Scattering Amplitudes in Particle Physics

University of Southampton

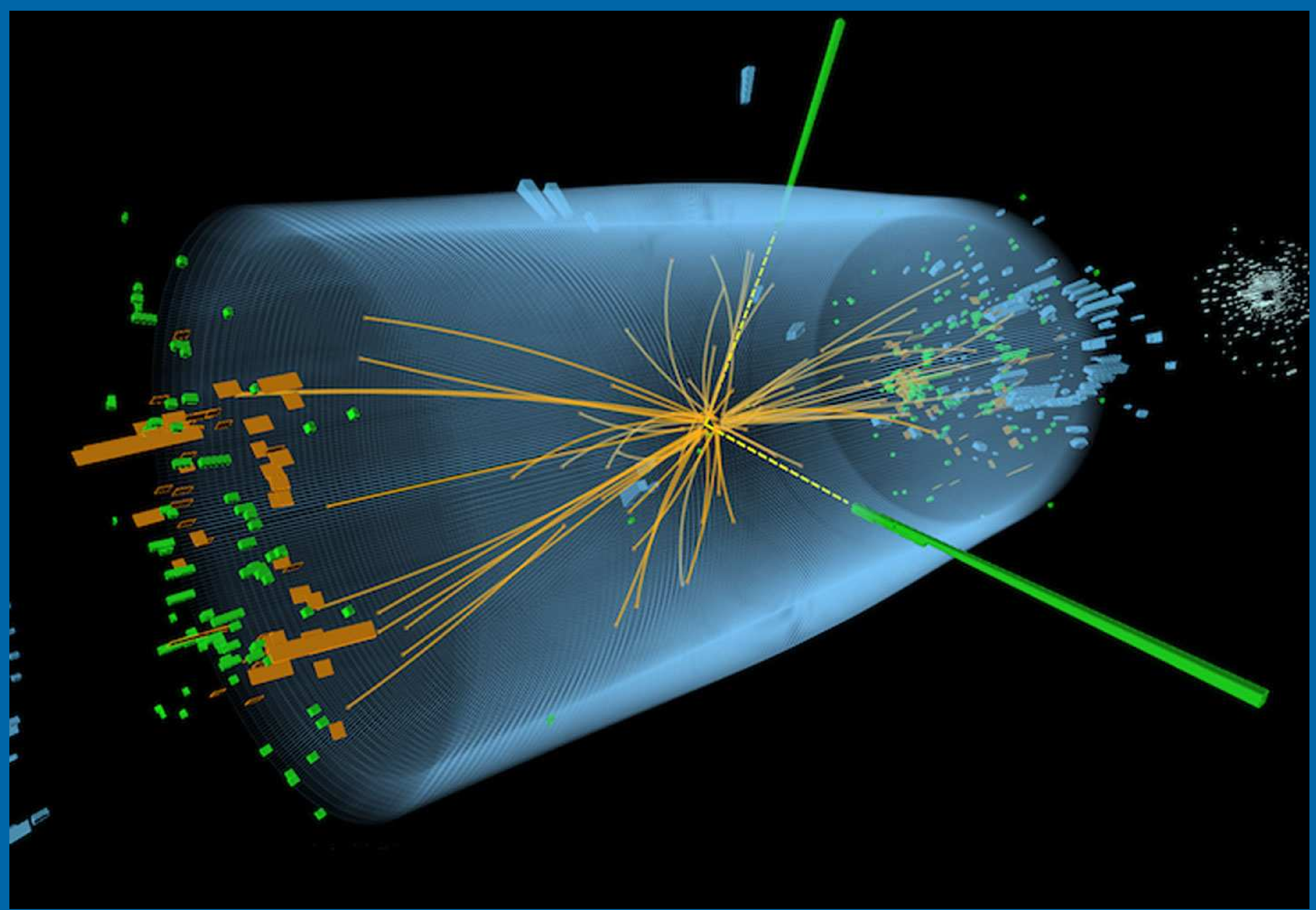
1) What is Particle Physics?

Particle physics is the study of the subatomic particles that make up the universe and how the interact. Theoretically this is done through Quantum Field Theory (QFT) which combines the topics of classical fields (i.e. electromagnetism), quantum mechanics (the very small), and special relativity (the very fast). Therefore it is the natural mathematical framework to study subatomic particles at high energies, such as those found at the Large Hadron Collider (LHC). It is also has uses in many other areas, nuclear physics, atomic physics, condensed matter physics, astrophysics, and cosmology. With its versatile applicability, understanding QFT is vital for understanding the nature of the universe. QFT has also opened up many new bridges between physics and mathematics.

2) What are Scattering Amplitudes?

When studying particle physics, the natural object to calculate is called a **scattering amplitude**. This is not a physical object but a purely mathematical one which can be used to calculate the cross-section of a particle process, this is the area in which the particles must meet in order to scatter/interact and is proportional to the probability that an interaction will occur. Cross-sections can be measured at particle accelerators, such as the LHC.

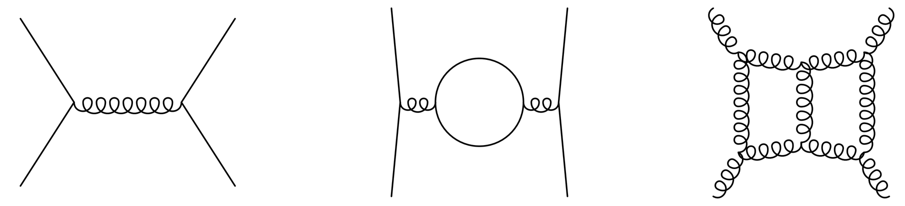
In the '60's a lot of work went into understanding the mathematical properties of amplitudes but the standard approaches quickly became intractable, even for the simplest theories. However, this didn't deter physicists and some of the work I do is in understanding the mathematical structures hidden within amplitudes.



A candidate Higgs Boson event from CMS. (Image: CERN/CMS)

3) How do we calculate Scattering Amplitudes?

The typical method for calculating scattering amplitudes, found in any QFT textbook, is through Feynman diagrams. These diagrams are very convenient pictorial representations of the mathematical expressions which describe particle interactions.



These diagrams are a great innovative tool for calculating Feynman diagrams but they do have their limitations. Increasing the number of external particles or the number of loops in a diagram quickly makes their calculation intractable. For example:

$gg \to gg$	4 diagrams
$gg \to ggg$	25 diagrams
$gg \to gggg$	220 diagrams

for $gg \to 8g$ there are approximately **a million** diagrams - and that's just at tree-level! Loops would be much, much worse...

Work done by Parke and Taylor in the '80's managed to reduce the 220 diagrams for the $2 \to 4$ gluon process to a single expression

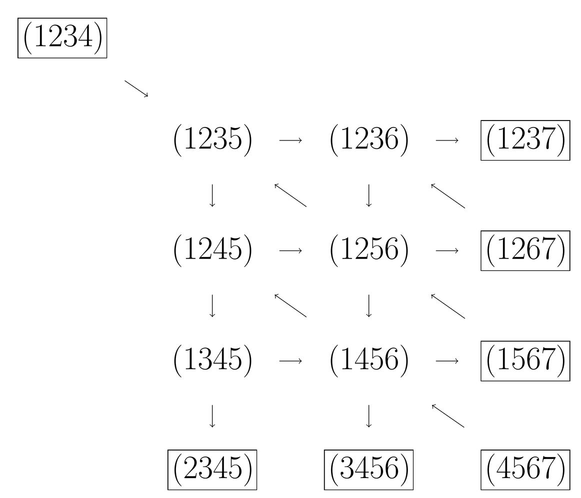
$$\mathcal{M}_6 = \frac{\langle 12 \rangle^3}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

which can be generalised to n external particles - a remarkable result!

So Feynman diagrams have their use but the Parke-Taylor formula suggests there may be more efficient methods of calculating scattering amplitudes.

5) My Research

In my research I investigate new methods to calculate scattering amplitudes, using $\mathcal{N} = 4$ SYM and its various symmetries to improve the calculations. Various interesting structures have been discovered in SYM, one of which I'm researching is **cluster algebras**. These cluster algebras have very interesting mathematical properties and there has been lots of work over the past few years into how they relate to amplitudes. In my research with my collaborators we have been exploring how to build amplitudes out of the nodes of these cluster algebras.



The symmetries of $\mathcal{N} = 4$ SYM are how these cluster algebras are connected to amplitudes and as supersymmetry and conformal symmetry have not been shown to be apparent in the real world, we can't necessarily use them for the theories which describe the real world. However, in understanding these properties we may discover a new method or structure which could be applied to the real world.

4) The Perfect Theory

The QFT used to study the processes at the LHC is called Quantum Chromodynamics (QCD), which describes the interactions between quarks and gluons, the particles which make up protons and neutrons inside atoms. Unfortunately Feynman diagrams can only be used at very small scales in QCD, other methods have to be employed at larger scales. To try to better understand QCD and the mathematics of scattering amplitudes we use a toy-model - a theory which doesn't describe the real world but can be used to develop new methods to apply to real-world calculations.

$\mathcal{N} = 4$ Supersymmetric Yang-Mills

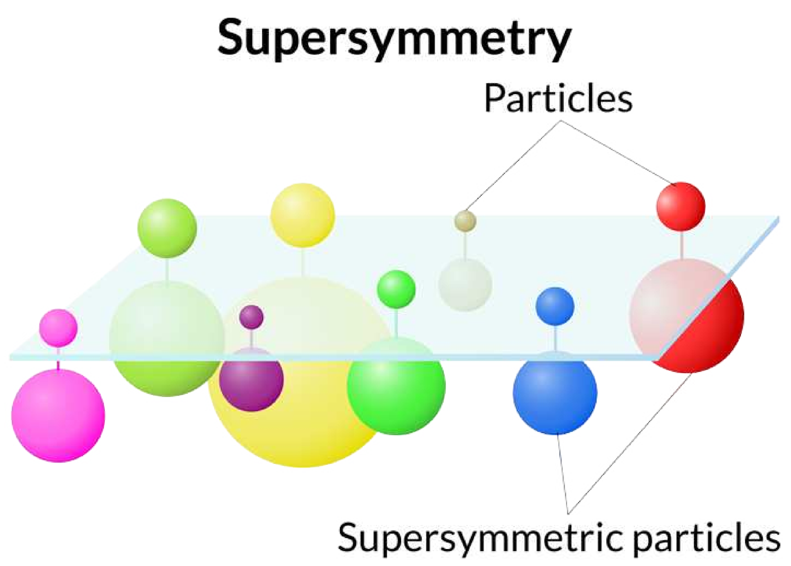
This theory (SYM) is the most symmetric QFT that doesn't include gravity and can be considered to be a supersymmetric generalisation of QCD, in fact, the tree-level amplitudes for the two theories coincide! The more symmetric a theory the more constrained it is - so the more symmetries the easier the calculations are!

Supersymmetry

This is a symmetry between types of particles:

force particles $\leftrightarrow$ matter particles

hence, if you build a supersymmetric QFT every particle in your theory obtains a supersymmetric partner of whichever type it isn't.



(Image : Science News)

Current experimental data has not yet shown any signs of supersymmetry in the real world but theoretical physicists still use it as a mathematical tool in calculations.

Conformal

This is another symmetry this theory posesses, one which means that the theory is the same at all scales therefore all particles are massless, making the calculations, once again, much easier.

These symmetries allow various modern techniques in amplitudes to be explored and developed in one theory, a "jack of all trades" theory, but a master of not-the-real-world, however it is useful for developing techniques to apply to the real world.

For more information on my research please have a look at my recent paper - **arXiv:1710.10953**.

University of Southampton Jack Foster j.a.foster@soton.ac.uk