Direct Numerical Simulation of a turbulent channel flow with 3D random roughness on a wall

Stefano Leonardi
University of Puerto Rico Mayaguez

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Introduction

• Turbulent flows over rough surfaces have many practical applications:
  – vegetation, topography, cities for ABL
  – heat exchangers or inside turbine blades
  – effect on wind turbine (near the sea)
  – Accumulation of algae and living organisms on ships and turbine blade

• Our knowledge of the effect of the flow over rough surfaces is incomplete…
Roughness Function

- **k-type**

\[ \Delta U^+ = \kappa^{-1} \ln k^+ + C_1 \]

- **d-type**

\[ \Delta U^+ = \kappa^{-1} \ln d^+ + C_2 \]

(\(d^+\) is the pipe diameter, channel half-width or b/\(\ell\) thickness)

Leonardi, Orlandi & Antonia PoF 2007
d-k type behavior is due to the different contributions of frictional and form drag. Vortex shedding out of the cavity (in our opinion) does not justify a k-type behavior.

<table>
<thead>
<tr>
<th>d-type</th>
<th>k-type</th>
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<tbody>
<tr>
<td>Dominated by friction drag</td>
<td>Dominated by pressure drag</td>
</tr>
<tr>
<td>Important Reynolds number</td>
<td>Almost no viscous scales</td>
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Speculations:
- a geometry can be both d or k depending on Re. In fact by increasing Re the viscous contribution decreases and drag can be dominated by pressure.
$\Delta U^+$ is correlated with the drag

For a k-type the drag is dominated by pressure drag

Pressure drag is then correlated with $uv$ at the crests plane, and then somehow with $vv$

successful correlation
• Where is the crests plane for a random rough wall?

• In a practical (engineering) application how do I get $\nuv$? (Southampton 2012)

• However I cannot get an equivalent sand grain roughness either without running the experiment first

• To close the loop we would need some correlation between geometrical quantities of the surface and $\nuv$ or drag coefficient or equivalent sand grain roughness
• Perhaps surprisingly, still we cannot answer a very basic question, I give you a rough surface you provide me with an approximation of the drag which I can use in regional simulations in meteorology or oceanography.

• Leonardi & Castro 2010 “One of the simplest questions (at least to ask) concerns the plan area density of the building array which yields maximum surface drag. The answer must naturally depend on the specific building shapes and orientations, but even for the arguably simplest case of uniformly staggered arrays of cubes there remains some uncertainty.”
• Most of the studies have dealt with laboratory roughness: square bars, sand grain, meshes, rods, cubes, chopped transverse bars.
• Are they representative of a real rough wall?
• “an understanding of idealized roughness may not properly extrapolate to more practical cases of highly irregular surface roughness” Bons 2002.
What is the way forward?

To study random roughness and try to understand the influence on the drag of the geometrical parameters (average, rms, skw)
Recent studies on random (real) rough wall

• Dynamic roughness model (Anderson & Meneveau 2011)
• DNS Chen, Cardillo, Araya, Castillo & Jansen APS2010-2011
• Low order representation of irregular roughness (Mejia-Alvarez & Christensen 2010)
• Extensive study on geometry moments to classify roughness (Flack & Schultz 2010)
Geometrical sketch

Random height wedges

Modified wall
(eliminated $k_i$ if $k_{i+1} - k_i > x_{i+1} - x_i$)
(30% eliminated)

- Same average height
- Different rms and skw

Uniform

- Two set of simulations with different average height ($k_{avg} = 0.1h$ $k_{avg} = 0.05h$)
Boundary Conditions

Free slip condition (and normal wall velocity zero) not shown here

Periodic in spanwise and streamwise direction (it is half channel)

Temperature $T=1$
Lower wall and roughness

Temperature $T=-1$

Only part of the domain is shown (half in spanwise dir. 1/6 in streamwise, ¼ in wall normal
Vectors are plotted every 8 in spanwise direction, every 2 in wall normal.
Law of the wall

\[ \overline{U}^+ = \kappa^{-1} \ln y^+ + C - \Delta \overline{U}^+ \]

- **Large elements** (average height=0.2)
  - Graph showing \( U^+ \) vs. \( y^+ \)
  - \( \Delta U^+ \) indicated

- **Small elements** (average height=0.1)
  - Graph showing \( U^+ \) vs. \( y^+ \)
  - \( \Delta U^+ \) indicated
• k-type

\[ \Delta U^+ = \kappa^{-1} \ln k^+ + C_1 \]
Perhaps surprisingly, same roughness function regardless of different layouts and rms of geometry \( h_{rms} \).

Total drag is dominated by form drag.

As a result of the simulations the 3 rough walls have approximately the same \( u\tau \) (3-4% difference), the form drag being the 90% of the total drag.

\[
P_d = q \sum_i C_{d,i} k_i \approx q \langle C_d \rangle \sum_i k_i \approx q \langle C_d \rangle k_{avg}
\]

Since all the roughness elements are triangular wedges in a dense array the variability of the drag coefficient of each element is not much and we can approximate

Same drag despite different geometrical layout brought to the same roughness function.

\[\Delta U^+ \text{ is correlated with the drag}\]
Flow structure

Random height

Uniform
Perhaps one would expect a recirculation as in 2D roughness, but spanwise streams induced ejection (similar to array of cubes Leonardi & Castro 2011, Lee & Sung 2011, Coceal et al. 2006)
Horizontal sections

Color contours of pressure (only ¼ of the domain is shown)
Horizontal sections

Color contours of pressure (only ¼ of the domain is shown)
Turbulent intensities

\[ \overline{u_i^2} \]
Equation for Reynolds stress

\[
\frac{D}{Dt} \left\langle u_i u_j \right\rangle = T_{ij} + \nu \nabla^2 \left\langle u_i u_j \right\rangle + P_{ij} + \Pi_{ij} + \varepsilon_{ij}
\]

\[
P_{ij} = -\left\langle u_i u_k \right\rangle \frac{\partial \left\langle U_j \right\rangle}{\partial x_k} - \left\langle u_j u_k \right\rangle \frac{\partial \left\langle U_i \right\rangle}{\partial x_k}
\]

\[
T_{ij} = -\frac{\partial}{\partial x_k} \left\langle u_i u_j u_k \right\rangle
\]

\[
\varepsilon_{ij} = -2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right\rangle
\]

\[
\Pi_{ij} = -\frac{1}{\rho} \left\langle u_i \frac{\partial p}{\partial x_j} + u_j \frac{\partial p}{\partial x_i} \right\rangle
\]

Budgets of uu normalised by \( u_i^4 / \nu \)

Averages in time and space

MKM99
Heat flux enhancement

Statistical convergence has to be improved, preliminary results only

Random height

Modified height

Uniform height

Smooth channel
Conclusions

• The three geometries present similar roughness function and drag coefficients. The dominant length scale appears to be the averaged surface despite different layouts and rms of the surface.

• Heat and mass transfer is increased over rough walls with respect to the smooth wall. Despite the same roughness function, random rough wall was enhancing the heat transfer the most.

• Is the roughness function a good quantity to characterize the flow?

• Energy is extracted from the streamwise component via the pressure velocity correlation and distributed to spanwise and wall normal velocity.

• Visualizations show the isotropy is better approximated over a rough surface. Iso-contours of velocity and vorticity are more inclined with respect to the wall normal direction.
Questions?