indicators. Finally, *Credit* was constructed using information from World Bank Financial Inclusion Database (FINDEX) (further details can be found in the Appendix).

[8]The list of the 92 countries along with their credit market classification is available upon request.

Table 1 - Cross-country growth regressions

	Full sample		GCD only		Mod. growth	
Independent var.	**Coef.**	**t-stat**	**Coef.**	**t-stat**	**Coef.**	**t-stat**
Constant	2.071	3.59**	1.928	2.79**	2.171	3.89**
$HExp$	-0.780	-3.89**	-0.678	-2.59*	-0.578	-2.88**
$GExp$	-0.082	-0.71	-0.096	-0.61	-0.096	-0.83
$HExp \times Credit$	1.203	4.99**	1.097	3.40**	0.838	3.11**
R^2_{adj}	0.204		0.135		0.072	
Obs	92		63		82	

Notes. GCD = Global Consumption Database. Mod. growth: $-2 <$ Growth < 4.5.

even after controlling for the public education factor using the share of public education expenditures in GDP, which turns out to be statistically insignificant. In particular, the impact of education expenditures on growth, $\hat{\alpha}_1 + \hat{\alpha}_3 Credit_i$, is negative at low levels of credit market development but changes in sign as credit market development is increased.[9] The critical value occurs at a credit index of 0.648 in the full sample case, and similar values are obtained for the restricted samples.[10] Given that the credit index takes on the five discrete values noted previously, the model predicts a positive relationship between the ratio of household expenditures on education to consumption and real GDP growth for countries with a credit index of 3/4 or 1 (i.e. countries with developed credit markets) and a negative relationship for countries with a credit index of 1/2 or less (i.e. countries with less developed credit markets). Therefore, higher private spending on education, as a percentage of consumption, may exert a negative influence on growth at low levels of credit market development, but the effect becomes unambiguously positive at a sufficiently high level of credit market development.

[9] We have conducted additional robustness checks, including alternative definitions of outliers and the credit index, and we found similar results to those presented in Table 1.

[10] The critical values for the GCD-only and Exc.-outliers samples are respectively 0.618 and 0.689. Note that the critical value is computed as the solution to $\hat{\alpha}_1 + \hat{\alpha}_3 Credit_i = 0$, or $Credit^* = -\hat{\alpha}_1/\hat{\alpha}_3$.

One of the main components of private financing of education are parental transfers towards children's education. Although there are surveys (e.g. the US Consumer Expenditure Survey (CEX), the UK Student income and expenditure survey) which provide panel data on parental transfers for children's education, cross-country data is very limited.[11] *The Value of Education* research study, commissioned by HSBC, provides evidence from a diversified sample of 15 countries and territories.[12] The study suggests that 86% of parents are paying for their children's education, while 84% of those with a child in university or college are paying towards their education. The survey also reports that, on average, 63% of parents pay for private tuition for their children, the highest percentages being reported in developing and East-Asian countries and the lowest in developed countries.[13] According to the survey, the highest proportions (22-39%) of university students paying their own education costs reside in developed countries, while the lowest (< 1-5%) reside in developing and East Asian countries.[14] Using the average total amount spent on a child's education by parents across the 15 countries expressed in USD, reported in HSBC's Value of Education survey (2017), we construct a measure of intensity of parental transfers, which allows us to do a comparison across countries.[15] The ideal would be to measure the intensity using the ratio

[11] Studies such as Zissimopoulos and Smith (2009) and Alessie et al (2014) provide evidence on cross-country inter-vivos parental transfers which are not solely focused on transfers for educational purposes.

[12] The survey represents the views of 8481 parents in 15 countries and territories. All information reported from the survey is reproduced with permission from The Value of Education Foundations for the Future, published in 2016 & 2017 by HSBC Holdings.

[13] The highest percentages are in China (93%), Indonesia (91%), Egypt (88%), Hong Kong (88%), India (83%), Singapore (82%) and Malaysia (81%), and the lowest in France (32%), Canada (31%), Australia (30%) and the UK (23%). Given that, on average, 22% of parents interviewed, admitted to not knowing how much they were spending on their childrens education (Financial Times, June 29, 2017), it is likely that the reported financial contribution of parents is underestimated.

[14] The highest proportions are in Canada (39%), USA (37%) and Australia (22%), and the lowest in Egypt (< 1%), India (1%), Hong Kong (4%) and Singapore (5%).

[15] The HSBC survey was conducted online in February 2017.

of the average of the total parental transfers towards children's education to the average of the total parental consumption or income. Since an accurate direct measure of cross-country parental consumption is not feasible, we replace the latter with the average per capita household consumption expressed in USD between years 2005 and 2017. Thus, our intensity measure corresponds to the ratio of average total parental transfers for children's education to the average per capita household consumption.[16] We approximate the degree of access to free public education using the ratio of government expenditures on education to GDP. To emphasize the relevance of the latter, table 2 also reports the cross-country measure of the average, between years 2005 and 2017, government expenditures on education as a percentage of GDP, using available information from widely used data banks.[17] The first column (Edu) of table 2, corresponds to the intensity of parental transfers for primary to undergraduate level education, the second column (Col/Uni) corresponds to the intensity of parental transfers for college and/or university education and the third column (GExp) corresponds to government expenditures on education as a percentage of GDP.

Although the sample of countries is small, the table is indicative as it covers countries with very different characteristics. The table clearly demonstrates the high variation of the intensity of parental transfers across highly heterogeneous countries. It highlights the relatively large parental transfers per household consumption unit in countries in Asia and Africa such as China, India, Indonesia, Egypt and Hong Kong, and the relatively small

[16] The annual per capita household consumption between years 2005 and 2017, expressed in 2010 USD, was obtained from World Bank's Data Bank, apart from Taiwan which was obtained from the Statistical Bureau of the Republic of China (Taiwan). Parental transfers, expressed in 2016 USD, were obtained from HSBC's survey, conducted in 2017.

[17] The share of government expenditures in GDP was obtained from IMF's, World Bank's and OECD's Data Banks for all countries apart from the USA which was obtained from the U.S. Bureau of Economic Analysis and Taiwan which was obtained from the Statistical Bureau, Republic of China (Taiwan).

Table 2 - Cross-country intensities of parental transfers toward children's education and government expenditures on education

country	Edu	Col/Uni	GExp	country	Edu	Col/Uni	GExp
Hong Kong	6.2	0.8	3.6	UK	1.0	0.3	4.6
UAE	4.8	0.9	0.5	Mexico	3.6	0.6	5.0
Singapore	4.0	0.9	2.9	Canada	0.8	0.2	4.8
USA	1.7	0.4	5.1	India	23.3	4.0	0.5
Taiwan	5.2	0.8	4.0	Indonesia	10.0	1.4	3.2
China	23.1	3.1	0.2	Egypt	8.4	0.6	3.7
Australia	1.2	0.2	2.8	France	0.7	0.2	5.5
Malaysia	5.4	1.8	5.3				

transfers in countries of the western world such as France, Canada the UK and Australia. Table 3 also suggests that parental transfers are significantly affected by the role of the government in offering free public education which minimizes the need of financial support from parents. Specifically, the cross-correlation between the intensity of parental transfers for all levels of education and the share of government spending on education is around -0.74, while the cross-correlation between the intensity of parental transfers for college/university and the share of government spending on education is -0.66. In other words, in countries where the government invests more in education (per unit of output), which would imply that free public education becomes more accessible, parents tend to transfer less funds (per unit of consumption) towards their children's education. Likewise, in countries where governments spend relatively little on education, parents devote a bigger share of funds towards the education of their children.

Free public education can be viewed as an alternative to a credit market for education loans where individuals privately borrow in order to finance their education. That is, the government pays for the education of individuals in exchange of future taxation, rather than individuals paying directly for their education by borrowing from a credit institution in exchange of future interest payments. We may also think of the government playing the role of an intermediary between borrowers/lenders and financial institutions, where the government borrows from financial institutions in order to finance public education and then taxes individuals in order to pay off the debt.[18] Public and private education loans for tertiary education are quite common in various countries. For instance, countries such as Denmark, Finland, France and Norway, offer low or no tuition fees and provide students access to generous public subsidies for higher education. On the other hand, in countries such as the US, the UK and Australia, where access to education (even in public institutions) is costly, there are well-developed credit markets for education loans. In the following section we focus on parental altruistic motives in financing children's education within an OLG framework and examine the implications on economic growth and dynamic efficiency at different levels of credit market development.

3 Economic Environment

3.1 Complete markets

We consider an OLG economy, populated by agents who live for three periods.[19] Within each generation, the agents are homogeneous and population increases at the rate $n > -1$.

[18]Equivalence of government with financial institutions would presumably require taxes to be lump-sum, because Ricardian equivalence-type results do not generally hold in OLG models.

[19]The model has a similar structure to that of Boldrin and Montes (2005) with the difference that parents are altruistic and care about their children's development of human capital while the children have the option to use the financial market not only as a credit market for funding their education but also as a financial investment opportunity.

An agent draws utilty from consumption, $c_{t,m}$, when middle age, consumption, $c_{t+1,o}$, when old age and the amount of transfers, ω_t, that the agent provides when middle aged to each of his children for the development of their human capital.[20] The consumption of the young is assumed to be incorporated in the consumption of the middle aged. The lifetime utility of a young agent born in period $t-1$ is defined as $U(c_{t,m}, c_{t+1,o}, \omega_t) = u(c_{t,m}) + \beta u(c_{t+1,o}) + \gamma u(\omega_t)$, where $\beta > 0$, $\gamma > 0$ and $u(\cdot)$ is an incresing and twice differentiable function with $u''(\cdot) < 0$.[21] Young agents born in period $t-1$, are endowed with $h_{t-1}^{y} > 0$ units of human capital that are invested in the production of next period human capital, along with additional resources denoted by $d_{t-1} > 0$. An agent's human capital, $h_t > 0$, evolves according to a smooth, homogeneous of degree one function $h\left(d_{t-1}, h_{t-1}^{y}\right)$. Aggregate output Y_t is produced in a perfectly competitive market which comprises of large number of homogeneous firms, each producing output using human and physical capital according to a smooth, concave and constant returns to scale production function F. The latter enables us to write $Y_t = F(H_t, K_t)$, where $H_t > 0$ and $K_t > 0$ correspond to aggregate human and physical capital, respectively. Firms maximize profits by taking as given the price of human capital, w_t, and the price of physical capital R_t^k,while the price of output is normalized to unity. This implies that w_t and R_t^k correspond to the marginal product of human and physical capital, respectively.

[20]The literature distinguishes between (i) *bequests*, which are transfers made upon death and which may be accidental; and (ii) *inter-vivos* transfers, which are made between living people. The empirical literature has found that inter-vivos transfers are a non-trivial fraction of total transfers from parents to children (Gale and Scholz, 1994; Cox and Raines, 1985).

[21]Lambrecht et al. (2005) assume that parents pay for the education of their children but derive utility from the total income of their children. In addition, children cannot borrow to fund their education and rely solely on parents. An alternative way to model altruism is to assume that the utility function of the children is an argument of the utility function of the parents (see Barro, 1974). Rangazas (2000) however, finds that a standard infinitely lived neoclassical model with Barro-type altruistic preferences is inconsistent with the data.

There is a frictionless and perfectly competitive financial market which serves as an intermediary between agents and firms, enabling them to borrow and lend (invest) at the same gross interest rate, R_t, as in Boldrin and Montes (2005). Due to perfect foresight, a simple arbitrage argument suggests that the gross interest rate, R_t must be equal to the return of physical capital R_t^k. Specifically, a young individual born in period $t-1$, will either borrow $\bar{b}_{t-1} > 0$ from the financial market, if the optimal investment in human capital, d_{t-1}, exceeds parental transfers that is, $\bar{b}_{t-1} = d_{t-1} - \omega_{t-1}$ or save (invest) $-\bar{b}_{t-1} > 0$ if parental transfers exceed the optimal investment in human capital that is $-\bar{b}_{t-1} = \omega_{t-1} - d_{t-1}$.[22] A middle age individual saves s_{t-1} for his retirement while firms borrow from the credit market in order to invest, I_{t-1}^k, in next period's physical capital. It is assumed that one unit of investment in physical capital corresponds to one unit of physical capital that is, $I_{t-1}^k = K_t$.[23]

In the second period of his life, a middle aged individual supplies labor in a perfectly competitive labor market at the wage rate w_t, per unit of human capital, and receives the revenue from his investment in the financial market (if $\bar{b}_{t-1} < 0$) or pays off the loan of the previous period (if $\bar{b}_{t-1} > 0$) at the gross interest rate R_t. Then, he contributes to his childrens' education and makes further personal consumption-saving decisions. In particular, the middle age agent transfers ω_t to each of his $1+n$ children and saves s_t in the financial market for his retirement. Since agents within each generation are homogeneous, the aggregate savings of the middle aged, the aggregate borrowing (saving) of the young

[22] Galor and Zeira (1993) have a similar setting where agents can borrow to invest in human capital if the parental endowment is small enough.

[23] Full depreciation of physical capital is a reasonable assumption, and empirically plausible for this model as the period may correspond to 30-40 actual years.

and the aggregate human capital can be written as $S_t = (1+n)^{t-1} s_t$, $\overline{B}_t = (1+n)^t \overline{b}_t$, and $H_t = (1+n)^{t-1} h_t$, respectively. The total assets held by financial intermediaries must be equal to the total liabilities recorded in their balance sheets that is, $S_t = B_t + K_{t+1}$, where $B_t = \overline{B}_t/(1+n)$. The latter expressed per middle aged individual is $s_t = b_t + k_{t+1}$, where $b_t = \overline{b}_t/(1+n)$, $k_{t+1} = (1+n)\widetilde{k}_{t+1}$ and $\widetilde{k}_{t+1}$ is physical capital per middle aged individual in period $t+1$.[24] Given that F is homogeneous of degree one, we can also express input prices as a function of $x_t = \widetilde{k}_t/h_t$ that is, $w_t = f(x_t) - x_t f'(x_t)$ and $R_t = f'(x_t)$, where $f(x_t) = F(1, x_t)$. Thus, the ratio of the gross interest rate to the wage rate can be written as a decreasing function of the factor intensity ratio that is, $R_t/w_t = \kappa(x_t)$. Finally, in old age, the agent consumes all his wealth. It follows that the budget constraints, respectively, of an agent in middle age and old age are the following: $c_{t,m} + s_t + R_t \overline{b}_{t-1} + (1+n)\,\omega_t = w_t h_t$ and $c_{t+1,o} = R_{t+1} s_t$. Then, the problem for an agent born in period $t-1$ is

$$\max_{d_{t-1}, s_t, \omega_t} \{ u\left(w_t h\left(d_{t-1}, h^y_{t-1}\right) - s_t - R_t\left(d_{t-1} - \omega_{t-1}\right) - (1+n)\,\omega_t\right)$$
$$+\beta u\left(R_{t+1} s_t\right) + \gamma u\left(\omega_t\right)\}.$$

Notice that production can be expressed in terms of output per middle aged agent of period t that is, $y_t = F\left(h_t, \widetilde{k}_t\right)$. In the analysis that follows, we consider the following parametric version of the economy where $u(\theta) = \ln(\theta)$, $F\left(h_t, \widetilde{k}_t\right) = A h_t^{\delta} \widetilde{k}_t^{1-\delta}$, $h\left(d_{t-1}, h^y_{t-1}\right) = B\left(d_{t-1}\right)^{\zeta}\left(h^y_{t-1}\right)^{1-\zeta}$, $h^y_{t-1} = \mu h_{t-1}$, with $A \geq 1$, $B \geq 1$, $\mu > 0$, $\delta \in (0,1)$ and $\zeta \in (0,1)$.

Equilibrium: *Given initial conditions* $\{d_{-1} > 0,\ b_{-1},\ h_0 > 0,\ k_0 > 0\}$, *there are sequences of prices* $\{R_t, w_t\}_{t=0}^{\infty}$ *and quantities* $\{d_t,\ \overline{b}_t,\ h_{t+1},\ k_{t+1},\ \omega_t\}_{t=0}^{\infty}$ *that satisfy the optimal*

[24] In Boldrin and Montes (2005), b_t is replaced with d_t which is restricted to always be non-negative.

conditions of the problem of agents, such that the resource constraint, $F\left(h_t, \widetilde{k}_t\right) = c_{t,m} + \frac{c_{t,o}}{1+n} + s_t + (1+n)\,\omega_t$ *and the balance sheet of financial intermediaries,* $s_t = b_t + k_{t+1}$, *hold for* $t \geq 0$.

The optimality condition with respect to ω indicates that γ corresponds to the intensity of parental transfers towards children's education:

$$\gamma = \frac{(1+n)\omega_t}{c_{t,m}}$$

We also interpret γ as the level of the parental altruism motive towards children's education. Manipulating the optimality conditions, it can be shown that $k_{t+1} = \widetilde{\Psi}\,(\gamma) = \Psi A^{-1} y_t$, $s_t = (1-\delta)^{-1}\left[1-\delta(1-\zeta)\right] k_{t+1}$, $d_t = \delta\zeta\Psi\left[A(1+n)(1-\delta)\right]^{-1} y_t$, $\omega_t = \gamma\left[\delta\zeta\left(\beta+\gamma\right)\right]^{-1}\left[1-\delta\left(1-\zeta\right)\right] d_t$ and $h_{t+1} = \Phi h_t^{1-\zeta(1-\delta)} \widetilde{k}_t^{\zeta(1-\delta)}$, where

$$\Psi = \frac{(1-\delta)[\delta\beta(1-\zeta)+\gamma]A}{\left[1-\delta(1-\zeta)\right](1+\beta+\gamma)} \text{ and } \Phi = \left(\frac{B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\delta\zeta\Psi}{(1-\delta)(1+n)}\right)^{\zeta}$$

Proposition 1: *There exists* $\gamma^* > 0$ *such that when* $\gamma < \gamma^*$ *then,* $\overline{b}_t > 0$; *when* $\gamma > \gamma^*$ *then,* $\overline{b}_t < 0$; *when* $\gamma = \gamma^*$ *then,* $\overline{b}_t = 0$, *where* $\gamma^* \equiv \beta\delta\zeta(1-\delta)^{-1}$.

Proof. *It follows from the fact that* $\overline{b}_{t-1} = [\beta\delta\zeta - \gamma\,(1-\delta)][\delta\zeta\,(\beta+\gamma)]^{-1} d_{t-1}$ *and* $d_{t-1} > 0$.

■

Given the characteristics of the economy, proposition 1 establishes the condition under which young agents borrow from the credit market (i.e. $\overline{b}_t > 0$) in order to fund their education. The proposition indicates that the young agents will borrow from the credit market in order to develop their human capital only if the level of altruism of parents is below a certain threshold. In other words, if parents' level of altruism is sufficiently high, young agents find it optimal to invest part of their endowment in the credit market. The intuition for this result is simply that if the parental altruism motive is below the threshold value, parents transfer to their children not enough to cover their children's desired level of investment in education, and thus they want to borrow from the credit market. If parents are more altruistic than this (i.e. γ exceeds the threshold value), they end up transferring more to their children than they would like to invest in education, and so the child places what is left over as savings in the financial intermediary.

It is straightforward to show that[25]

$$x_t = \left(\frac{x_0 (1+n)\,(1+g_{h0})}{\Psi} \right)^{(1-\delta)^t} \prod_{i=0}^{t} \left[\frac{\Psi}{(1+n)\,(1+g_{ht-i})} \right]^{(1-\delta)^i} . \tag{1}$$

As in Boldrin and Montes (2005), the only rest point of (1) is the origin. In other words, when $g_{ht} = g_h$, the only case where $x_t = x$ for all $t \geq 1$ is when $x = x_0$.

Definition 1 *At the Balanced Growth Path (BGP), there are constants x, g and g_y such that, $\frac{K_t}{H_t} = x$, $\frac{K_t}{K_{t-1}} = \frac{H_t}{H_{t-1}} = \frac{Y_t}{Y_{t-1}} = 1 + g_y$, and $\frac{k_t}{k_{t-1}} = \frac{y_t}{y_{t-1}} = \frac{h_t}{h_{t-1}} = 1 + g$ where $1 + g_y = (1 + n)(1 + g)$.*

It follows that the ratio of physical to human capital and the growth rate at the unique

[25] Notice that $k_{t+1} = \Psi A^{-1} y_t$ implies $x_t = \left[(1+n)\,(1+g_{ht})\right]^{-1} \Psi x_{t-1}^{1-\delta}$, where g_{ht} is the time t growth rate of human capital. Then the latter can be solved backwards and be reduced to (1).

BGP are given by,

$$x=\left[\frac{\Psi}{(1+n)(1+g)}\right]^{\frac{1}{\delta}}=\left(\frac{\Psi}{\Phi(1+n)}\right)^{\frac{1}{\delta+\zeta(1-\delta)}}\quad \text{and}\quad 1+g=\left(\frac{\Psi}{1+n}\right)^{\frac{\zeta(1-\delta)}{\delta+\zeta(1-\delta)}}\Phi^{\frac{\delta}{\delta+\zeta(1-\delta)}},$$

respectively. The latter implies that g is a monotonically increasing function of γ.

As noted by Abel et al. (1989), an equilibrium path is dynamically inefficient if the economy is consistently investing more in capital than it earns in profit. The laissez-faire BGP is dynamically efficient if investment cannot be reallocated between physical and human capital in a way that a social planner can achieve a non-negative gain in welfare for all generations living on the existing or the new BGP. Following Del Rey and Lopez-Garcia (2013, 2016), we assume that the social planner preserves the functional form of individual preferences, while treating generations equally across time. Along the BGP, the objective of the social planner is to pick stationary values for $\widehat{c}_m = c_{t,m}/h_t$, $\widehat{c}_o = c_{t+1,o}/h_t$ and $\widehat{\omega} = \omega_t/h_t$ that maximize the utility function, given by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega}) = u(\widehat{c}_m) + \beta u(\widehat{c}_o) + \gamma u(\widehat{\omega})$, subject to the balanced growth version of the resource constraint, as demonstrated in the proof of proposition 2. Then dynamic (in)efficiency of the BGP is formally defined as follows.

Definition 2 *A laissez-faire BGP is dynamically inefficient if a reduction in x increases the welfare, as measured by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$, of generations living on the existing or a new BGP. Otherwise the BGP is dynamically efficient.*[26]

Proposition 2: *The laissez-faire complete markets BGP, is dynamically efficient if* $1 \leq$

[26] As shown in the proof of proposition 2, the social planner will set the investment variable $\widehat{d} = \widehat{\omega}$. Unlike in Del Rey and Lopez-Garcia (2016), $\widehat{d}$ becomes an argument of the utility function just like $\widehat{c}_y$ and $\widehat{c}_o$, and so it is treated as such.

$R/(1+n)(1+g) \leq \gamma_R^c$, where $\gamma_R^c = \gamma/\delta\zeta\beta \geq 1$. *Equivalently, it is dynamically efficient if* $\gamma \in \Omega^c \equiv \{\gamma \geq \delta\zeta\beta : \gamma_1^c \leq \gamma \leq \gamma_2^c\}$, *where* $\gamma_1^c = \frac{A\zeta\beta\delta(1-\delta)}{\Psi}$, $\gamma_2^c = \frac{[1-\delta(1-\zeta)](1+\beta)-\delta\beta(1-\zeta)}{\delta(1-\zeta)}$, *and* $\Omega^c \neq \{\emptyset\}$ *only if* $\delta < (1+\beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$.

Proof. *See the appendix.* ■

Proposition 2 establishes that the BGP is dynamically efficient under complete markets if the level of the parental altruism motive lies between two thresholds. Contrary to the model where the parental altruistic motive is absent (see Boldrin and Montes, 2005), we demonstrate that condition $R \geq (1+n)(1+g)$ is not sufficient for dynamic efficiency. This is due to the fact that for relatively small levels of the parental altruism motive (lower than γ_1^c), investment in human capital is small which induces an inefficiently high ratio physical to human capital. The latter also occurs at relatively high levels of the parental altruism motive (higher than γ_2^c). That is, parents give an inefficiently large transfer to their children that exceeds the amount necessary for optimal investment in human capital. As a result, young agents save the remainder, or over-invest in the financial market in a socially inefficient manner. This leads to an over-accumulation of physical capital since x exceeds the maximum socially optimal level. When $\gamma > \gamma_2^c$ ($\gamma < \gamma_1^c$), the welfare of all generations living on the BGP can be increased via a reallocation of investment from physical to human capital. The reallocation decreases the ratio of physical to human capital relative to that of the laissez-faire equilibrium, leading to an increase (decrease) of the interest rate relative to the growth rate. Note that if $\gamma_1^c \leq \gamma^* < \gamma \leq \gamma_2^c$, the BGP path is dynamically efficient when $\bar{b}_t < 0$. The latter demonstrates that strictly positive investments in the financial market that lead to increases in physical capital need not always lead to dynamic inefficiency.

3.2 Incomplete Markets

3.2.1 No Credit Market

First, we consider the case where the credit market is absent. Thereby young agents neither can borrow in order to fund educational investment nor can invest in the financial market, i.e. $b_t = 0$ for all $t \geq 0$. As a result, investment in education will be funded entirely by transfers, i.e. $d_t = \omega_t$. The firms continue to have access to credit. It follows that the problem solved by an agent born in period $t-1$ is identical to the case of complete markets, except that d_{t-1} is no longer a choice variable while $s_t \equiv k_{t+1}$. The optimality conditions for this problem imply that $k_{t+1} = s_t = \overline{\Psi} A^{-1} y_t$, $d_t = \omega_t = \gamma\overline{\Psi}\left[\beta A(1+n)\right]^{-1} y_t$ and $h_{t+1} = \overline{\Phi} h_t^{1-(1-\delta)\zeta} \widetilde{k}_t^{(1-\delta)\zeta}$, where $\overline{\Psi} = \beta\delta A\left(1+\beta+\gamma\right)^{-1}$ and $\overline{\Phi} = \left[B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\gamma\overline{\Psi}\left[\beta(1+n)\right]^{-1}\right]^{\zeta}$. As in the case of complete markets, it can be shown that the ratio of physical to human capital, $\overline{x}_t$, satisfies (1), where Ψ is replaced with $\overline{\Psi}$ and the only rest point is the origin, i.e. $\overline{x}_t = \overline{x} = \overline{x}_0$ for all $t \geq 1$.[27] The definition of the BGP is the same as definition 1 as well as the functional forms of the ratio of physical to human capital and the the growth rate along the BGP are the same as those in the case of complete markets with $\overline{x}$, $\overline{g}$, $\overline{g}_y$ and $\overline{\Psi}$, replacing x, g, g_y and Ψ. Contrary to the case of complete markets, the growth rate of the BGP may be either increasing, decreasing or unchanged in response to an increase in the level of the parental altruism motive. Specifically, $\partial\overline{g}/\partial\gamma > 0$ if $\gamma < \gamma^{\overline{g}}$, $\partial\overline{g}/\partial\gamma = 0$ if $\gamma = \gamma^{\overline{g}}$ and $\partial\overline{g}/\partial\gamma < 0$ if $\gamma > \gamma^{\overline{g}}$, where $\gamma^{\overline{g}} \equiv \delta(1-\delta)^{-1}(1+n)^2(1+\beta)$. Contrary to the case of complete

[27] Using the equations for the saving rates of sections 3.1 and 3.2.1 and letting $[s/y]^{MC}$ and $[s/y]^{CM}$ denote the saving rates in the economies with missing and complete markets, respectively, it can be shown that for any $\gamma < \gamma^*$, $[s/y]^{MC} > [s/y]^{CM}$, whereas for any $\gamma > \gamma^*$ $(\gamma = \gamma^*)$, $[s/y]^{MC} < [s/y]^{CM}$ $\left([s/y]^{MC} = [s/y]^{CM}\right)$.

markets, the ratio of investment in physical capital to output in incomplete markets, $\widetilde{\widetilde{\Psi}}(\gamma)$, is a monotonically decreasing function of γ. Then, proposition 3 follows from definition 2.

Proposition 3: *With no credit market, the laissez-faire BGP is dynamically efficient if* $1 \leq R/(1+n)(1+\overline{g}) \leq \delta\gamma_R^c + 1$, *where* $\gamma_R^c = \gamma/\delta\zeta\beta$. *Equivalently, it is dynamically efficient if* $\gamma \in \Omega^{in} \equiv \{\gamma > 0; \gamma \geq \gamma^{in}\} \neq \varnothing$, *where* $\gamma^{in} = max\{\gamma_1^{in}, \gamma_2^{in}\}$, *with* $\gamma_1^{in} \equiv \frac{\beta\delta-(1-\delta)(1+\beta)}{1-\delta}$ *and* $\gamma_2^{in} \equiv \frac{\zeta[1-\delta(1+\beta)]}{1-\zeta(1-\delta)}$.

Proof. *See the appendix.* ■

Proposition 3, establishes that the laissez-faire BGP will be dynamically efficient under no credit markets if the parental altruism motive exceeds a threshold value. Intuitively, since the young cannot borrow to fund human capital investment, dynamic efficiency can be achieved only if parental transfers are large enough to ensure that human capital is not under-accumulated in a socially inefficient manner. This, in turn, requires a high degree of altruism. In other words, when $\gamma < \gamma^{in}$, parental transfers are relatively small and since the young are credit constrained, the allocation (k, h) is such that x exceeds the maximum socially optimal level and the interest rate is strictly smaller than the growth rate of aggregate output. As in the case of complete markets, if part of investment is reallocated from physical capital to human capital, the resulting increase in the interest rate will lead to an increase in the welfare of all generations living on the current or new BGP. The result implies that economies with sufficiently high levels of altruism might accumulate human capital in an efficient manner despite the absence of credit markets for funding investment in education. Proposition 4 lays out conditions which relate to the level of parental altruism, γ, the degree of intergenerational persistence in human capital, $1-\zeta$, under which an a economy with missing financial markets outperforms or underperforms, in terms of growth along the BGP, an otherwise identical economy with complete financial markets.

Proposition 4: *There are thresholds* $\overline{\gamma} > 1$ *and* $\underline{\gamma} < 1$*, where* $0 < \underline{\gamma} < \gamma^* < \overline{\gamma}$ *and* $\widetilde{\zeta} \equiv \delta(1+\delta)^{-1}$ *such that:* ***(a)*** *If* $\zeta < \widetilde{\zeta}$ *then* ***(i)*** $g = \overline{g}$ *if* $\gamma = \gamma^*$ *or* $\gamma = \overline{\gamma}$*,* ***(ii)*** $g < \overline{g}$ *if* $\gamma^* < \gamma < \overline{\gamma}$ *and* ***(iii)*** $g > \overline{g}$ *if* $\gamma < \gamma^*$*or* $\gamma > \overline{\gamma}$*.* ***(b)*** *If* $\zeta > \widetilde{\zeta}$ *then* ***(i)*** $g = \overline{g}$ *if* $\gamma = \underline{\gamma}$ or $\gamma = \gamma^*$*,* ***(ii)*** $g < \overline{g}$ *if* $\underline{\gamma} < \gamma < \gamma^*$ *and* ***(iii)*** $g > \overline{g}$ *if* $\gamma < \underline{\gamma}$ *or* $\gamma > \gamma^*$*.* ***(c)*** *If* $\zeta = \widetilde{\zeta}$ *then* ***(i)*** $g = \overline{g}$ *if* $\gamma = \gamma^*$ *and* ***(ii)*** $g > \overline{g}$ *if* $\gamma < \gamma^*$*and if* $\gamma > \gamma^*$*.*

Proof. *See the appendix.* ■

Proposition 4 establishes that (i) it is possible for the growth rate under no credit markets to exceed that under complete markets, and (ii) that whether it does can be linked to the degree of the parental altruism motive. The first part of the result indicates that the absence of credit markets need not imply lower economic growth. This is again a result that goes contrary to conventional wisdom on the role of credit markets. The second part of the result tells us that part (i) pertains specifically to economies with sufficiently low or high levels of the parental altruism motive.[28] Note also that the Proposition 4 depends on the intergenerational persistence in human capital, $(1-\zeta)$: an economy with a missing credit market for education loans outperforms, in terms of growth, an otherwise identical economy with complete credit markets if the level of intergenerational correlation in human capital and the level of the parental altruism motive are both relatively high, or if they are both relatively low. In the first case, under complete markets and a specific range of high levels

[28]Let MC and CM denote the economy with a missing credit market and the economy with a complete market. Then note that proposition 4 also implies that $\overline{g} > g$ and $[s/y]^{MC} > [s/y]^{CM}$ if $\underline{\gamma} < \gamma < \gamma^*$ and $\zeta > \widetilde{\zeta}$, whereas $\overline{g} > g$ and $[s/y]^{MC} < [s/y]^{CM}$ if $\gamma^* < \gamma < \overline{\gamma}$ and $\zeta < \widetilde{\zeta}$.

of the parental altruism motive, young agents choose to save part of the parental transfer than exploiting the relatively high intergenerational persistence in human capital to generate more human capital for future generations. Higher savings by the young lead to an over-accumulation of physical capital relative to human capital along the BGP. The economy avoids the latter when the young are restricted from accessing the financial market. In the second case, under complete markets and a range of low levels of the parental altruism motive, the young use the financial market to borrow a relatively large amount to complement the parental transfer in investing in their human capital development, rather than saving part of the transfer, allowing the economy to generate a growth-optimal ratio of physical to human capital. When the young are restricted from accessing the credit market for education loans, excess borrowing is prevented and so the economy avoids over-accumulation of human capital. Notice that threshold $\widetilde{\zeta}$ is positively related to the intensity of the use of labor in production. It follows that as the degree of labor intensity increases, the more likely it becomes that the economy with a missing credit market outperforms an economy with complete markets at relatively high levels of the parental altruism motive than low levels.[29]

Boldrin and Montes (2005), argue that a model without a credit market delivers a dynamically inefficient equilibrium, as a natural implication of their model in which human capital is solely financed via a credit market. We show that government intervention is not necessary for an economy with a missing credit market to achieve a dynamically efficient equilibrium as long as parental altruism is sufficiently high. Our findings may also help to

[29]This result can be compared to the finding of de la Croix and Michel (2007) who demonstrate that the maximum growth rate is achieved in a borrowing constrained regime as long as the elasticity of earnings to education is high enough. The elasticity of earnings to education (assuming that d captures the level of education) corresponds to $\delta\zeta$.

explain the mixed results that have been found in empirical analyses of the effects of borrowing constraints on growth. For instance, while Japelli and Pagano (1994) found that the presence of borrowing constraints tend to raise growth rates, the opposite result was found by De Gregorio (1996). Proposition 4 offers a possible explanation for the fact that several Asian economies with extremely high intensity of parental investment towards children's education (e.g. China and India) have been able to grow at fast rates over recent decades despite the absence of well-developed credit markets for education and low public investment in education. A numerical illustration of the findings is displayed in Figure 1. Here, we set $\beta = 0.3$, $\zeta = 0.60$, $\delta = 2/3$, and $n = 1/2$. The scaling parameters A and B are set at 10 and 2.5. As indicated by the analytical results, the growth rate under complete markets

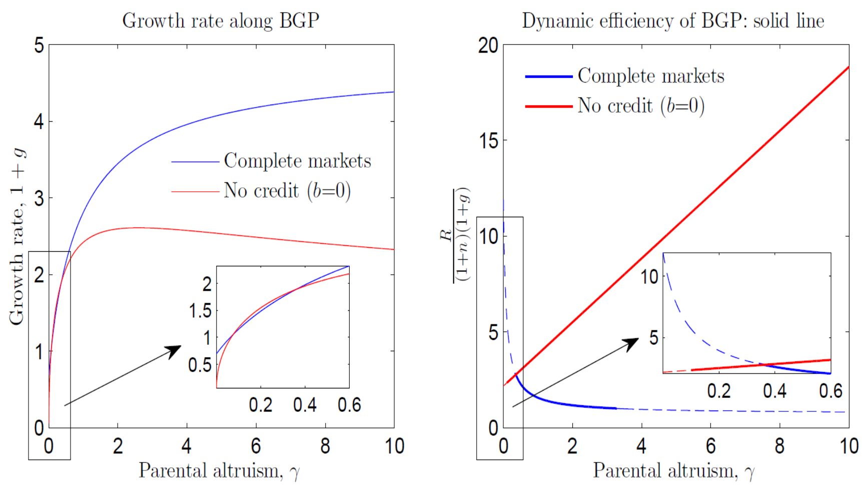

Figure 1: Growth and efficiency vs the parental altruism motive (Case of $\zeta > \widetilde{\zeta}$)

increases monotonically with γ, while in the no-credit market case growth initially increases with γ before reaching a maximum and declining. The relationships between growth and the parental altruistic motive, γ, displayed in figure 1, are consistent with the findings presented in section 2. In section 2, we found that high growth is associated with high intensity of household spending on education at high levels of credit market development, but not at low levels. Here, the intensity of household spending on education when financial markets are missing, coincides with the intensity of parental transfers for education (γ). Under complete markets, growth increases with γ, as suggested by the empirical findings, because a higher percentage of parental transfers, increase human capital and thus output. That is, at low levels of the parental altruism motive, the absence of financial markets does not restrain growth. At high levels of parental altruism however, the intensity of parental transfers is high and when financial markets are absent, the high volume of transfers, leads to under-accumulation of physical capital. In other words, the young are unable to invest the funds that exceed the optimal investment in human capital to the financial market, and indirectly channel the investment to physical capital. Notice also that the growth rate when the credit market is absent, exceeds the growth rate of the economy with complete markets for a range of γ values, consistent with Proposition 4. Since $\zeta < \tilde{\zeta}$ under the above calibration, we are in the first case of Proposition 4: the growth rates intersect when $\gamma = \gamma^*$ $(= 0.15)$ and when $\gamma = \overline{\gamma}$ (=0.63), and for $\gamma \in (\gamma^*, \overline{\gamma})$ the growth rate is higher under no-credit market.

The second panel of figure 1 displays the required conditions for dynamic efficiency in the economies with complete and no credit markets. Note that only the solid part of the curves corresponds to dynamically efficient allocations. As Propositions 2 and 3 suggest, the complete markets laissez-faire BGP is dynamically efficient only if γ falls within a restricted range of values, between γ_1^c and γ_2^c, while the corresponding BGP of the economy without a credit market is always dynamically efficient as long as γ is greater or equal than threshold

γ_1^{in}. When the intensity parental transfers is low, the BGP of both the complete markets economy and the no-credit market economy is dynamically inefficient, whereas when the intensity of parental transfers is high ($> \gamma_2^c$), the BGP of the complete markets economy is always dynamically inefficient while the BGP of the no-credit market economy is always dynamically efficient.

3.2.2 Credit Market with Limited Access

We now consider the intermediate case where young households may borrow up to a limit in order to invest in education. As Coeurdacier et al. (2015), we assume, in particular, that they can borrow up to a fraction $\lambda > 0$ of the present value of their labor income when middle aged, or $\bar{b}_{t-1} \leq \lambda w_t h_t / R_t$.[30] The Kuhn-Tucker conditions with a binding borrowing constraint imply that $k_{t+1} = A^{-1}\overline{\overline{\Psi}} y_t$, $s_t = (1-\delta)^{-1}\left[1-\delta(1-\lambda)\right] k_{t+1}$, $d_t = \overline{\overline{\Psi}}[\gamma\left[1-\delta(1-\lambda)\right] + \beta\delta\lambda][A\beta(1-\delta)(1+n)]^{-1} y_t$, $\omega_t = \overline{\overline{\Psi}}[1-\delta(1-\lambda)][A\beta(1-\delta)]^{-1} y_t$ and $h_{t+1} = \overline{\overline{\Phi}} h_t^{1-(1-\delta)\zeta} \widetilde{k}_t^{(1-\delta)\zeta}$ where[31]

$$\overline{\overline{\Psi}} = \frac{\beta\delta(1-\delta)(1-\lambda)A}{(1+\beta+\gamma)\left[1-\delta(1-\lambda)\right]} \quad \text{and} \quad \overline{\overline{\Phi}} = \left[\frac{B^{\frac{1}{\zeta}} \mu^{\frac{1-\zeta}{\zeta}} \overline{\overline{\Psi}}\left[\gamma\left[1-\delta(1-\lambda)\right] + \beta\delta\lambda\right]}{\beta(1-\delta)(1+n)}\right]^{\zeta}.$$

For the reasons stated in the previous sections, the functional forms of the ratio of physical to human capital and the growth rate at the unique BGP with a binding borrowing limit are the same as those in the case of complete markets with $\overline{\overline{x}}$, $\overline{\overline{g}}$, $\overline{\overline{g_y}}$ and $\overline{\overline{\Psi}}$, replacing x, g,

[30]Investing in the credit market (i.e. $\bar{b}_{t-1} < 0$) is assumed to be unconstrained, as in the case of complete markets.

[31]It is worth noting that the equation for the savings rate implies that the savings rate is higher for economies with tighter borrowing constraints (smaller λ). Since the borrowing constraint binds for relatively small γ. this result is consistent with the result on savings rate of section 3.2.1 - see footnote 27.

g_y and Ψ, respectively.

Proposition 5: *Along the BGP, for any $\lambda > 0$ such that $\lambda < \zeta$, there exists $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \gamma^{bin}\} \neq \varnothing$ such that the borrowing limit is binding only if $\gamma \in \Omega^{bin}$, where $\gamma^{bin} \equiv [\beta\delta(\zeta - \lambda)] [1 - \delta(1 - \lambda)]^{-1}$, while for any $\lambda \geq \zeta$, $\Omega^{bin} = \varnothing$ and the borrowing limit does not bind.*

Proof. *See the appendix.* ■

Proposition 5 states that for a high enough level of the parental altruism motive ($\gamma \geq \gamma^{bin}$), the young will receive large enough parental transfers that the borrowing limit is not binding. Then the first-order conditions collapse to those under complete markets, and hence the economy is on the complete markets BGP, i.e. $g = \overline{\overline{g}}$. As long as the intergenerational correlation of human capital, $1 - \zeta$, is lower than threshold $(1 - \lambda)$, the borrowing constraint binds if and only if the parental altruism motive is relatively small. Proposition 6 also demonstrates that the threshold γ^{bin}, below which the borrowing constraint binds, is strictly smaller than the threshold γ^*, below which the young borrow in order to fund their education under complete markets. This implies that even if $\lambda < \zeta$, for any $\gamma \in \left\{\gamma > 0;\ \gamma^{bin} < \gamma < \gamma^*\right\}$, the borrowing constraint does not have any adverse effect on young agents as they can borrow the same amount they would have borrowed if markets were complete. Notice also that $\overline{\overline{g}}$ is always increasing in γ under reasonable parameter values.

Having identified the conditions under which the borrowing constraint is binding, we now proceed to an analysis of growth rates and dynamic efficiency. The main results are summarized in Propositions 6 and 7.

Proposition 6: *When the borrowing limit is binding, the laissez-faire BGP is always dynamically inefficient, whereas when the borrowing limit is not binding, the BGP is dynamically efficient according to Proposition 2.*

Proof. *See the appendix.* ■

When the young wish to borrow in order to invest in their human capital but are restricted from doing so (i.e. the borrowing limit binds), the laissez-faire BGP is always dynamically inefficient as the ratio of physical to human capital is too high. This means that along a binding borrowing limit, a central planner can always find alternative allocations of physical and human capital that increase the welfare of all generations leaving on a new BGP. Proposition 6 is consistent with proposition 3, as both indicate that under incomplete markets, the laissez-faire BGP is dynamically inefficient at relatively low levels of the parental altruism motive. Proposition 7 lays out conditions which relate the degree of tightness of the borrowing constraint and the levels of parental altruism and intergenerational persistence of human capital, under which an economy with limited access to education loans, outperforms or underperforms, in terms of growth, an otherwise identical economy with complete financial markets.

Proposition 7: *For $\lambda < \zeta$,* ***(a)*** *if $\lambda \geq \widetilde{\lambda}$, then* ***(i)*** *$g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and* ***(ii)*** *$g < \overline{\overline{g}}$ if $\gamma < \gamma^{bin}$;* ***(b)*** *if $\lambda < \widetilde{\lambda}$ then for $\zeta \leq \widetilde{\zeta}^{*}$,* ***(i)*** *$g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and* ***(ii)*** *$g > \overline{\overline{g}}$ if $\gamma < \gamma^{bin}$, while for $\zeta > \widetilde{\zeta}^{*}$, there exists $\underline{\gamma}_2 \in \Omega^{bin}$, as long as $\lambda < \lambda_{\gamma}$, such that* ***(i)*** *$g = \overline{\overline{g}}$ either if $\gamma = \underline{\gamma}_2$ or $\gamma \geq \gamma^{bin}$,* ***(ii)*** *$g < \overline{\overline{g}}$ if $\underline{\gamma}_2 < \gamma < \gamma^{bin}$ and* ***(iii)*** *$g > \overline{\overline{g}}$ if $\gamma < \underline{\gamma}_2$, where $\widetilde{\lambda} = (1-\delta)(1-\zeta)\left[1-\delta(1-\zeta)\right]^{-1}$, $\widetilde{\zeta}^{*} = \delta(1-\delta)(1-\lambda)\left[1-\delta^2(1-\lambda)\right]^{-1}$ and $\lambda_{\gamma} = (1-\zeta)\left[1-\delta(1-\lambda)\right]$.*

Proof. *See the appendix.* ■

Proposition 7 suggests that along the BGP an economy with limited access to education loans exhibits higher growth than an otherwise identical economy with complete credit markets either when (i) credit constraints are loose or (ii) credit constraints are tight while the degree intergenerational correlation of human capital, $(1-\zeta)$, is relatively low and the parental altruism motive falls within a range of relatively high values. In both cases, under complete markets, young agents end up over-accumulating human capital relative to physical capital. Specifically, in the first case, the loose credit constraints prevent young individuals from over-borrowing and over-investing in human capital, keeping the ratio of physical to human capital at a growth-optimal level, relative to that in complete markets. The latter is too low as the young over-borrow to invest in human capital. In the second case, under complete markets, young individuals choose to borrow and complement the parental transfer for the development of their human capital. However, since the degree of intergenerational correlation of human capital is low, they end up over-accumulating human capital relative to physical capital.

Empirical studies like De Gregorio (1996) and Aghion et al. (2010) suggest that the relationship between growth and credit constraints may be negative. However, there are cases of countries with undeveloped credit markets exhibiting high economic growth rates. For instance, economic growth in China has proceeded at a fast rate despite the absence of well developed credit markets. Proposition 7 provides a possible explanation of these observations that relates the restrictions to borrowing with the intergenerational correlation of human capital and the level of the parental altruism motive.[32]

Figure 2 provides a numerical illustration of the theoretical results. The calibration is the same as in section 3.2.1, except that to illustrate two different cases we consider two values for ζ and fix the borrowing constraint parameter λ at 0.1. In Case I, we set $\zeta = 0.60$, which corresponds to case (b) of Proposition 7. As expected, growth is higher for $\gamma < \gamma^{bin}$ (=0.25) in the economy with borrowing limit and coincides with the complete markets growth rate for $\gamma \geq \gamma^{bin}$. In Case II, we set $\zeta = 0.90$, which produces the final sub-case of Proposition 7. The growth rate starts out higher in the economy with borrowing limit but then falls below the growth rate in the complete markets economy. Once $\gamma \geq \gamma^{bin}$ (=0.40), the growth rates in the complete markets and borrowing limit cases coincide because the borrowing limit ceases to be binding. In both cases the economy with the binding borrowing constraint is always dynamically inefficient, as indicated by proposition 6.

Several studies examine the relationship between borrowing constraints, the savings rate and economic growth.[33] The long-run gross growth rate in our model can be expressed as $(1+g) = \varkappa(s/y)(x)^{-\delta}$ where where $\varkappa$ is a constant. Thus, growth depends positively on the savings effect, s/y, and negatively on the capital composition effect.[34] The dominance

[32] Using the growth rate equations along the BGP, it is straightforward to show that there exists a threshold $\tilde{\tilde{\gamma}} > 0$ which depends on λ with $\lim_{\lambda \to 0} \tilde{\tilde{\gamma}}(\lambda) = 1$, such that for any $\gamma \in \Omega^{bin}$, $\overline{\overline{g}} > \overline{g}$ if $\gamma > \tilde{\tilde{\gamma}}(\lambda)$, $\overline{\overline{g}} < \overline{g}$ if $\gamma < \tilde{\tilde{\gamma}}(\lambda)$ and $\overline{\overline{g}} = \overline{g}$ if $\gamma = \tilde{\tilde{\gamma}}(\lambda)$. In other words, the BGP growth rate in the economy with a missing market is strictly greater than the growth rate in the economy with limited access to credit, at low levels of the parental altruism motive.

[33] In a model without altruistic motives, Jappelli and Pagano (1994) show that borrowing constraints increase the savings rate and thereby raise economic growth whereas De Gregorio (1996) shows that once investment in human capital is introduced, borrowing constraints lower economic growth because of the negative effect on human capital accumulation. In the latter borrowing constraints affect human capital only indirectly via the incentive to work.

[34] In the model of Jappelli and Pagano (1994), only the savings effect is present (raising the savings rate, borrowing constraints raise growth) while in the model of De Gregorio (1996) the growth rate is related to the capital composition effect (borrowing constraints raise the ratio of physical to human capital which lowers economic growth).

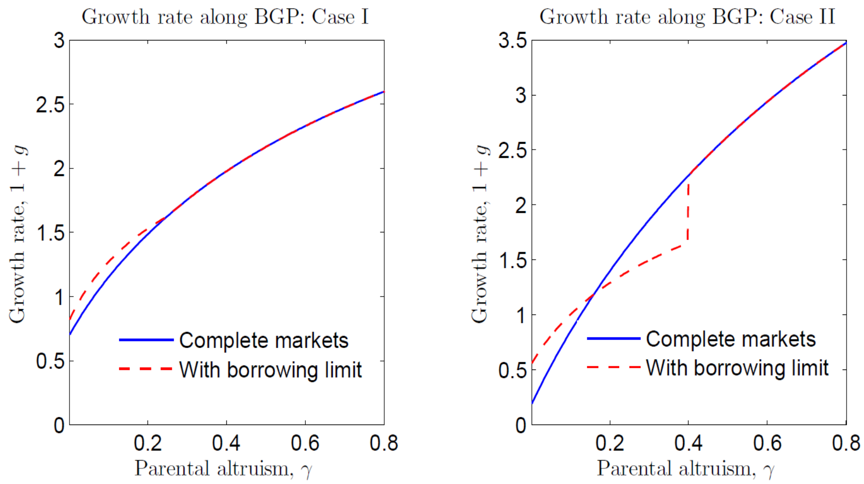

Figure 2: Growth vs parental altruism motive

of either the savings effect or the capital composition effect hinges on a number of factors, among which the parental altruism motive and the degree of credit markets imperfection. Our theoretical results indicate a non-monotonic relationship between borrowing constraints and growth that depends on the intensity of parental transfers or, equivalently, the parental altruism motive. Among others, our model provides theoretical justification of the finding of Jappelli and Pagano (1994) that the positive effect of liquidity constraints on the savings rate cannot be rejected by the data.[35] Our model also demonstrates conditions, which relate to the degree of intergenerational correlation of human capital and the degree of the parental altruism motive, that justify the observation of Coeurdacier et al. (2015) that emerging Asian economies exhibit higher growth rates and savings than those of advanced economies. Our results suggest that the latter may occur either under the existence of an imperfect

[35] As noted in footnote 31, the savings rate in the economy with a missing credit market is strictly greater than the savings rate in the economy with complete markets for relatively low levels of the parental altruism motive. Moreover, the savings rates of sections 3.1 and 3.2 along with proposition 5, imply that that for any $\gamma \in \Omega^{bin}$ along the BGP , $[s/y]^{MC} > [s/y]^{LABC} > [s/y]^{CM}$, where $[s/y]^{LABC}$ denotes the savings rate in the economy with limited access to the credit market for education loans.

credit market for education loans or when credit markets are completely absent.[36]

In summary, this section has presented several findings that challenge the conventional wisdom that the absence of credit markets or imperfections in credit markets are likely to hinder efficiency and growth.[37] Consequently, the results may help to shed light on empirical observations which relate credit constraints and growth, as well as the surprisingly strong growth performance of several Asian economies.

4 Conclusion

We present cross-country empirical evidence which shows that, after controlling for government spending on education, the intensity of household spending on education is positively related to economic growth at relatively high levels of credit market development, and negatively related at relatively low levels. We demonstrate that parental financing of children's education, one of the main components of household spending on education, exhibits a high intensity for countries such as China and India, which are characterized by high growth rates, relatively low shares of government spending on education and highly constrained markets for education loans. We also find that the intensity of parental transfers for children's education is significantly low for western countries, which are characterized by high shares of government spending and well-developed credit markets for education loans. Contrary

[36]Specifically, proposition 4 suggests that $\overline{g} > g$ and $[s/y]^{MC} > [s/y]^{CM}$ if $\underline{\gamma} < \gamma < \gamma^*$ and $\zeta > \widetilde{\zeta}$, while proposition 7 suggests that for any $\gamma \in \Omega^{bin}$, $[s/y]^{LABC} > [s/y]^{CM}$ and $\overline{\overline{g}} > g$ if either $\lambda \geq \widetilde{\lambda}$ or $\lambda < \widetilde{\lambda}$ and $\zeta > \widetilde{\zeta}^*$ and $\gamma > \underline{\gamma}_2$.

[37]If we instead considered a small open economy that takes the world real interest rate as given, as in De Gregorio (1996), our key theoretical results relating to *relative* growth rates and dynamic efficiency remain intact. These results are available upon request.

to the aggregate intensity of household spending on education, the intensity of parental financing of children's education is highly negatively correlated with the share of government spending on education. We argue that government spending on education, which facilitates accessibility to education, can be viewed as an alternative of a credit market for education loans, and that our results can be generalized to cover a government that acts as an intermediary between individual borrowers/lenders and financial institutions. Thus, without loss of generality, we focus our analysis on the credit market for education loans. We show that an OLG framework of endogenous growth that features altruistic transfers from parents to children, nesting 'missing credit markets' for education loans, can replicate and explain the empirical finding of the negative relationship between the intensity of household spending on education and growth at low levels of credit market development. Using the OLG framework, we also establish conditions under which missing or imperfect credit markets increase economic growth and do not hinder dynamic efficiency. We show that, as long as parental altruism is sufficiently high, government intervention is not necessary for an economy with a missing credit market to achieve dynamic efficiency. The parental altruism motive plays a key role in the results; however its implications for growth and dynamic efficiency are not monotonic, but depend crucially on the extent of credit market development. We thus argue that parental altruistic motives may be a factor behind cross-country differences in growth which the literature has hitherto had difficulty explaining.

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Appendix

Credit Index Construction

Our credit index is based on four indicators of credit provision and financial development: (1) Borrowed for Education or School Fees;[38] (2) Borrowed from a financial institution in the past year;[39] (3) Credit card ownership;[40] (4) Domestic Credit to the Private Sector / GDP (Average 1990-2010, %).[41] The credit index is calculated as Credit $\text{Index}_i \equiv 1 - \frac{1}{4}\sum_{j=1}^{4} FD_{i,j}$, where $FD_{i,j}$ is a dummy equal to 1 if indicator j in country i is below a chosen threshold for the given indicator. Our chosen thresholds were, respectively, 4%, 6%. 9%, 25%.

[38] % respondents (age 15+) who report borrowing any money for education or school fees in the past 12 months; year 2014.

[39] % (age 15+) of respondents who report borrowing any money from a bank or another type of financial institution in the past 12 months. An average value is computed based on years 2011 and 2014.

[40] % (age 15+) respondents who report having a credit card. Average value based on years 2011 and 2014.

[41] Domestic Credit to the Private Sector includes financial resources provided to the private sector, including loans, purchases of nonequity securities, and trade credits and other accounts receivable, that establish a claim for repayment. For some countries these claims include credit to public enterprises. Average value from 1990-2010 is used.

Table 1A - Descriptive Statistics

	Growth	*HExp*	*GExp*	*Credit*	*HExp* × *Credit*
Mean	1.75	2.18	4.18	0.65	1.43
Median	1.83	1.92	4.13	0.75	1
Maximum	6.47	8.34	7.84	1	7.03
Minimum	-2.77	0.09	1.60	0	0
Std. Dev.	1.61	1.64	1.34	0.28	1.35
Skewness	-0.11	1.63	0.30	-0.40	1.89
Kurtosis	3.67	6.39	2.87	2.20	7.16

Proofs

Proof of Proposition 2. Since $s_t = b_t + k_{t+1} = (1+n)\overline{b}_t + k_{t+1} = (1+n)d_t - (1+n)\omega_t + k_{t+1}$, savings per efficient labor at the balanced growth path reduce to $\widehat{s} = (1+n)\widehat{d} - (1+n)\widehat{\omega} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$, where $\widehat{d} = d_t/h_t = \widehat{\omega} + \widehat{\overline{b}}$, $\widehat{\omega} = \omega_t/h_t$, $\widehat{\overline{b}} = \overline{b}/h_t$ and $B\mu^{1-\zeta}\widehat{d}^{\zeta} = 1+g$. The balanced growth version of the resource constraint is obtained by dividing all terms of (6) by h_t: $Ax^{1-\delta} = \widehat{c}_m + \frac{\widehat{c}_o}{B\mu^{1-\zeta}\widehat{d}^{\zeta}}\frac{1}{1+n} + (1+n)\widehat{d} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$. The social planner maximizes $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$ subject to the latter. Note that the $\widehat{\overline{b}}$ terms cancel out in the BGP resource constraint, which implies that the planner will set $\widehat{d} = \widehat{\omega}$, and so the planner's choice variables reduce to $\left\{\widehat{c}_m,\, \widehat{c}_o,\, x,\, \widehat{d}\right\}$. The optimal conditions of the social planner can be summarized as follows:

$$R = (1+n)(1+g), \tag{A.1}$$

$$\frac{\gamma\widehat{c}_m}{\widehat{d}} = -\frac{\zeta\widehat{c}_o}{(1+n)(1+g)\widehat{d}} + (1+n) + \frac{\zeta x(1+n)(1+g)}{\widehat{d}}, \tag{A.2}$$

$$\widehat{c}_o = \beta(1+n)(1+g)\widehat{c}_m, \tag{A.3}$$

where (A.1) is the optimal condition for x, (A.2) is implied by the optimal conditions for $\widehat{c}_m$ and $\widehat{d}$ and (A.3) is implied by the optimal conditions for $\widehat{c}_o$ and $\widehat{c}_m$. According to definition 2, the BGP is dynamically inefficient if a reduction in x induces a strictly positive change in either $\widehat{c}_m$ or $\widehat{c}_o$ or $\widehat{d}$ of current generations as well as generations of transient periods. Using the BGP resource constraint, it can be shown that $\frac{\partial \widehat{c}_m}{\partial x}\Big|_{\widehat{c}_o,\widehat{d}} = (1-\delta)Ax^{-\delta} - (1+n)B\mu^{1-\zeta}\widehat{d}^{\zeta} \equiv R-(1+n)(1+g)$, $\frac{\partial \widehat{c}_o}{\partial x}\Big|_{\widehat{c}_y,\widehat{d}} = (1+n)(1+g)[R-(1+n)(1+g)]$ and $\frac{\partial \widehat{d}}{\partial x}\Big|_{\widehat{c}_y,\widehat{c}_o} = \frac{R-(1+n)(1+g)}{\Phi}$, where Φ is equal to the right hand side of (A.2). Condition $R \geq (1+n)(1+g)$ ensures that $\frac{\partial \widehat{c}_m}{\partial x}\Big|_{\widehat{c}_o,\widehat{d}} \geq 0$ and $\frac{\partial \widehat{c}_o}{\partial x}\Big|_{\widehat{c}_y,\widehat{d}} \geq 0$, when they are evaluated at the laissez-faire equilibrium. The only case where $R \geq (1+n)(1+g)$ may not be sufficient for dynamic efficiency of the BGP is when $R > (1+n)(1+g)$ and $\Phi < 0$, where Φ is evaluated at the laissez-faire equilibrium. $\Phi < 0$ only if $R/(1+n)(1+g) > \gamma/\delta\zeta\beta$. Therefore, a sufficient condition for dynamic efficiency of the laissez-faire BGP is $1 \leq R/(1+n)(1+g) \leq \gamma/\delta\zeta\beta$, for $\gamma/\delta\zeta\beta > 1$. If the left inequality holds but the right does not then the BGP is not dynamically efficient because $R/(1+n)(1+g) \geq 1$ implies that $R/(1+n)(1+g) > \gamma/\delta\zeta\beta$ for $\gamma/\delta\zeta\beta < 1$. Since $R/(1+n)(1+g) = (1-\delta)A\Psi^{-1}$, using the BGP laissez-faire condition, $x = \Psi^{1/\delta}/[(1+n)(1+g)]^{1/\delta}$, the sufficient condition for dynamic efficiency of the BGP, in terms of γ, is $\gamma_1^c \equiv \frac{A\zeta\beta\delta(1-\delta)}{\Psi} \leq \gamma \leq \frac{[1-\delta(1-\zeta)](1+\beta)-\delta\beta(1-\zeta)}{\delta(1-\zeta)} \equiv \gamma_2^c$ for any $\gamma \geq \delta\zeta\beta$, where $\gamma_2^c > 0$ only if $\delta < (1+\beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$. ■

Proof of Proposition 3. Following the proof of proposition 2, using the functional form of the optimal $\widehat{d}$, at the laissez-faire BGP with no-credit market, the functional form of $\overline{\Psi}$, and replacing x with $\overline{x}$ and g with $\overline{g}$, it is straightforward to show that $\Phi < 0$ only if $R/(1+n)(1+\overline{g}) > \delta\gamma_R^c + 1$. Therefore, the sufficient condition for dynamic efficiency of the laissez-faire BGP of the economy with no-credit market is $1 \leq R/(1+n)(1+\overline{g}) \leq \delta\gamma_R + 1$. Since $R/(1+n)(1+\overline{g}) = (1-\delta)A\overline{\Psi}^{-1}$, the left-hand side of the inequality reduces to $\gamma \geq \gamma_1^{in} \equiv \frac{\beta\delta-(1-\delta)(1+\beta)}{1-\delta}$, while the right-hand side reduces to $\gamma \geq \gamma_2^{in} \equiv \frac{\zeta[1-\delta(1+\beta)]}{1-\zeta(1-\delta)}$. It follows that the laissez-faire BGP of the economy with no-credit market is dynamically efficient if $\gamma \in \Omega^{in} \equiv \{\gamma > 0; \gamma \geq \gamma^{in}\} \neq \emptyset$, where $\gamma^{in} = max\{\gamma_1^{in}, \gamma_2^{in}\}$. ■

Proof of Proposition 4. Along the BGP, we would like to examine the conditions under which $g = \overline{g}$, $g > \overline{g}$ and $g < \overline{g}$. The latter is equivalent to $\Upsilon_1(\xi) = \Upsilon_2(\xi)$, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ and $\Upsilon_1(\xi) < \Upsilon_2(\xi)$, respectively, where $\xi = \gamma(1-\delta)(\beta\delta\zeta)^{-1}$, $\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta) + \zeta[1-\delta(1-\zeta)]^{-1}\xi$ and $\Upsilon_2(\xi) = \xi^{\delta}$, using the equations for $(1+g)$ and $(1+\overline{g})$ of sections 3.1 and 3.2.1. Notice that $\Upsilon_1(\xi)$ is a linear and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta)$, while $\Upsilon_2(\xi)$ is a concave and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_2(\xi) = 0^+$ and $\lim_{\xi\to +\infty}\Upsilon_2(\xi) = +\infty$. Since $\lim_{\xi\to 0}\Upsilon_1(\xi) > \lim_{\xi\to 0}\Upsilon_2(\xi)$, $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ do not intersect for any value of ξ if and only if $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. The latter is the case only if $\Upsilon_1(\xi^*) > \Upsilon_2(\xi^*)$, where $\partial\Upsilon_1(\xi^*)/\partial\xi = \partial\Upsilon_2(\xi^*)/\partial\xi$ which implies $\xi^* = [\delta(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. In other words, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ only if $X^{1-\delta} > 1 - \left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right]X$, where $X = [(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. Let the left hand side of the $X-$inequality be denoted by X_1 and the right hand side by X_2. Notice that it cannot be the case that $X \leq 1$ because that would imply that $\zeta \geq 1$. X_1 is an increasing and concave function of X which starts *almost* (since $X > 0$) from the origin. X_2 is a linear and decreasing function of X with $\lim_{X\to 0^+} X_2(X) = 1$. It follows that the $X-$inequality may hold only in the region on the right of the intersection point of X_1 and X_2. In this region, $X_2 < 1$ which implies that $\left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right]X > 0$. Given that the term in brackets is negative, the only way the latter holds is when $X < 0$ which cannot hold since $X > 0$. Thus, it cannot be the case that $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. It follows that $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ have at least one, and at most two intersection points. At least one of the points of intersection is the point where $\xi = 1$ since $\Upsilon_1(1) = \Upsilon_2(1) = 1$. It can be shown that there are three feasible cases. In *case 1*, the slope of $\Upsilon_2(\xi)$ is greater than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta < \delta(1+\delta)^{-1}$. Then, there exist $\overline{\xi} > 1$ such that $\Upsilon_1\left(\overline{\xi}\right) = \Upsilon_2\left(\overline{\xi}\right)$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < 1$ and $\xi > \overline{\xi}$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $1 < \xi < \overline{\xi}$. Therefore, under case 1 the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = 1$ *or* $\xi = \overline{\xi}$ ($\gamma = \gamma^*$ or $\gamma = \overline{\gamma}$); (ii) $g < \overline{g}$ *if* $1 < \xi < \overline{\xi}$ ($\gamma^* < \gamma < \overline{\gamma}$); (iii) $g > \overline{g}$ *if* $\xi < 1$ *or* $\xi > \overline{\xi}$ ($\gamma < \gamma^*$ or $\gamma > \overline{\gamma}$). In *case 2*, the slope of $\Upsilon_2(\xi)$ is smaller than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta > \delta(1+\delta)^{-1}$. Then, there exist $\underline{\xi} < 1$ such that $\Upsilon_1\left(\underline{\xi}\right) = \Upsilon_2\left(\underline{\xi}\right)$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < \underline{\xi}$ and $\xi > 1$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $\underline{\xi} < \xi < 1$. Therefore, under case 2, the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = \underline{\xi}$ *or* $\xi = 1$ ($\gamma = \underline{\gamma}$ or $\gamma = \gamma^*$); (ii) $g < \overline{g}$ *if* $\underline{\xi} < \xi < 1$ ($\underline{\gamma} < \gamma < \gamma^*$); (iii) $g > \overline{g}$ *if* $\xi < \underline{\xi}$ *or* $\xi > 1$ ($\gamma < \underline{\gamma}$ or $\gamma > \gamma^*$). In *case 3*, the slope of $\Upsilon_2(\xi)$ is equal to the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta = \delta(1+\delta)^{-1}$. In this case, $\xi = 1$ is the single point of contact between $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ while in all other cases, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$. Therefore, under case 3 the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = 1$ ($\gamma = \gamma^*$); (ii) $g > \overline{g}$ *if* $\xi < 1$ *and if* $\xi > 1$ ($\gamma < \gamma^*$ and if $\gamma > \gamma^*$). Note that the relationships in brackets are due to the fact that for $\overline{\xi}$ and $\underline{\xi}$, there are unique thresholds $\overline{\gamma}$ and $\underline{\gamma}$ such that $\beta\delta\zeta\overline{\xi} = \overline{\gamma}(1-\delta)$ and $\beta\delta\zeta\underline{\xi} = \underline{\gamma}(1-\delta)$. Since $\overline{\xi} > 1$ and $\underline{\xi} < 1$, the latter implies that $\overline{\gamma} > 1$, $\underline{\gamma} < 1$ and $0 < \underline{\gamma} < \gamma^* < \overline{\gamma}$. ■

Proof of Proposition 5 . Complementary slackness conditions imply that the Lagrange multiplier on the borrowing constraint, $\overline{\mu}_t > 0$, and $\overline{b}_{t-1} = \lambda w_t h_t / R_t$ when the borrowing constraint binds. Along the BGP, the optimality condition $w_t h_1\left(d_{t-1}, h^y_{t-1}\right) - R_t = \overline{\mu}_t$ reduces to $\zeta\left(1+\beta+\gamma\right)\left(1+\frac{\delta\lambda}{(1-\delta)}\right)\overline{\overline{\Psi}} > (1-\delta)A(1-\lambda)(\gamma+(\beta+\gamma)\frac{\delta\lambda}{1-\delta})$, which then collapses to $\gamma < \gamma^{bin} \equiv \gamma^*\left[\frac{(1-\delta)(\zeta-\lambda)}{[1-\delta(1-\lambda)]\zeta}\right] = \frac{\beta\delta(\zeta-\lambda)}{1-\delta(1-\lambda)}$. It follows that if $\lambda < \zeta$ then $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \gamma^{bin}\} \neq \varnothing$ and the borrowing constraint binds only if $\gamma \in \Omega^{bin}$. If $\lambda \geq \zeta$, then $\gamma^{bin} \leq 0$ and thus $\Omega^{bin} = \varnothing$ since it violates the assumption that $\gamma > 0$, and the borrowing constraint does not bind. ■

Proof of Proposition 6 . Following the proof of proposition 2, using the functional form of the optimal $\widehat{d}$, at the laissez-faire BGP with a binding borrowing constraint, the functional form of $\overline{\overline{\Psi}}$, and replacing x with $\overline{\overline{x}}$ and g with $\overline{\overline{g}}$, it is shown that $\Phi < 0$ only if $\frac{R}{(1+n)(1+\overline{\overline{g}})} > \frac{\gamma[(1-\delta)(\zeta\beta+\gamma)+(\beta+\gamma)\delta\lambda]}{\zeta\beta[\gamma(1-\delta)+(\beta+\gamma)\delta\lambda]} \equiv \overline{\gamma}_R$. Therefore, the sufficient condition for dynamic efficiency of the laissez-faire BGP is $1 \leq \frac{R}{(1+n)(1+\overline{\overline{g}})} \leq \overline{\gamma}_R$ (notice that when $\lambda = 0$, which implies that $\overline{b} \leq 0$, $\overline{\gamma}_R = \delta\gamma_R + 1$). Since $\frac{R}{(1+n)(1+\overline{\overline{g}})} = \frac{(1-\delta)A}{\overline{\overline{\Psi}}}$, the latter inequality can also be written as $\overline{\gamma}^{in}_1 \equiv \beta\frac{\delta(1-\lambda)}{1-\delta(1-\lambda)} - (1+\beta) \leq \gamma \leq \frac{\gamma\delta(1-\lambda)}{\zeta[1-\delta(1-\lambda)]}\frac{\beta[\delta\lambda+(1-\delta)\zeta]+\gamma[1-\delta(1-\lambda)]}{\beta\delta\lambda+\gamma[1-\delta(1-\lambda)]} - (1+\beta) \equiv \overline{\gamma}^{in}_2$. Since $\overline{\gamma}^{in}_2$ is increasing in γ and $\gamma \in \Omega^{bin}$, the upper limit of the inequality for dynamic efficiency is $\overline{\gamma}^{in}_2 \mid_{\gamma=\gamma^{bin}} = \beta\frac{\delta(\zeta-\lambda)(1-\lambda)}{\zeta[1-\delta(1-\lambda)]^2} - (1+\beta)$. Thus, dynamic efficiency is possible when $\gamma \in \Omega^{bin}$ as long as, (i) $\zeta \geq \lambda/\delta(1-\lambda) \equiv \overline{\zeta}_{low}$, which implies that $\overline{\gamma}^{in}_2 \mid_{\gamma=\gamma^{bin}} \geq \overline{\gamma}^{in}_1$, (ii) $\beta\left[\frac{\delta(\zeta-\lambda)(1-\lambda)}{\zeta[1-\delta(1-\lambda)]^2} - 1\right] > 1$, which implies that $\overline{\gamma}^{in}_2 \mid_{\gamma=\gamma^{bin}} > 0$. In addition, dynamic efficiency under a binding constraint requires $\gamma^{bin} \geq \overline{\gamma}^{in}_1$, which implies that $\beta\delta\frac{(1-\zeta)+(1-\lambda)}{1-\delta(1-\lambda)} \leq 1$. The latter along with (ii) imply that $\beta_{low} \equiv \frac{\zeta[1-\delta(1-\lambda)]^2}{\delta(\zeta-\lambda)(1-\lambda)-\zeta[1-\delta(1-\lambda)]^2} < \beta \leq \frac{1-\delta(1-\lambda)}{\delta[(1-\zeta)+(1-\lambda)]} \equiv \beta_{high}$. Then, it must be the case that $\beta_{high} > \beta_{low}$, which holds only if $\zeta < \frac{\delta(\zeta-\lambda)(1-\lambda)-\zeta[1-\delta(1-\lambda)]^2}{\delta[1-\delta(1-\lambda)][(1-\zeta)+(1-\lambda)]} \equiv$

ζ_{high}. The latter implies that $\delta(\zeta-\lambda)(1-\lambda)-\zeta[1-\delta(1-\lambda)]^2>0$, since $\zeta>0$, which then reduces to $\zeta > \frac{\delta\lambda(1-\lambda)}{\delta(1-\lambda)-[1-\delta(1-\lambda)]^2} \equiv \zeta_{low}$, where $\delta(1-\lambda)>[1-\delta(1-\lambda)]^2$. Since $\zeta_{low} > \overline{\zeta}_{low}$, $\beta_{high} > \beta_{low}$ when $\zeta_{low} < \zeta < \zeta_{high}$. Note that ζ_{high} can be either monotonically decreasing or monotonically increasing, depending on the values of the parameters. Since $\lim_{\zeta\to 0^+}\zeta_{high} < 0$, if ζ_{high} is monotonically decreasing in ζ, then dynamic efficiency is impossible under a binding borrowing constraint as $\zeta > 0$. The only feasible case of dynamically efficient BGP with a binding constraint is when ζ_{high} is monotonically increasing in ζ. Then, since $\lambda < \zeta$, under a binding constraint, it must be that $\lambda < \lim_{\zeta\to 1^-}\zeta_{high} = \frac{\delta(1-\lambda)^2-[1-\delta(1-\lambda)]^2}{\delta(1-\lambda)[1-\delta(1-\lambda)]}$, which can be rewritten as $\phi_1(x) \equiv 1-x+(1-\lambda)x^2 < 2(1-\lambda)x \equiv \phi_2(x)$, where $x=\delta(1-\lambda)$. Note that $\phi_1(x)$ is a convex function that reaches a minimum at $x=1/2(1-\lambda)$ while $\phi_2(x)$ is a linear function, with a positive slope, that passes through the origin. Since $\phi_1(0)=1$, there are two intersection points between $\phi_1(x)$ and $\phi_2(x)$ that lie in the positive area of x. Specifically, $x_{1,2} = 1+\frac{1\pm\sqrt{4(1-\lambda)^2+1}}{2(1-\lambda)}$. It follows that $\phi_1(x)$ lies below $\phi_2(x)$ only if $x_1 < x < x_2$. Since $x_1 > 1$, there is no $x \equiv \delta(1-\lambda) < 1$ such that $x_1 < x < x_2$. Therefore, dynamic efficiency of the laissez-faire BGP is impossible when the borrowing constraint binds. If the borrowing constraint is slack, the complete markets laissez-faire BGP and associated condition for dynamic efficiency apply (Proposition 2). ■

Proof of Proposition 7 . Along the BGP, when $\lambda < \zeta$ and $\gamma \in \Omega^{bin}$, we would like to examine the conditions under which $g = \overline{\overline{g}}$, $g > \overline{\overline{g}}$ and $g < \overline{\overline{g}}$. The latter is equivalent to $\overline{\Upsilon}_1\left(\overline{\xi}\right) = \overline{\Upsilon}_2(\overline{\xi})$, $\overline{\Upsilon}_1\left(\overline{\xi}\right) > \overline{\Upsilon}_2(\overline{\xi})$ and $\overline{\Upsilon}_1\left(\overline{\xi}\right) < \overline{\Upsilon}_2(\overline{\xi})$, respectively, where $\overline{\xi}$ $= [\gamma\,[1-\delta(1-\lambda)]+\beta\delta\lambda]/\beta\delta\zeta$, $\overline{\Upsilon}_1\left(\overline{\xi}\right) = \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\lambda)[1-\delta(1-\zeta)]} + \left(\frac{\zeta}{(1-\lambda)[1-\delta(1-\zeta)]}\right)\overline{\xi}$ and $\overline{\Upsilon}_2(\overline{\xi}) = \overline{\xi}^{\delta}$, using the equations for $(1+g)$ and $(1+\overline{\overline{g}})$ of sections 3.1 and 3.2.2. Notice that $\overline{\Upsilon}_1(\overline{\xi})$ is a linear and increasing function of $\bar{\xi}$, while $\overline{\Upsilon}_2(\bar{\xi})$ is a concave and increasing function of $\bar{\xi}$, with $\lim_{\bar{\xi}\to 0^+} \overline{\Upsilon}_2(\bar{\xi}) = 0^+$ and $\lim_{\bar{\xi}\to +\infty} \Upsilon_2(\bar{\xi}) = +\infty$. Since $\bar{\xi}$ is a function of γ, $\overline{\Upsilon}_1$ and $\overline{\Upsilon}_2$ can be written as $\overline{\Upsilon}_1(\gamma)$ and $\overline{\Upsilon}_2(\gamma)$. The properties of $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ and $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ imply that there might be either zero or, at most, two intersection points between $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ and $\overline{\Upsilon}_2\left(\bar{\xi}\right)$. A sufficient condition for no intersection points between $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ and $\overline{\Upsilon}_2\left(\bar{\xi}\right)$, i.e. $\overline{\Upsilon}_1\left(\bar{\xi}\right) > \overline{\Upsilon}_2\left(\bar{\xi}\right)$ for all values of $\bar{\xi}$, is that $\overline{\Upsilon}_1\left(\bar{\xi}^*\right) > \overline{\Upsilon}_2\left(\bar{\xi}^*\right)$ for $\bar{\xi}^*$ such that $\partial\overline{\Upsilon}_1\left(\bar{\xi}^*\right)/\partial\bar{\xi} = \partial\overline{\Upsilon}_2\left(\bar{\xi}^*\right)/\partial\bar{\xi}$. The latter reduces to $\bar{\xi}^* = (\delta(1-\lambda)[1-\delta(1-\zeta)]/\zeta)^{\frac{1}{1-\delta}}$. Thus, $\overline{\Upsilon}_1\left(\bar{\xi}^*\right) > \overline{\Upsilon}_2\left(\bar{\xi}^*\right)$ imply that $\overline{X}^{\frac{\delta}{1-\delta}} < \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\delta)(1-\lambda)[1-\delta(1-\zeta)]}$, where $\overline{X} = \delta(1-\lambda)[1-\delta(1-\zeta)]/\zeta$. Since, $\overline{X} > 0$, it must be the case that $\lambda < (1-\zeta)[1-\delta(1-\lambda)]$ which also implies that $\zeta < 1-\delta(1-\zeta)$ since $\lambda > 0$. Then, it follows that $\lambda < 1-\delta(1-\zeta)$ and thus, $\lambda < (1-\zeta)[1-\delta(1-\lambda)] < (\lambda-\zeta)/\lambda$. Since $\lambda > 0$, the latter can hold only if $\lambda > \zeta$, which cannot be the case since $\lambda < \zeta$. Therefore, it cannot be the case that $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ and $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ have no intersection points. In what follows, we focus on the cases where there is either one or two intersection points. At least one of the points of intersection is the point where $\bar{\xi} = 1$ since $\overline{\Upsilon}_1(1) = \overline{\Upsilon}_2(1) = 1$. Following the proof of proposition 5, it can be shown that there are four feasible cases. In *case 1*, there is a single intersection point only when the intercept of $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ is negative, i.e. $\lambda \geq (1-\delta)(1-\zeta)[1-\delta(1-\zeta)]^{-1} \equiv \widetilde{\lambda}$. Note that the unique intersection point must be 1. Since $\bar{\xi} = 1$ implies that $\gamma = \gamma^{bin}$, it follows that (i) $g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and (ii) $g < \overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$. For *cases 2-4*, the intercept $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ is strictly positive, i.e. $\lambda < \widetilde{\lambda}$. For *case 2*, recall that when $\bar{\xi} = 1$, $\gamma = \gamma^{bin}$. Thus, when the slope of $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ is greater than the slope of $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ at $\bar{\xi} = 1$, i.e. $\zeta < \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1} \equiv \widetilde{\zeta}^*$, then $\overline{\Upsilon}_1\left(\bar{\xi}\right) \geq \overline{\Upsilon}_2\left(\bar{\xi}\right)$ for any $\bar{\xi} \leq 1$ or equivalently, (i) $g = \overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ and (ii) $g > \overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$, since the borrowing constraint will not bind if $\gamma \geq \gamma^{bin}$ and the economy will behave as in the case of complete markets. In *case 3*, the slope of $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ is smaller than the slope of $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ at $\bar{\xi}=1$, i.e. $\zeta > \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. Then, there exist $0 < \underline{\xi}^* < 1$ such that $\overline{\Upsilon}_1\left(\underline{\xi}^*\right) = \overline{\Upsilon}_2\left(\underline{\xi}^*\right)$, $\overline{\Upsilon}_1\left(\bar{\xi}\right) > \overline{\Upsilon}_2\left(\bar{\xi}\right)$ for $\bar{\xi} < \underline{\xi}^*$, and $\overline{\Upsilon}_1\left(\bar{\xi}\right) < \overline{\Upsilon}_2\left(\bar{\xi}\right)$ for $\underline{\xi}^* < \bar{\xi} < 1$. For any $\bar{\xi} > 1$, the borrowing constraint will not bind and thus it will behave as in the case of complete markets. It follows that there is $\underline{\gamma}_2$, as long as $\lambda < (1-\zeta)[1-\delta(1-\lambda)] = \lambda_\gamma$, such that $\underline{\gamma}_2 < \gamma^{bin}$ Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g = \overline{\overline{g}}$ if $\gamma = \underline{\gamma}_2$ or $\gamma \geq \gamma^{bin}$, (ii) $g < \overline{\overline{g}}$ *if* $\underline{\gamma}_2 < \gamma < \gamma^{bin}$ and (iii) $g > \overline{\overline{g}}$ *if* $\gamma < \underline{\gamma}_2$. Finally, in *case 4*, the slope of $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ is equal to the slope of $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ at $\bar{\xi}=1$, i.e. $\zeta = \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. In this case, $\bar{\xi}=1$ is the single point of contact between $\overline{\Upsilon}_1\left(\bar{\xi}\right)$ and $\overline{\Upsilon}_2\left(\bar{\xi}\right)$ while in all other cases, $\overline{\Upsilon}_1\left(\bar{\xi}\right) > \overline{\Upsilon}_2\left(\bar{\xi}\right)$. Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g = \overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ and (ii) $g > \overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$. ■