



Scalarization of compact objects

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In collaboration with

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Plan of the talk:

- Scalarization – general idea
- Neutron star scalarization in scalar tensor theories
- Black hole scalarization in extended scalar-tensor theories
- Conclusions

Why generalized theories of gravity?

There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.

- **Hamlet (1.5.167-8), Hamlet to Horatio**

Observations

- Strong field regime not well constrained
- Dark Energy
- Dark Matter (?)

Theory

- Theories trying to unify all the interactions
- Attempts to quantize gravity
- Examining GR from a broader perspective help us understand it better

Scalarization – general idea

- General idea:
 - ✓ **Modified theory** of gravity **possessing** an additional mediator of the gravitational interaction – a **scalar field φ**
 - ✓ **Perturbative equivalent to GR in the weak field**
 - ✓ Nonlinear effects for strong fields – **scalarization**
 - ✓ **Not possible** to be modelled directly by **approximate metrics or perturbative techniques.**
- Conditions for scalarization
 - ✓ A **gravitational theory** where **$\varphi = 0$ is always a solution** of the field equations
 - ✓ A **source of the scalar field** should be present leading to **negative values** of the **square of the effective mass**, i.e. tachyonic instability

Scalarization – general idea

- Einstein frame action (Scalar-tensor theories)

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{g} [R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 4V(\varphi)] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

- Field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_* T_{\mu\nu} + 2\partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - 2V(\varphi)g_{\mu\nu}$$

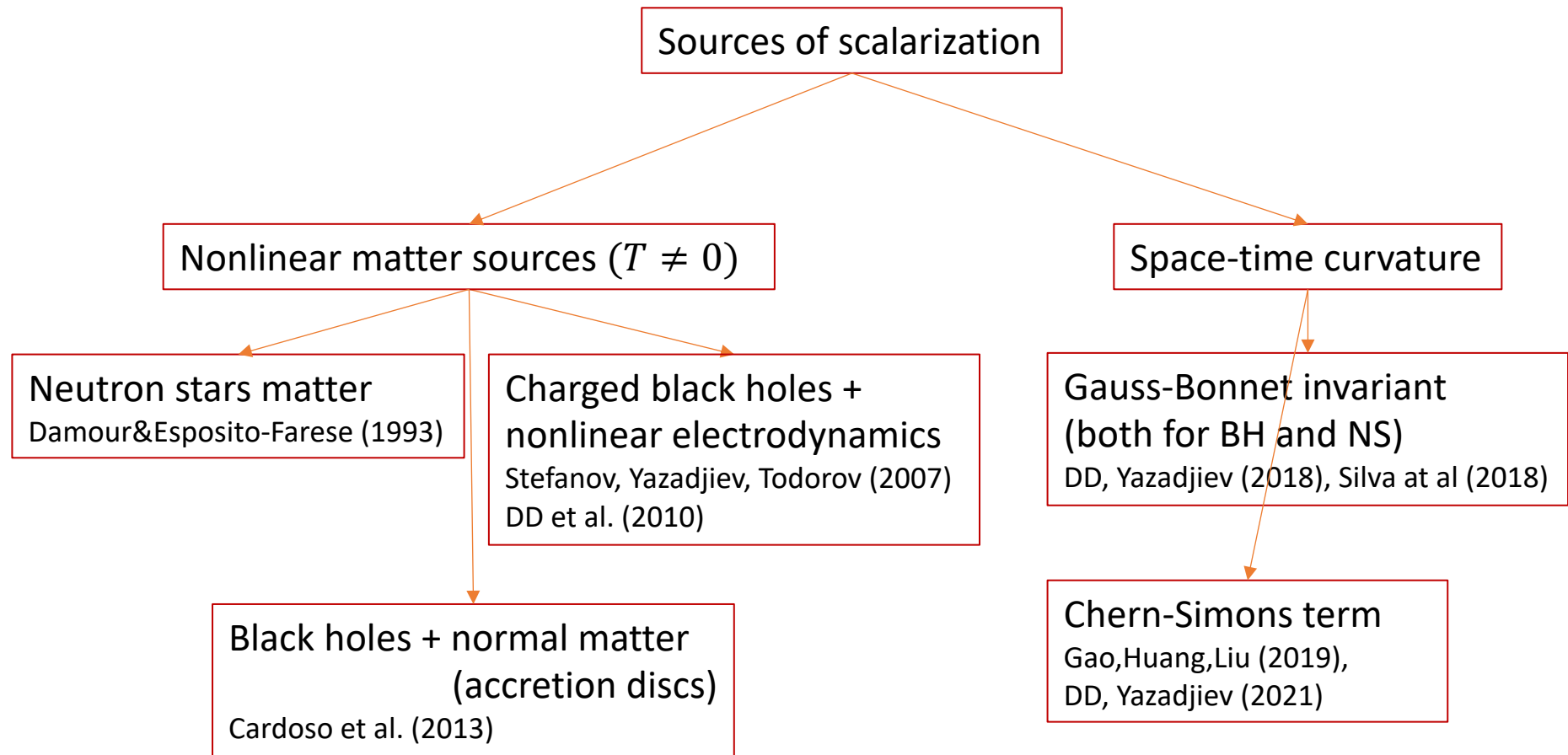
$$\square\varphi = -4\pi G_* \alpha(\varphi) T + \frac{dV(\varphi)}{d\varphi}$$

- **Spontaneous scalarization** occurs if the pure **GR** solution is **unstable against scalar perturbations** $\delta\varphi$

$$(\square - \mu_{\text{eff}}^2)\delta\varphi = 0, \text{ where } \mu_{\text{eff}}^2 = \left. \frac{d\alpha}{d\varphi} \right|_{\varphi=0} 4\pi G_* T$$

- If $\mu_{\text{eff}}^2 < 0$ a **tachyonic instability** is present leading to a development of the scalar field
- If φ grows the **nonlinear terms** start operating and instability **settles to a stable scalarized compact object**

Scalarization – general idea



Neutron star scalarization

Scalarized neutron stars – DEF model

- **Scalarization of neutron stars** Damour&Esposito-Farese (1993) due to a **nonzero trace** of the energy momentum tensor. **Energetically more favorable** over the GR solutions.

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- Assume the following form of the **coupling function**:

$$\alpha(\varphi) = \frac{d \ln(A(\varphi))}{d\varphi} = \alpha_0 + \beta_0 \varphi$$

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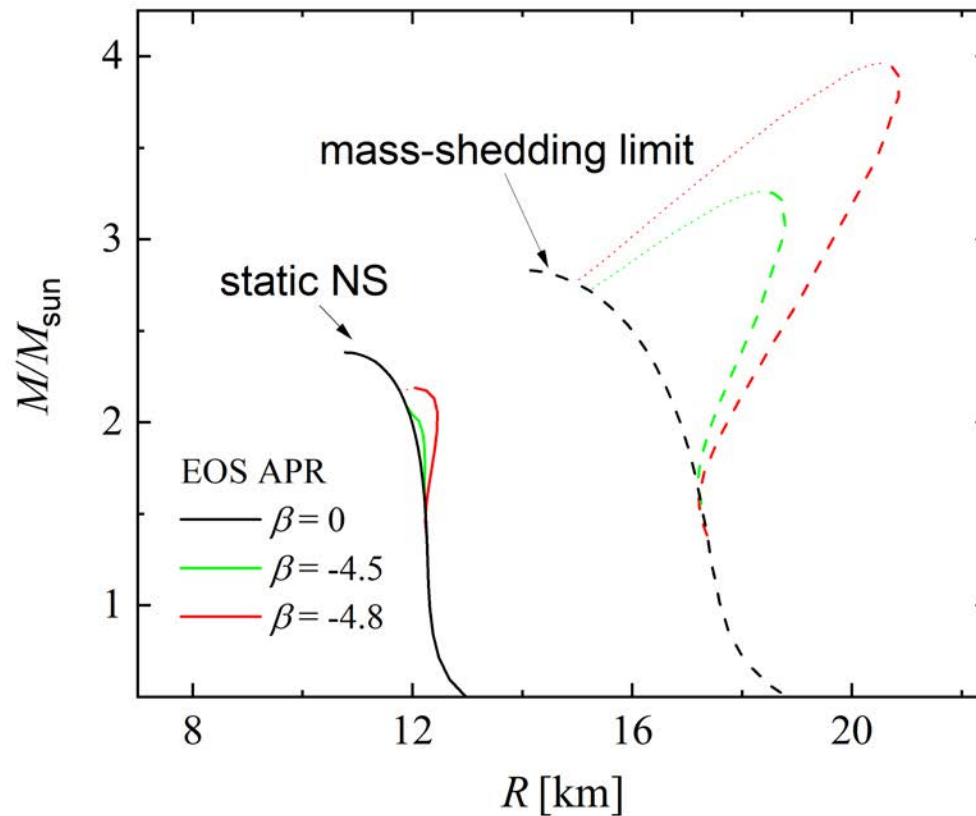
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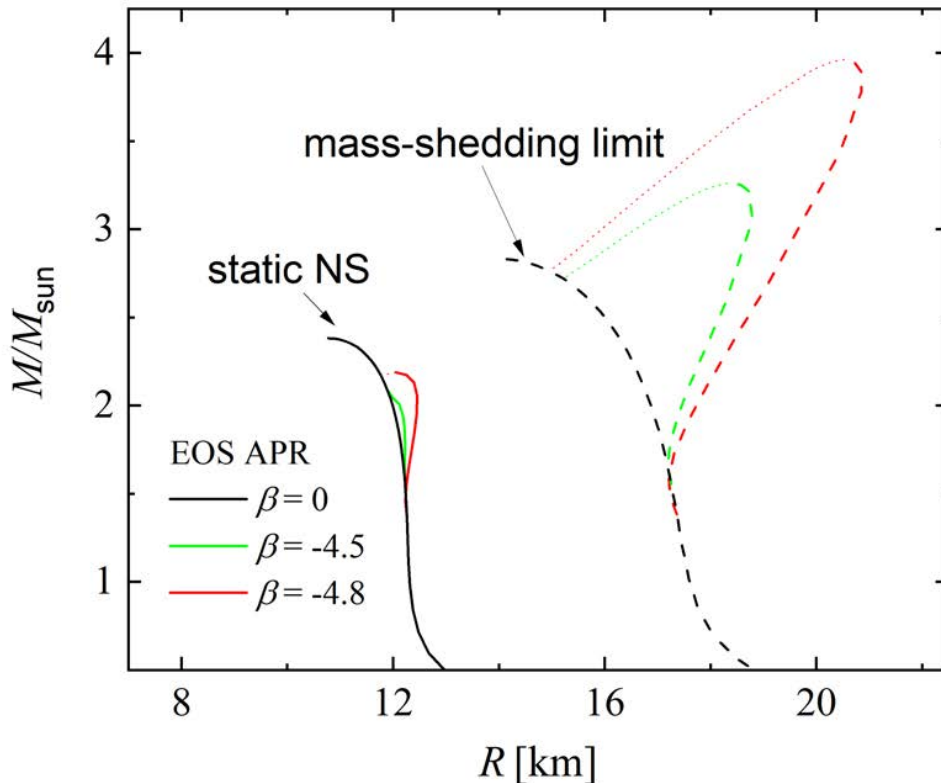
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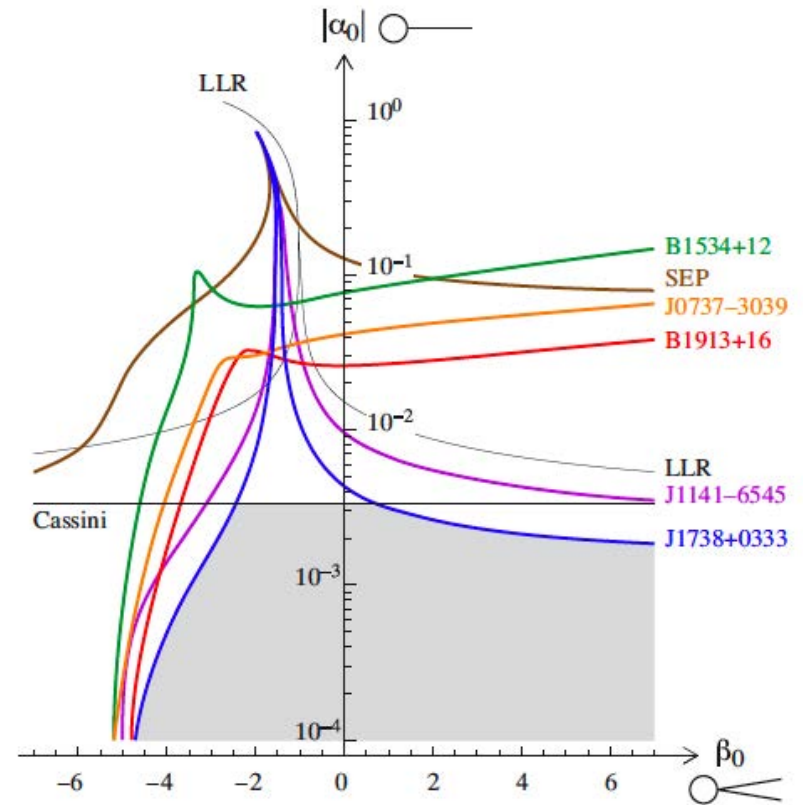
Neutron star scalarization

Scalarized neutron stars – DEF model

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- For $r \rightarrow \infty, \varphi \sim \frac{D}{r}$, where D is the scalar charge. If $|D| > 0$ – **scalar dipole radiation**



DD, Yazadjiev, Stergioulas, Kokkotas (2013,2014)



Freire et al (2012)

Scalar field potential

$$\square\varphi = -4\pi G_* \alpha(\varphi) T + \frac{dV(\varphi)}{d\varphi}$$

$$V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \lambda \varphi^4$$

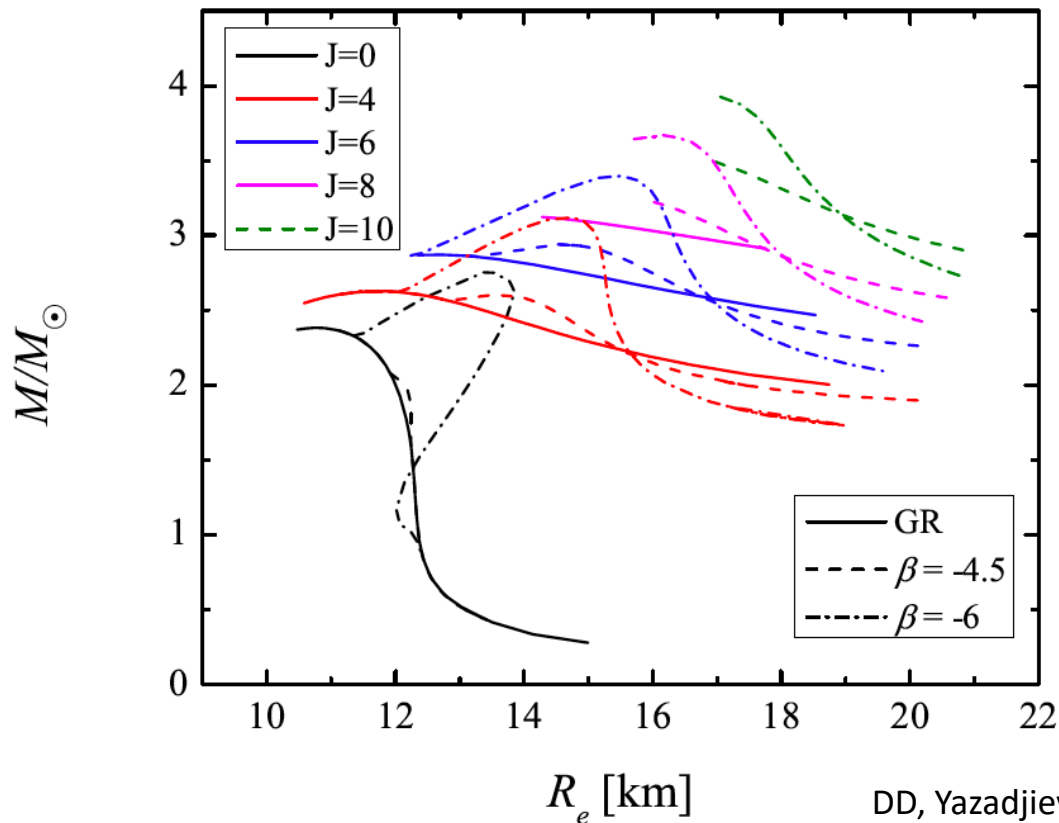
Scalar field mass

Self-interaction term

- Introduces an **effective range of the scalar field** connected to its **Compton wavelength** $\lambda_\varphi = \frac{2\pi}{m_\varphi}$. For $r \gg \lambda_\varphi$ the scalar field drops exponentially.
- For $m_\varphi \gg 10^{-16}$ eV practically **not constraints on β_0** can be imposed on the basis of the binary pulsar observations Ramazanoglu,Pretorius(2016), Yazadjiev,DD(2016), Rosca-Mead et al. (2020)
- The scalar field mass suppressed the scalar field, but larger β_0 are allowed. Thus, very large deviations from GR can be achieved Yazadjiev,DD(2016), Rosca-Mead et al. (2020)

Differential rotation

- ***j*-constant differential rotation** law $u_t u^\phi = F(\Omega) = A_{\text{diff}}^2 (\Omega_c - \Omega)$ with $A_{\text{diff}} = 1.225$
- **Higher *J*** reached for the scalarized NS
- The **maximum mass** of the possible supramassive configurations **increases** significantly.



DD, Yazadjiev, Stergioulas, Kokkotas (2018)

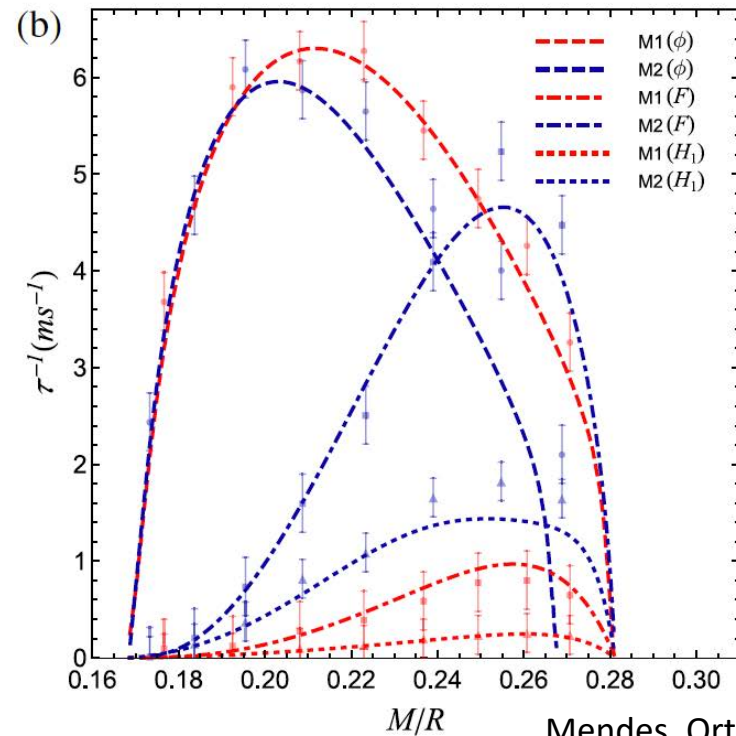
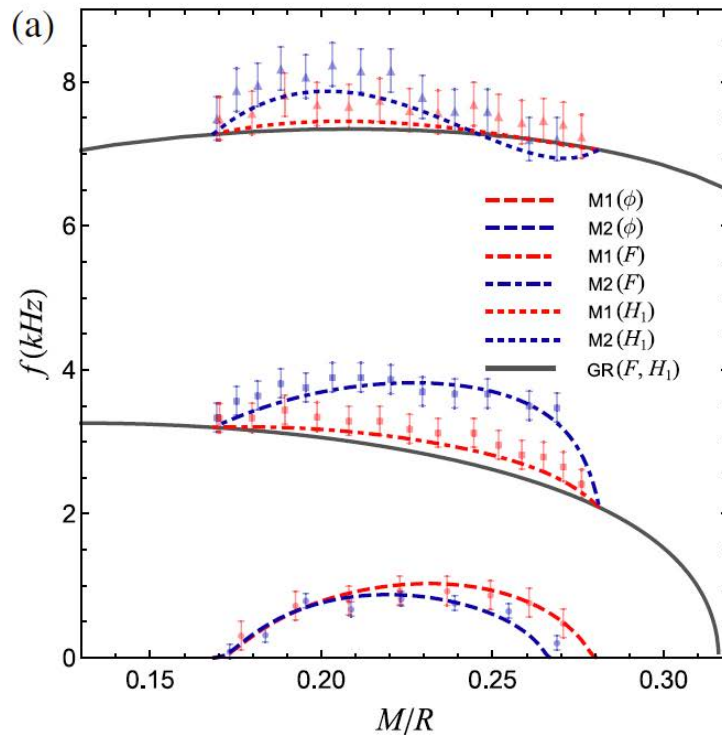
Dynamics of scalarized neutron star

Radial perturbations

- The scalar field **radiates energy** and thus the **oscillations are damped**
- A **new class of scalar modes** appear Mendes, Ortiz (2018)

$$e^{3\lambda_{(0)}}(\tilde{\epsilon}_{(0)} + \tilde{p}_{(0)})\ddot{\xi} - \left(\frac{\Gamma_1 \tilde{P}_{(0)}}{a_{(0)}^4 r^2} e^{\lambda_{(0)} + 3\nu_{(0)}} (e^{-\nu_{(0)}} a_{(0)}^4 r^2 \xi)' \right)' + A_\xi \xi + A_{\delta\phi} \delta\phi + A_{\delta\phi'} \delta\phi' = 0,$$

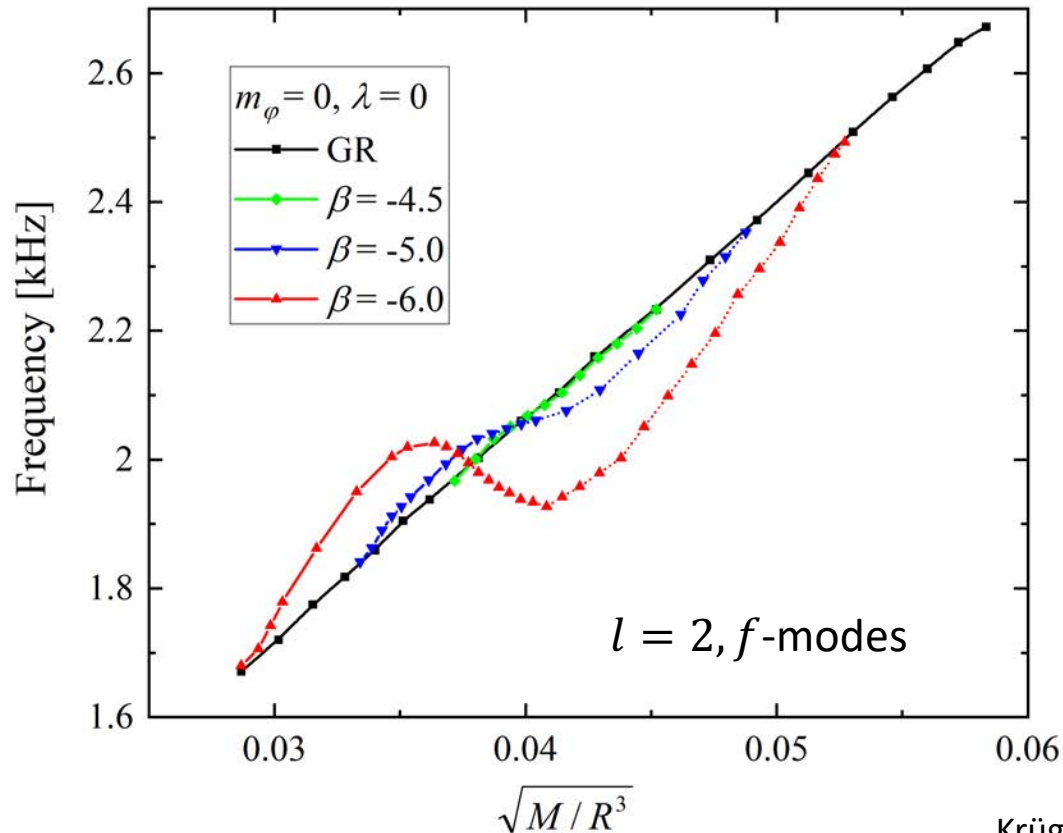
$$e^{2\lambda_{(0)} - 2\nu_{(0)}} \delta\ddot{\phi} - \delta\phi'' + B_{\delta\phi'} \delta\phi' + B_{\delta\phi} \delta\phi + B_{\xi'} \xi' + B_\xi \xi = 0$$



Mendes, Ortiz (2018)

Dynamics of scalarized neutron star

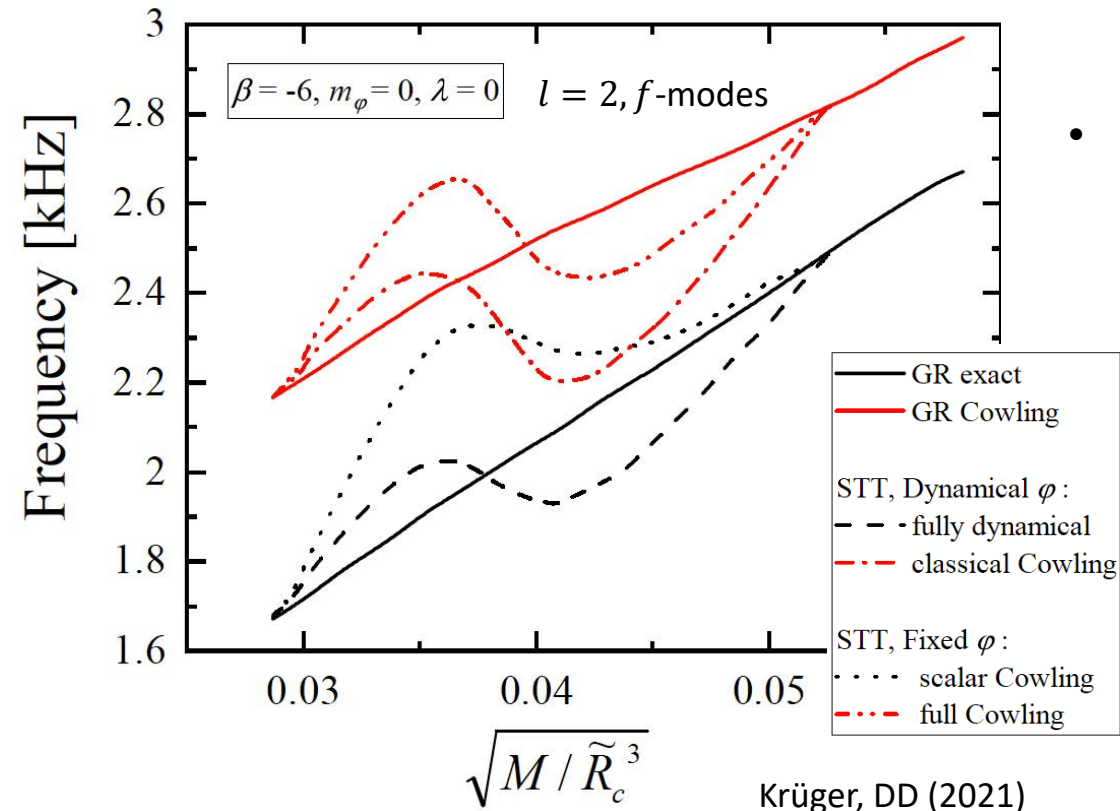
- **Nonradial stellar oscillations** ($l = 2, f$ -modes) Krüger, DD (2021).
- Time evolution of the coupled system of equations for the metric, fluid and the scalar field perturbations – allows to handle the boundary conditions easier.



Krüger, DD (2021)

Dynamics of scalarized neutron star

- **Accuracy of the Cowling approximation** for scalarized neutron stars (Sotani, Kokkotas (2004), Sotani (2014))
 - ✓ **full Cowling** – fixed **spacetime & scalar field**
 - ✓ **classical Cowling** – fixed **spacetime**
 - ✓ **scalar Cowling** – fixed **scalar field**



- **Hypothesis** – the Cowling approximation will give **good results** also in other **more complicated alternative theories** of gravity

Neutron star scalarization possible also in **other alternative theories** of gravity:

- **Gauss-Bonnet** gravity DD,Yazadjiev (2018)
- **Chern-Simons** gravity (not yet constructed)
- Other types of **Horndeski** theories Andreou et al. (2019)
- Tensor-**multi-scalar** theories DD,Yazadjiev (2020)

Scalarized black holes

Gauss-Bonnet theory - basics

- **Quadratic gravity** – the action is supplemented with all possible curvature invariants of second order. Motivated by the **attempts to quantize gravity**.

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left[R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) \right. \\ \left. + f_1(\varphi) R^2 + f_2(\varphi) R_{\mu\nu} R^{\mu\nu} + f_3(\varphi) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + f_4(\varphi) {}^*RR \right] + S_{\text{matter}}[\chi, \gamma(\varphi)g_{\mu\nu}]$$

- **Drawback** – in its general form it leads to field equations that are of order higher than two and ghosts can appear.

Gauss-Bonnet theory - basics

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- **Gauss-Bonnet gravity** – the equations are of second order

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \right].$$

Gauss-Bonnet invariant:

$$\mathcal{R}_{GB}^2 = R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$$

Quadratic theory

- **Quadratic gravity** – the action is supplemented with all possible curvature invariants of second order. Motivated by the **attempts to quantize gravity**.

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- **(dynamical) Chern-Simons gravity** – deviated from GR only in the presence of a parity-odd source such as rotation

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \varphi \nabla^\mu \varphi + 8\lambda^2 f(\varphi)^* \mathcal{R} \mathcal{R} \right].$$

Pontryagin invariant.

Black hole scalarization in Gauss-Bonnet gravity

- **Field equations** – Gauss-Bonnet gravity:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_{\mu}\varphi\nabla_{\nu}\varphi - g_{\mu\nu}\nabla_{\alpha}\varphi\nabla^{\alpha}\varphi - \frac{1}{2}g_{\mu\nu}V(\varphi),$$

$$\nabla_{\alpha}\nabla^{\alpha}\varphi = \frac{1}{4}\frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}_{GB}^2,$$

- **Conditions for the existence** of scalarized solutions

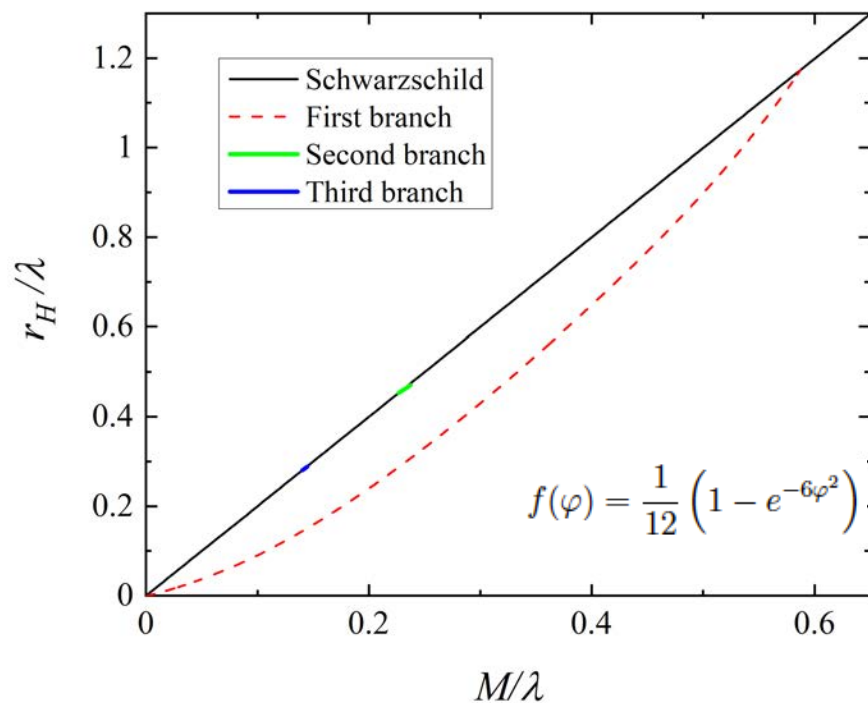
$$(\square - \mu_{\text{eff}}^2)\delta\varphi = 0 \text{ with } \mu_{\text{eff}}^2 = -\frac{\lambda^2}{4}\frac{d^2f}{d\varphi^2}(0)R_{GB}^2 < 0$$

$$\text{Scalarization possible for: } \frac{df}{d\varphi}(0) = 0, \frac{d^2f}{d\varphi^2}(0) > 0$$

- **Scalarization** in a similar way possible also in **Chern-Simons gravity**.

Black hole scalarization in Gauss-Bonnet gravity

- **Scalarized solutions** with different number of nodes exist that **bifurcate from the Schwarzschild solution** DD, Yazadjiev (2018), Silva et al. (2018), Antoniou, Bakopoulos, Kanti (2018) and the first $n = 0$ branch can be stable Blazquez-Salcedo, DD, Kunz, Yazadjiev (2018), Blazquez-Salcedo, DD, Kahlen, Kunz, Yazadjiev (2019,2020)
- **Energetically more favorable** over the Schwarzschild black hole.
- For low black hole masses - **loss of hyperbolicity**.



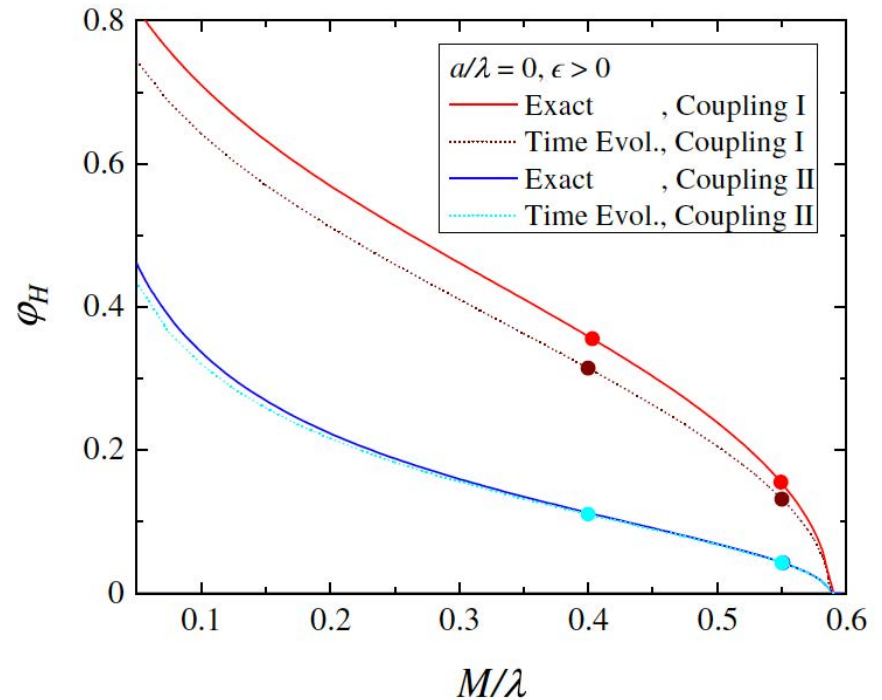
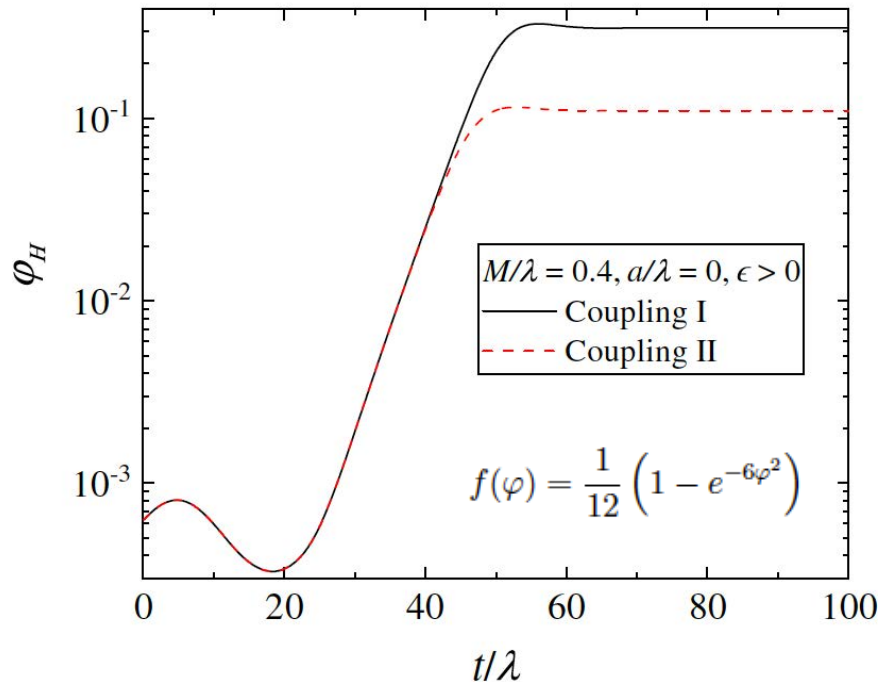
DD, Yazadjiev (2018)

- The stability is dependent on the exact form of the **coupling function** and possibly the scalar field **potential**. Minamitsuji, Ikeda (2019), Silva et al. (2019), Macedo et al. (2019), DD, Staykov, Yazadjiev (2019)
- **Rotation downsizes the phenomenological effects of scalarization** Cunha, Herdeiro, Radu (2019), Collodel et al (2019).
- **Non-GR effects** are only **significant for low spin** (BH shadow, QPO oscillations)

Decoupling limit

- Study only the **scalar field evolution** on a **fixed spacetime background**

$$\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} R_{GB}^2$$

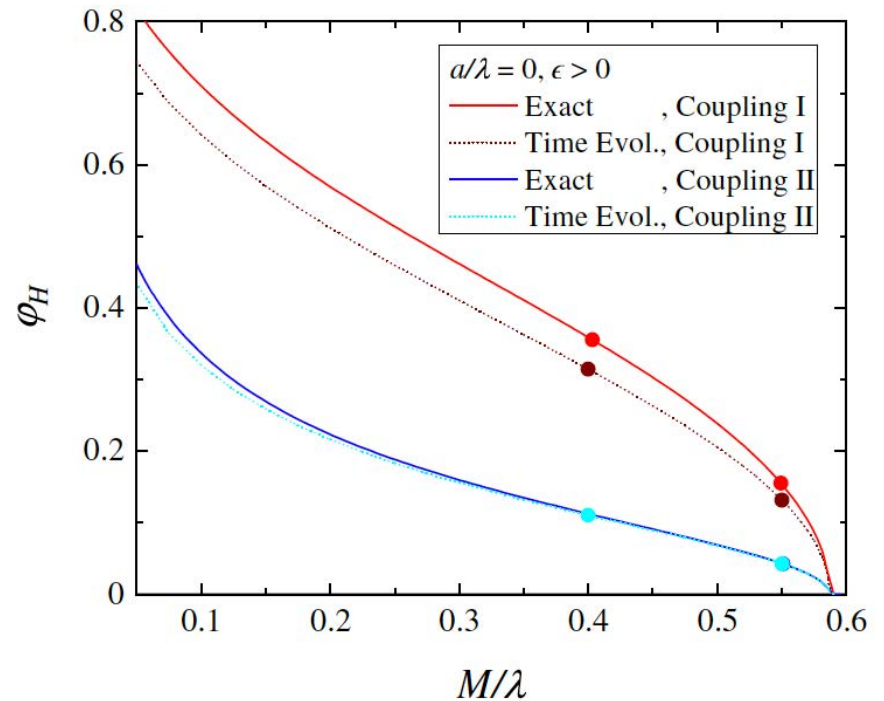
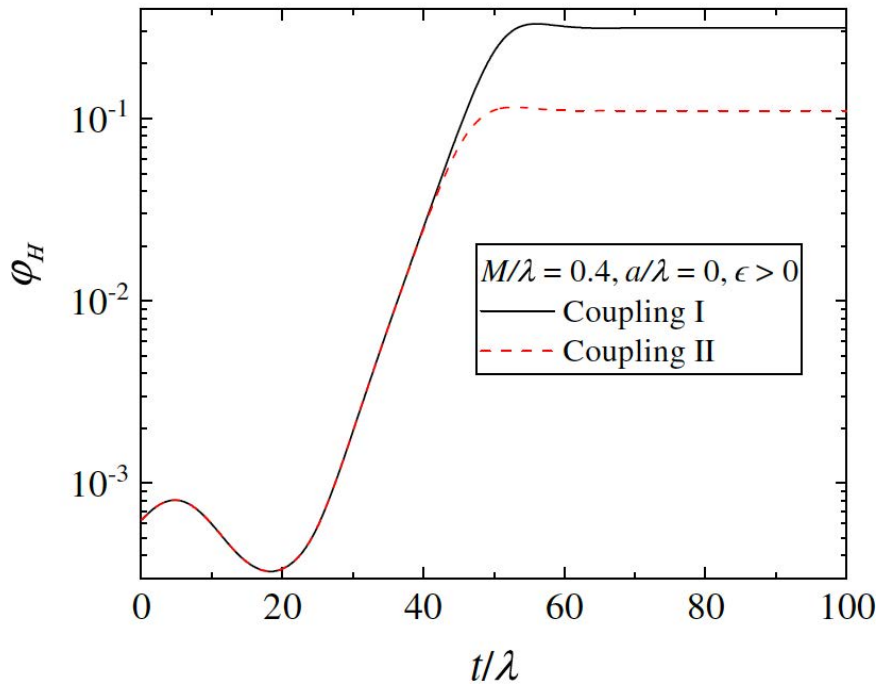


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DD, Yazadjiev (2021)

Decoupling limit – Rotating solutions

- Study only the **scalar field evolution** on a **fixed spacetime background**

$$\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} R_{GB}^2$$

- Scalar field equation on a Kerr background $ds^2 = -\frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 - 2a \sin^2 \theta \frac{r^2 + a^2 - \Delta}{\Sigma} dt d\phi$
 $+ \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2$

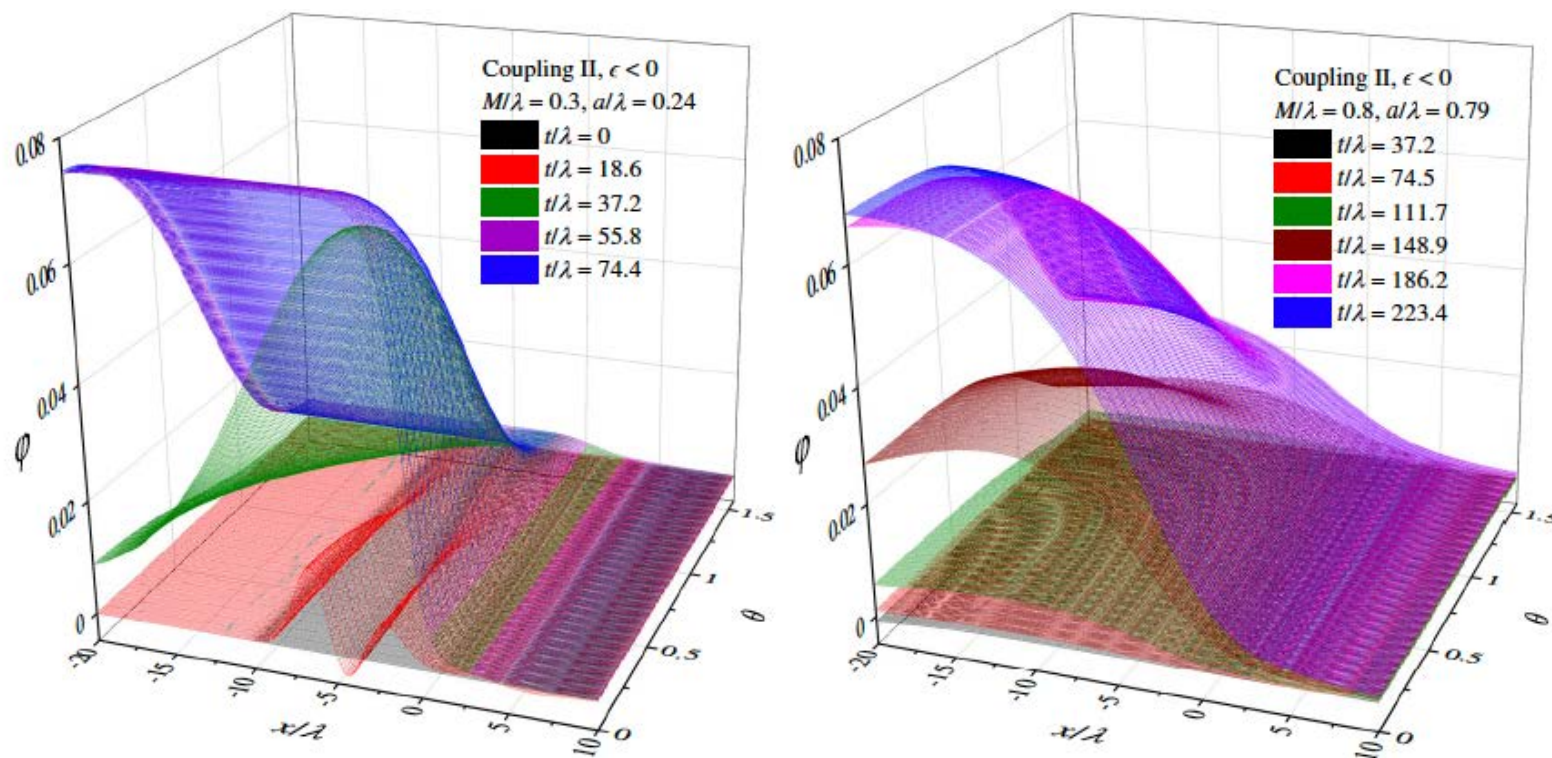
$$\begin{aligned} & - [(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta] \partial_t^2 \varphi + (r^2 + a^2)^2 \partial_x^2 \varphi + 2r\Delta \partial_x \delta \varphi - 4Mar \partial_t \partial_{\phi_*} \varphi \\ & + 2a(r^2 + a^2) \partial_x \partial_{\phi_*} \varphi + \Delta \left[\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \varphi) + \frac{1}{\sin^2 \theta} \partial_{\phi_*}^2 \varphi \right] \\ & = -\lambda^2 \frac{12M^2 \Delta}{\Sigma^5} (r^2 - a^2 \cos^2 \theta) (r^4 - 14a^2 r^2 \cos^2 \theta + a^4 \cos^4 \theta) \frac{df(\varphi)}{d\varphi}. \end{aligned}$$

$$\frac{1}{4} R_{GB}^2$$

Decoupling limit – Rotating solutions

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$$\nabla_{\alpha} \nabla^{\alpha} \varphi = -\frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} R_{GB}^2$$



DD, Yazadjiev (2021)

Dynamical Chern-Simons gravity (decoupling limit)

- Only the rotating black holes differ from the Kerr solutions

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \varphi \nabla^\mu \varphi + 8\lambda^2 f(\varphi) {}^*RR \right]$$

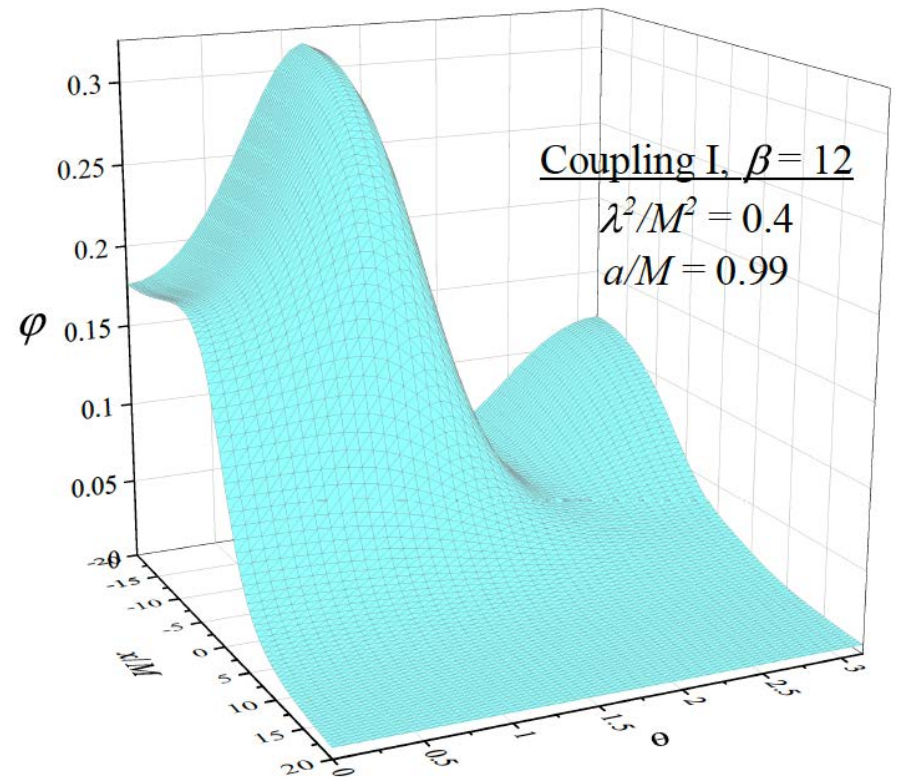
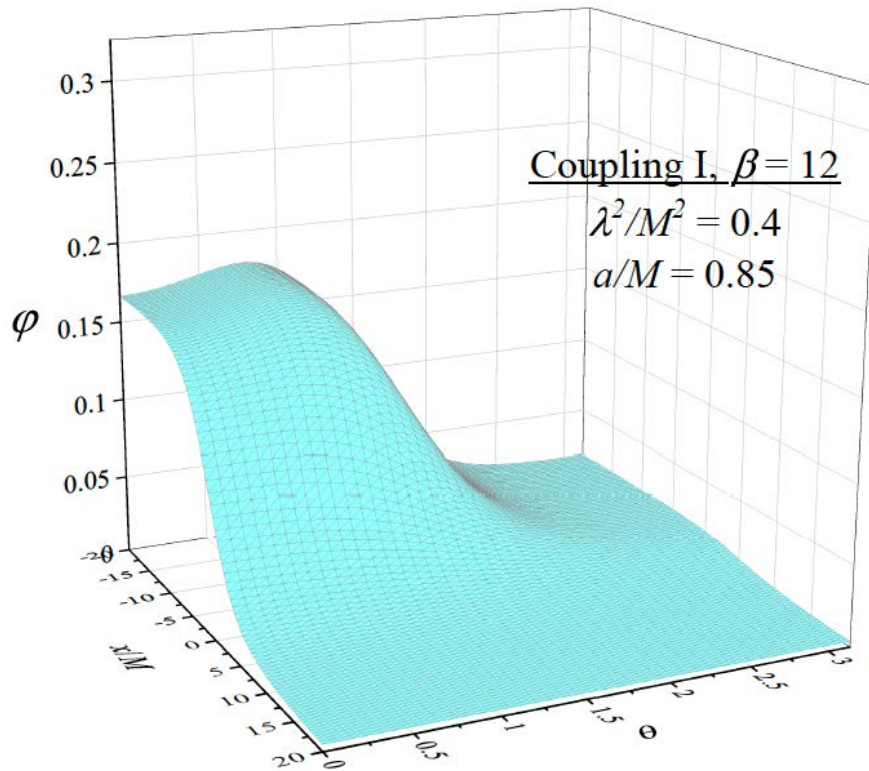
$$\nabla_\alpha \nabla^\alpha \varphi = -2\lambda^2 \frac{df(\varphi)}{d\varphi} {}^*RR$$

$$\begin{aligned} & - \left[(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta \right] \partial_t^2 \varphi + (r^2 + a^2)^2 \partial_x^2 \varphi + 2r\Delta \partial_x \varphi - 4Mar \partial_t \partial_{\phi_*} \varphi \\ & + 2a(r^2 + a^2) \partial_x \partial_{\phi_*} \varphi + \Delta \left[\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \varphi) + \frac{1}{\sin^2 \theta} \partial_{\phi_*}^2 \varphi \right] \\ & = -\lambda^2 \frac{192aM^2\Delta}{\Sigma^5} r \cos \theta (3r^2 - a^2 \cos^2 \theta) (r^2 - 3a^2 \cos^2 \theta) \frac{df(\varphi)}{d\varphi}. \end{aligned}$$

$2 {}^*RR$ Pontryagin invariant.

Dynamical Chern-Simons gravity (decoupling limit)

- Scalar field profiles of the scalarized black holes.



DD, Yazadjiev (2021)

Conclusions and future perspectives

- Scalarization is a very interesting nonlinear effect allowing for large deviations from GR while keeping the weak field regime unaltered.
- Can be sourced by the curvature of the spacetime itself, matter, exotic fields, nonlinear electrodynamics, etc.
- Interesting observational signatures are expected that can help us further constrain the strong field regime of gravity.

Future perspectives:

- ❖ Dynamics of scalarized BH and NS should be further studied.
 - ❖ Construction of scalarized BH and NS in other classes of alternative theories
 - ❖ Spontaneous vectorization and tensorization
 - ❖ Further understanding of the problems appearing for certain theories allowing scalarization and determining whether they are viable and how to overcome the problems.
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THANK YOU!
