UNIVERSITY OF SURREY

Self-consistent gaps and equation of states for neutron star applications

Ding, Rios, *et al.*, *Phys Rev C* **94** 025802 (2016)
Sellahewa & Rios, *in preparation*

Arnau Rios Huguet
Lecturer & STFC Advanced Fellow
Department of Physics
University of Surrey

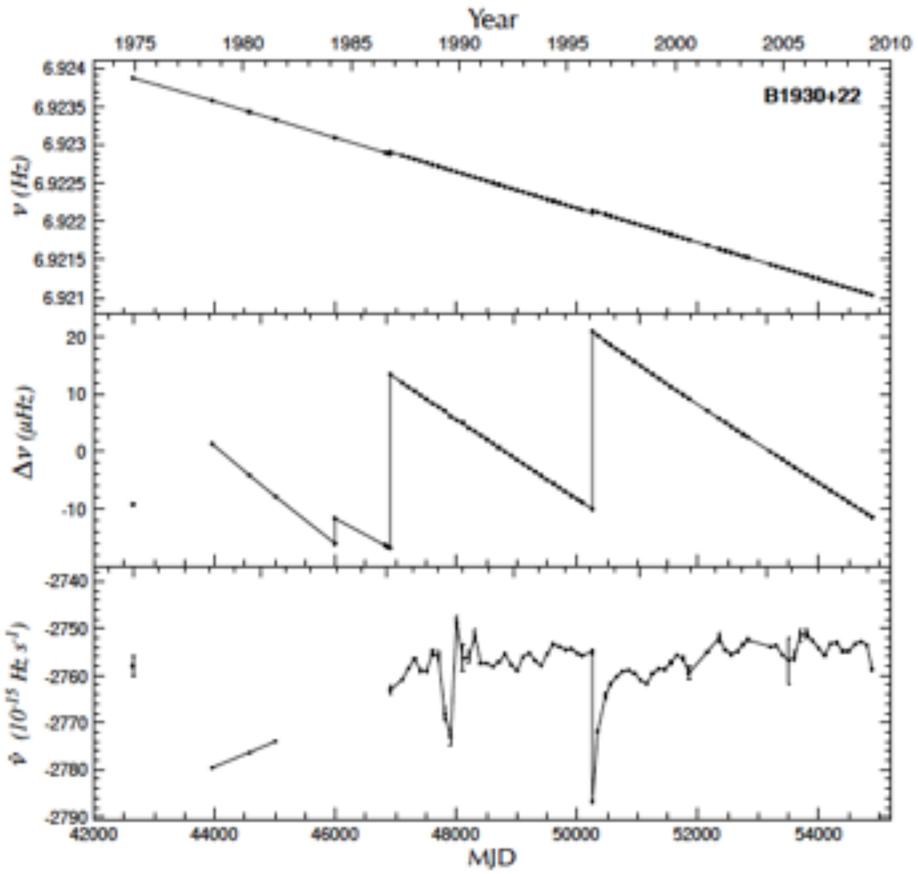
Southampton Workshop, 13 September 2016

Summary

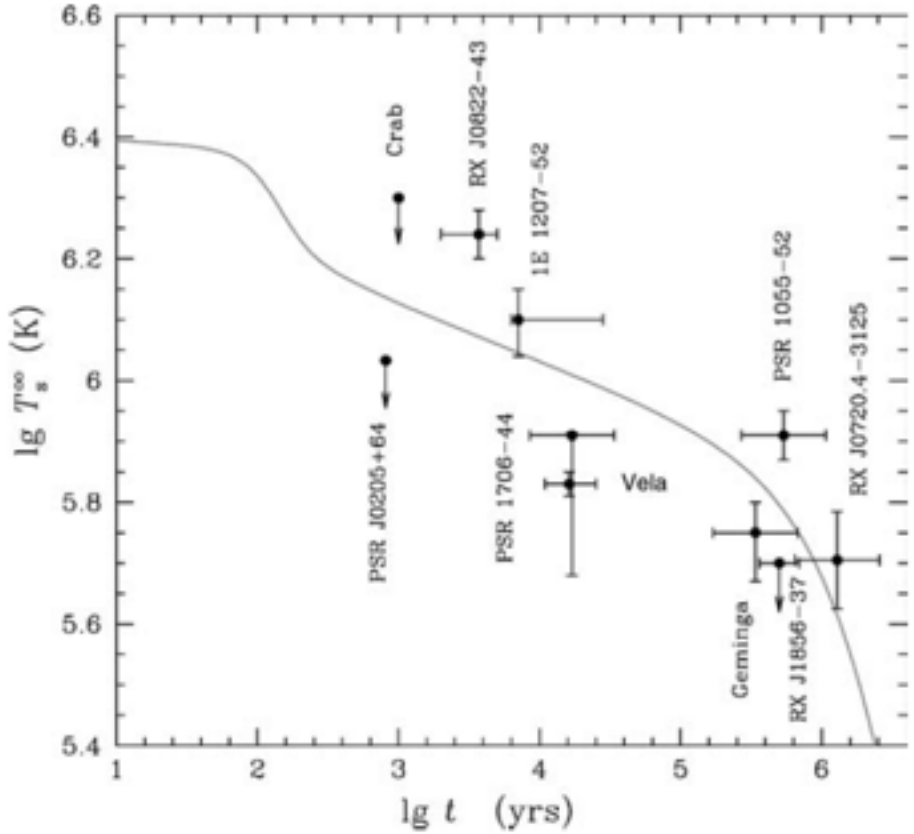
1. NS model with effective forces
2. NS model with realistic forces
3. Correlations and systematic errors

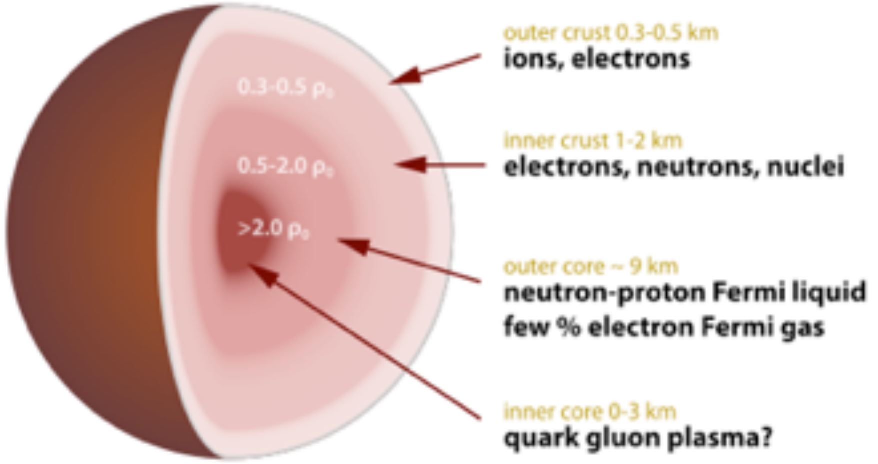
NS data!

Glitches



ν cooling





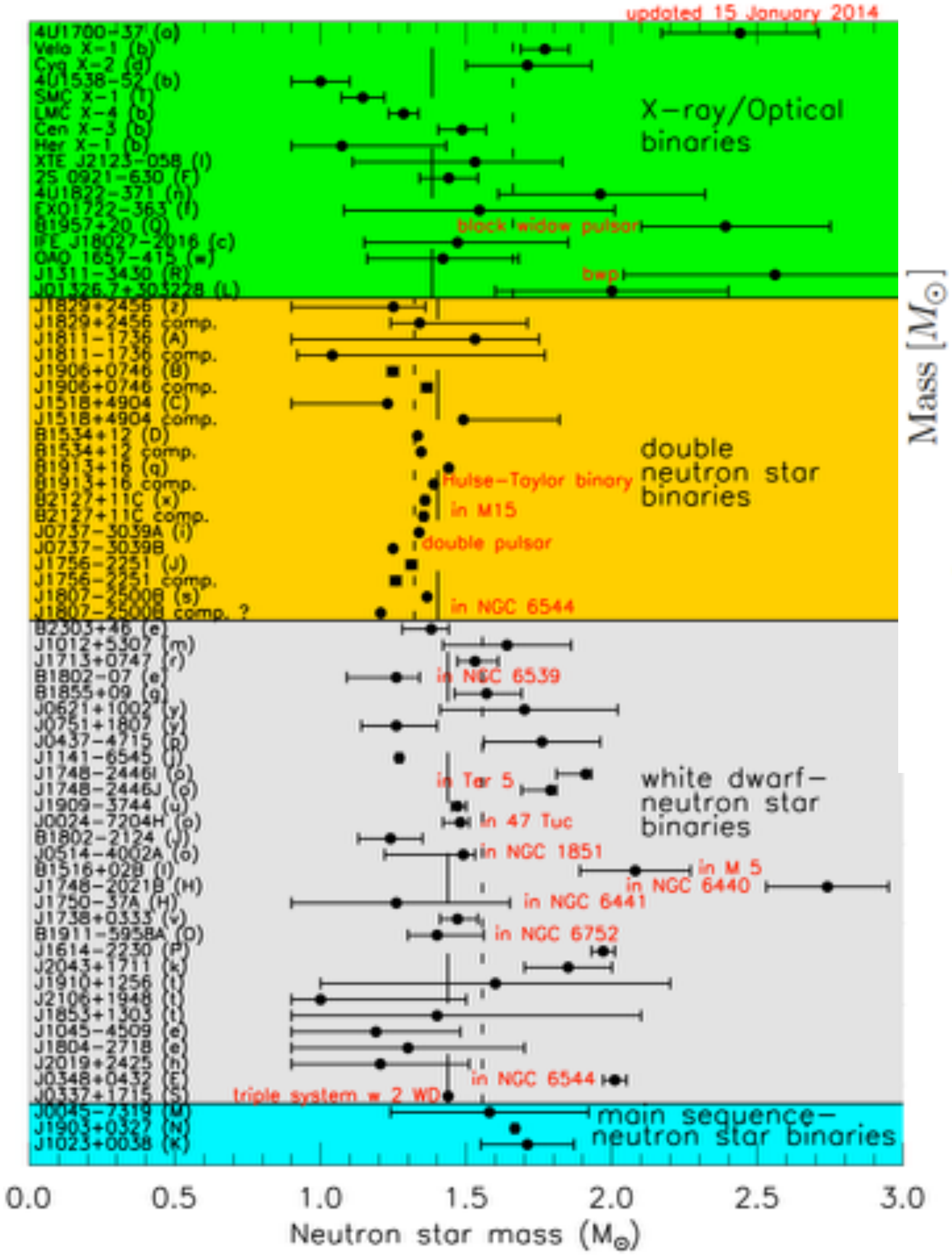
outer crust 0.3-0.5 km
ions, electrons

inner crust 1-2 km
electrons, neutrons, nuclei

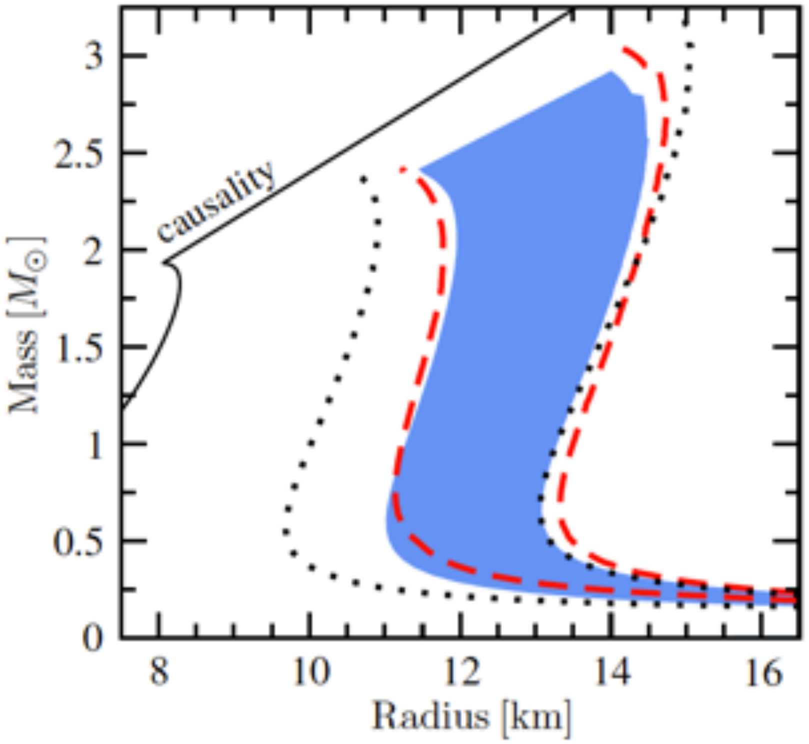
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas

inner core 0-3 km
quark gluon plasma?

Masses



Radii

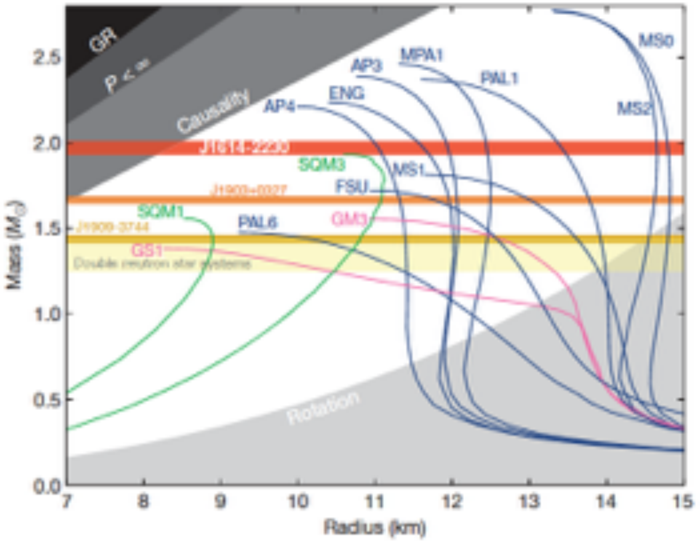


Hebeler, Lattimer, Pethick, Schwenk
ApJ **773** 11 (2013)

Neutron star modeling

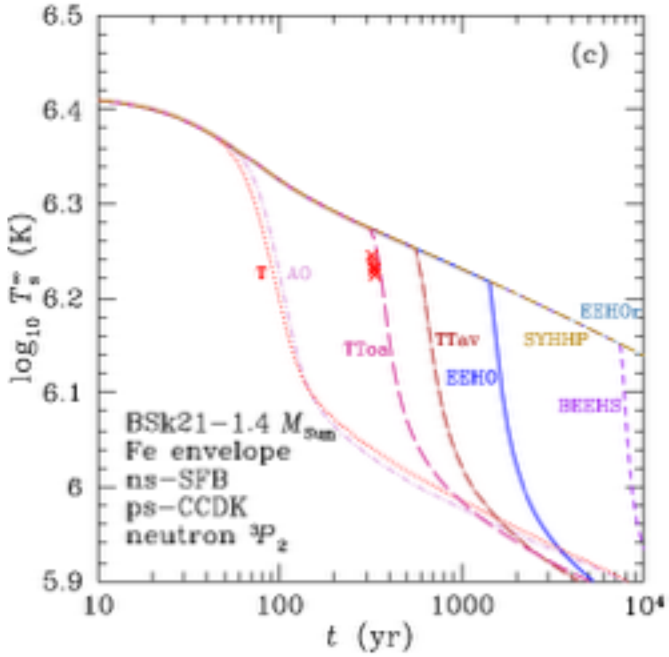
Input #1
EoS

Observable #1
Mass-Radius relation



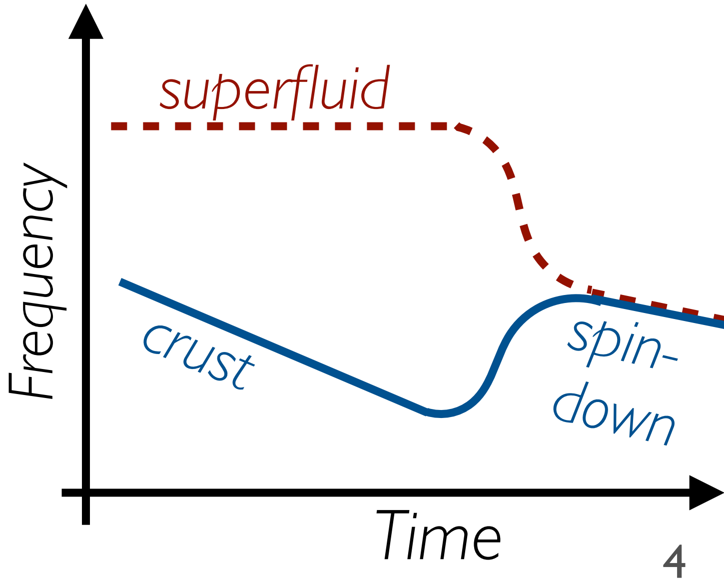
Input #2
Pairing gap

Observable #2
Cooling curve

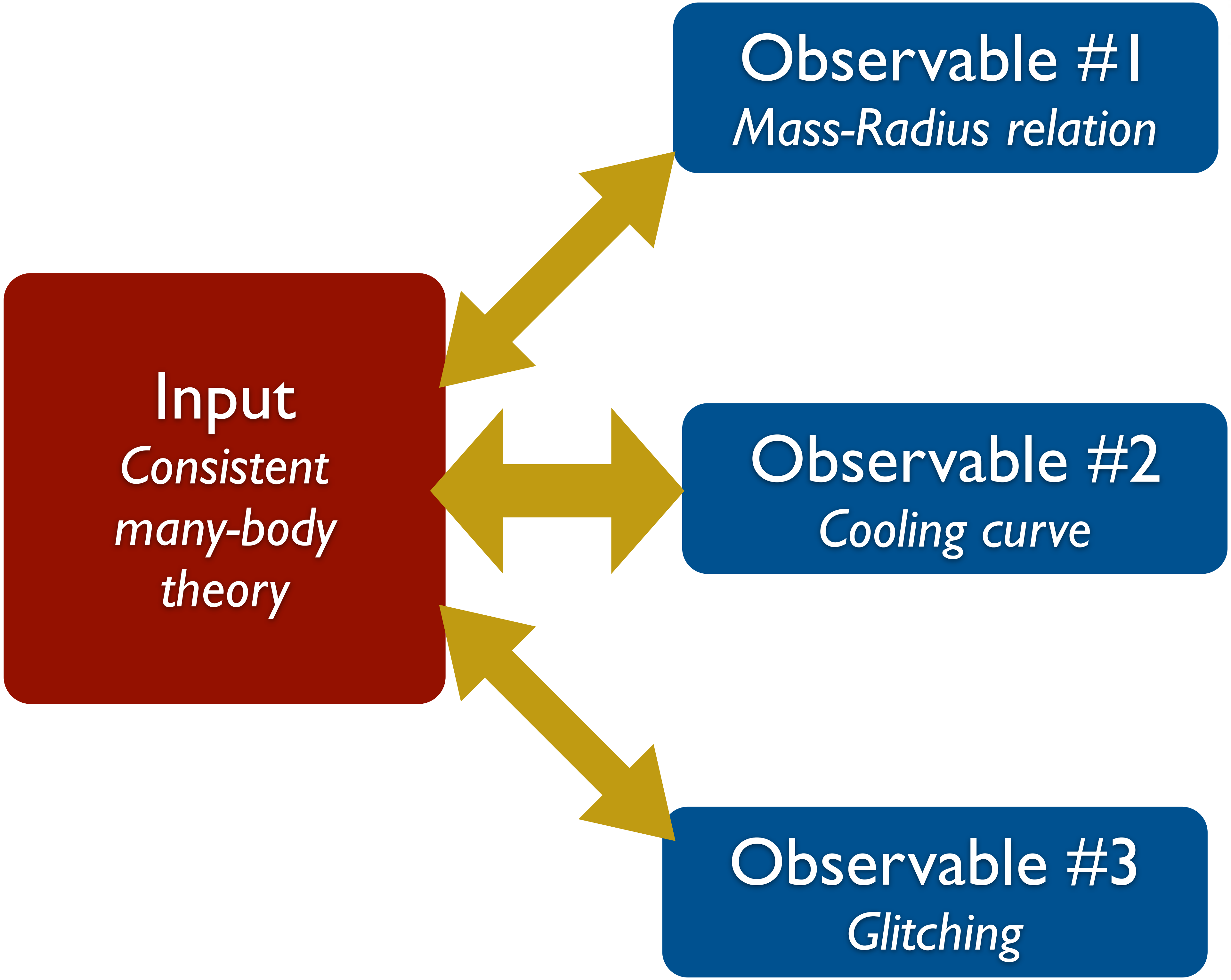


Input #3
Crust-core

Observable #3
Glitching

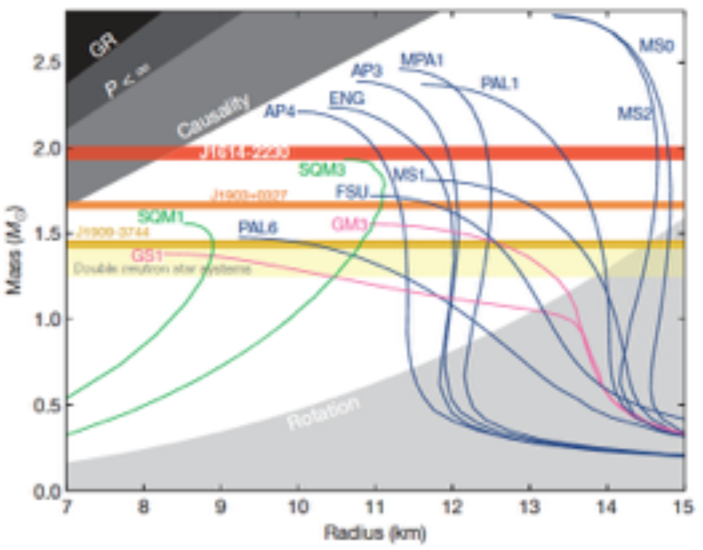


Neutron star modeling

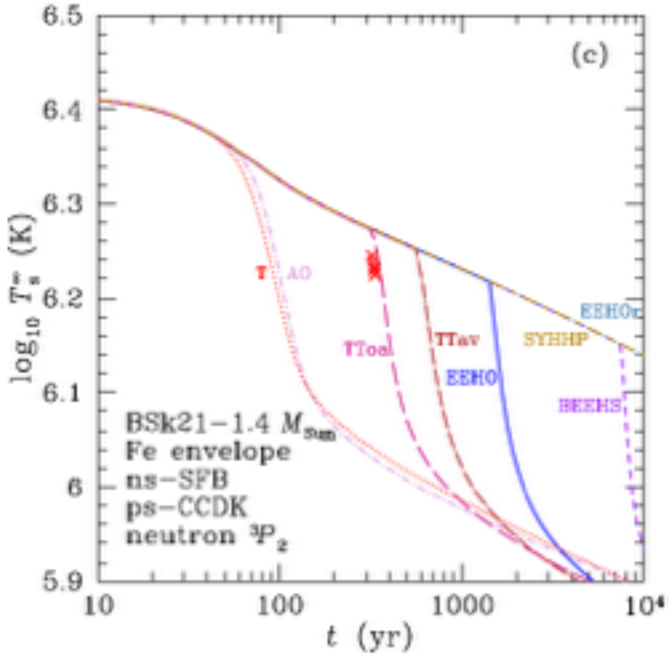


Input
Consistent many-body theory

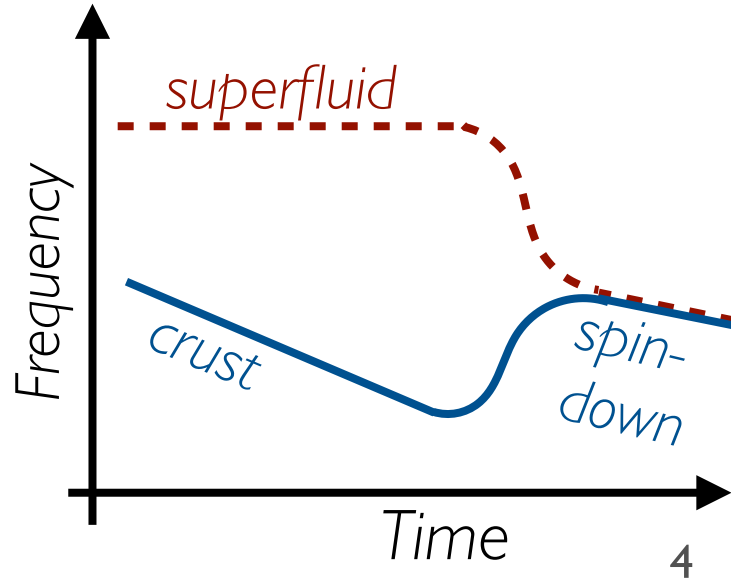
Observable #1
Mass-Radius relation



Observable #2
Cooling curve



Observable #3
Glitching



A step forward towards consistency

The Gogny force

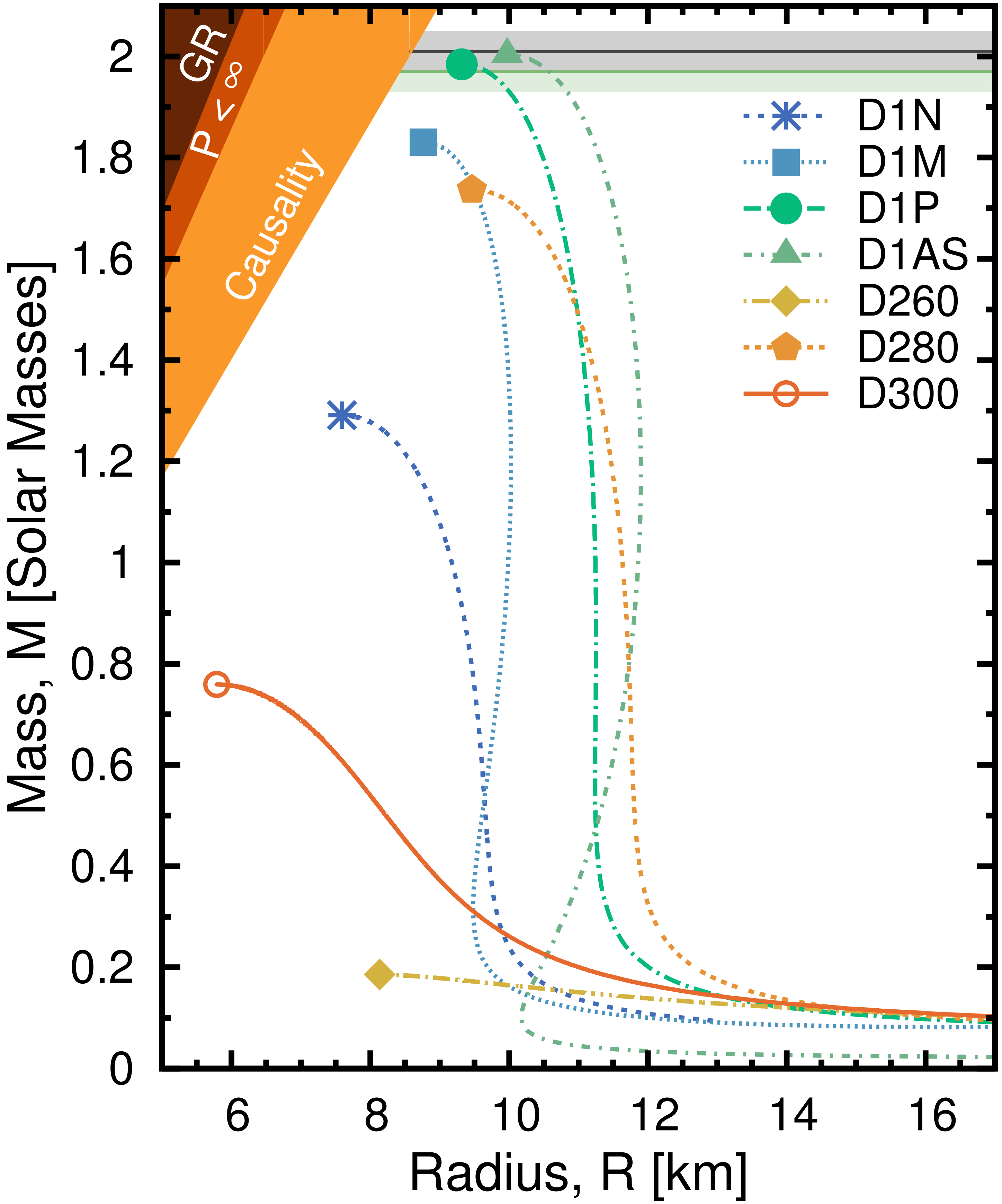
$$
\begin{aligned}
V(\vec{r}) = & \sum_{i=1,2} \Big(W_i + B_i\, P_\sigma - H_i\, P_\tau - M_i\, P_\sigma\, P_\tau \Big) e^{-\frac{r^2}{\mu_i}} \\
& + \sum_{i=1,2} t_0^i\, \left(1 + x_0^i\, P_\sigma\right) \rho^{\alpha_i}\, \delta(\vec{r}) \\
& + iW_0(\sigma_1 + \sigma_2)[\vec{k}' \times \delta(\vec{r})\vec{k}]
\end{aligned}
$$

- 11 parametrizations
- Popular in **fission** studies

- Any good for isospin?
- Only 7 give stable NSs!

- **Pairing**?

comp star
compstar-esf.org



R. Sellahewa & A. Rios, *PRC* **90**, 054327 (2014)

A step forward towards consistency

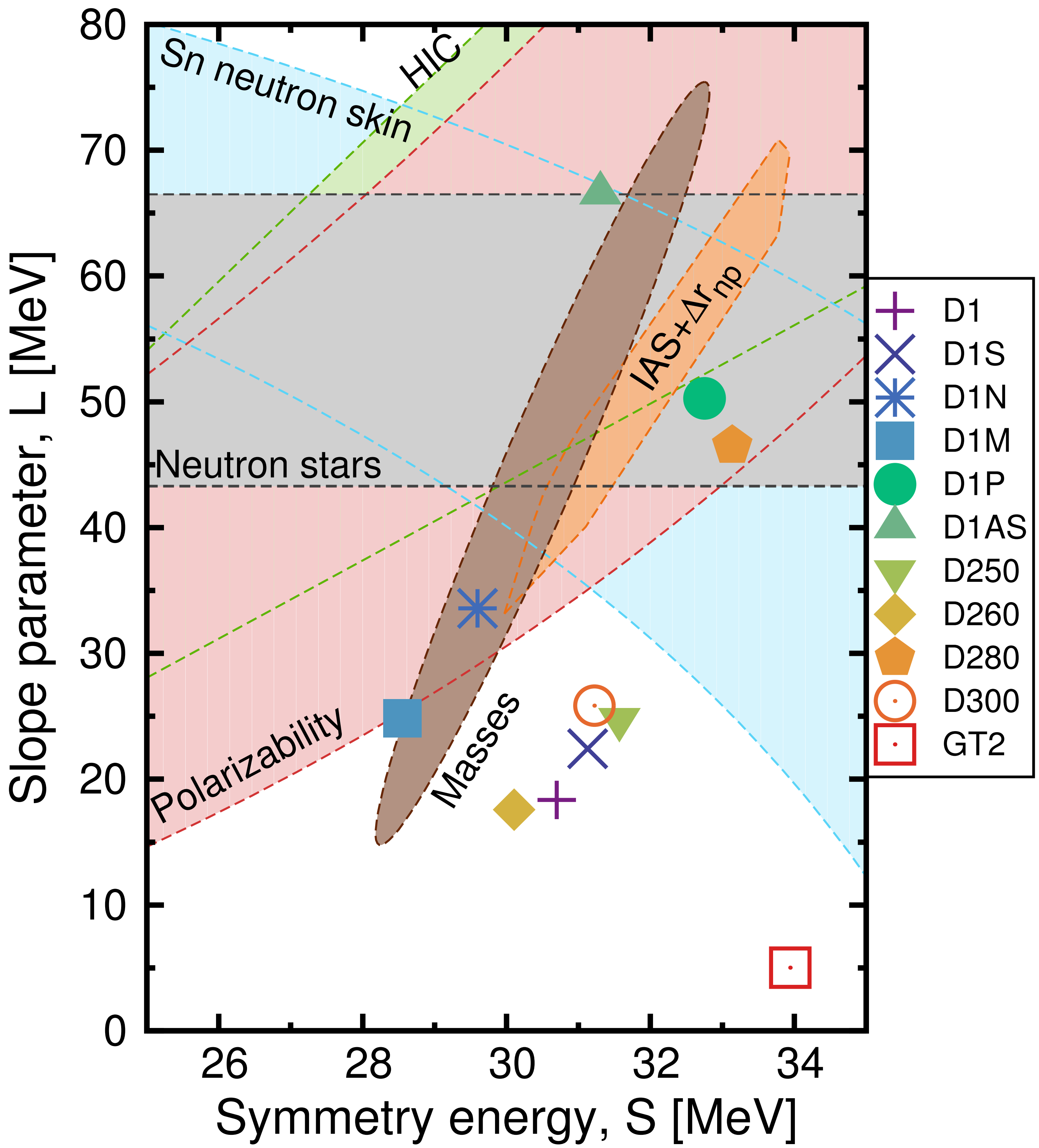
The Gogny force

$$\begin{aligned} V(\vec{r}) = & \sum_{i=1,2} \Big(W_i + B_i\, P_\sigma - H_i\, P_\tau - M_i\, P_\sigma\, P_\tau \Big) e^{-\frac{r^2}{\mu_i}} \\ & + \sum_{i=1,2} t_0^i\ (1 + x_0^i\, P_\sigma)\ \rho^{\alpha_i}\ \delta(\vec{r}) \\ & + iW_0(\sigma_1 + \sigma_2)[\vec{k}' \times \delta(\vec{r})\vec{k}] \end{aligned}$$

- 11 parametrizations
- Popular in **fission** studies

- Any good for isospin?
- Only 7 give stable NSs!

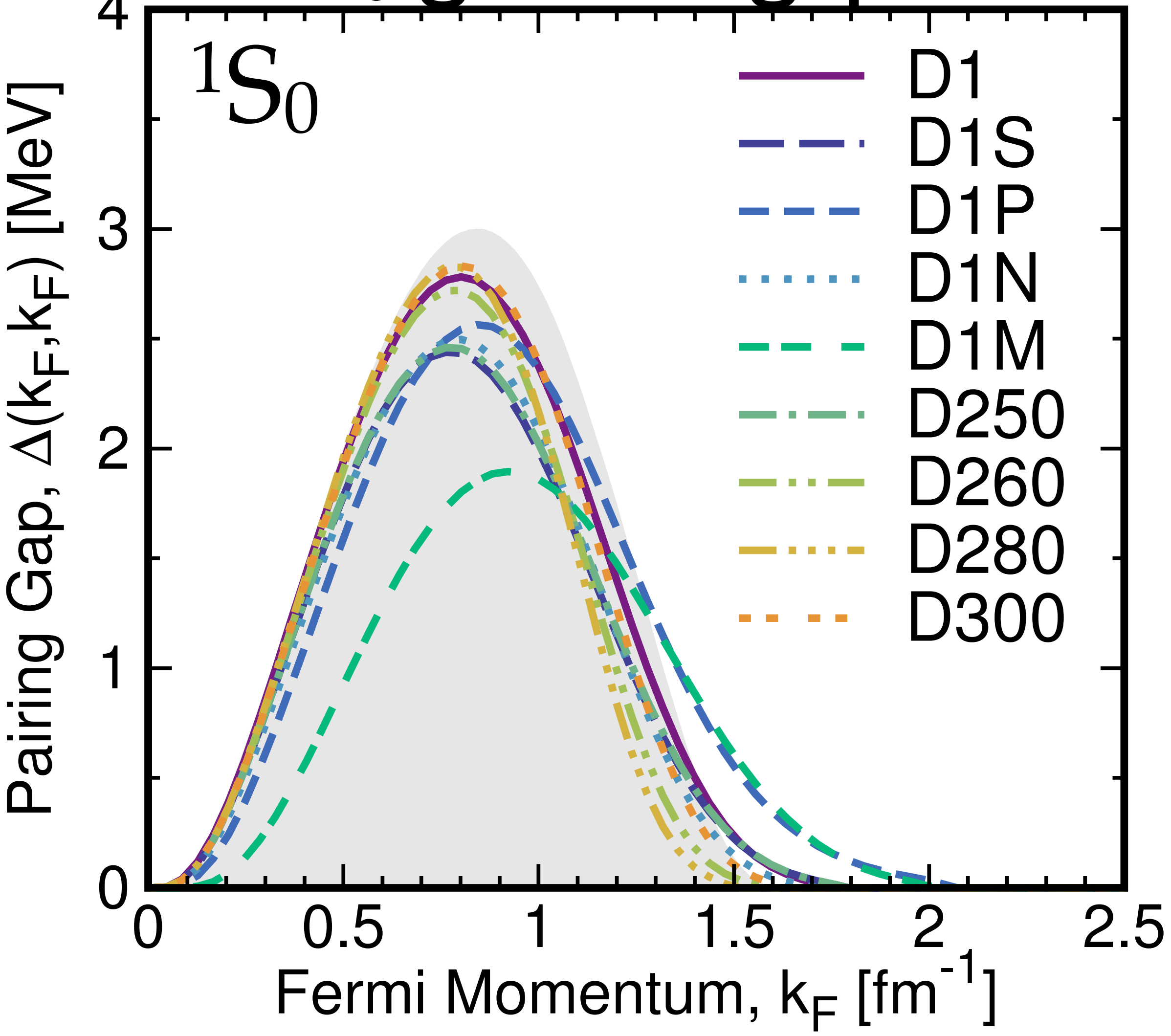
- **Pairing**?



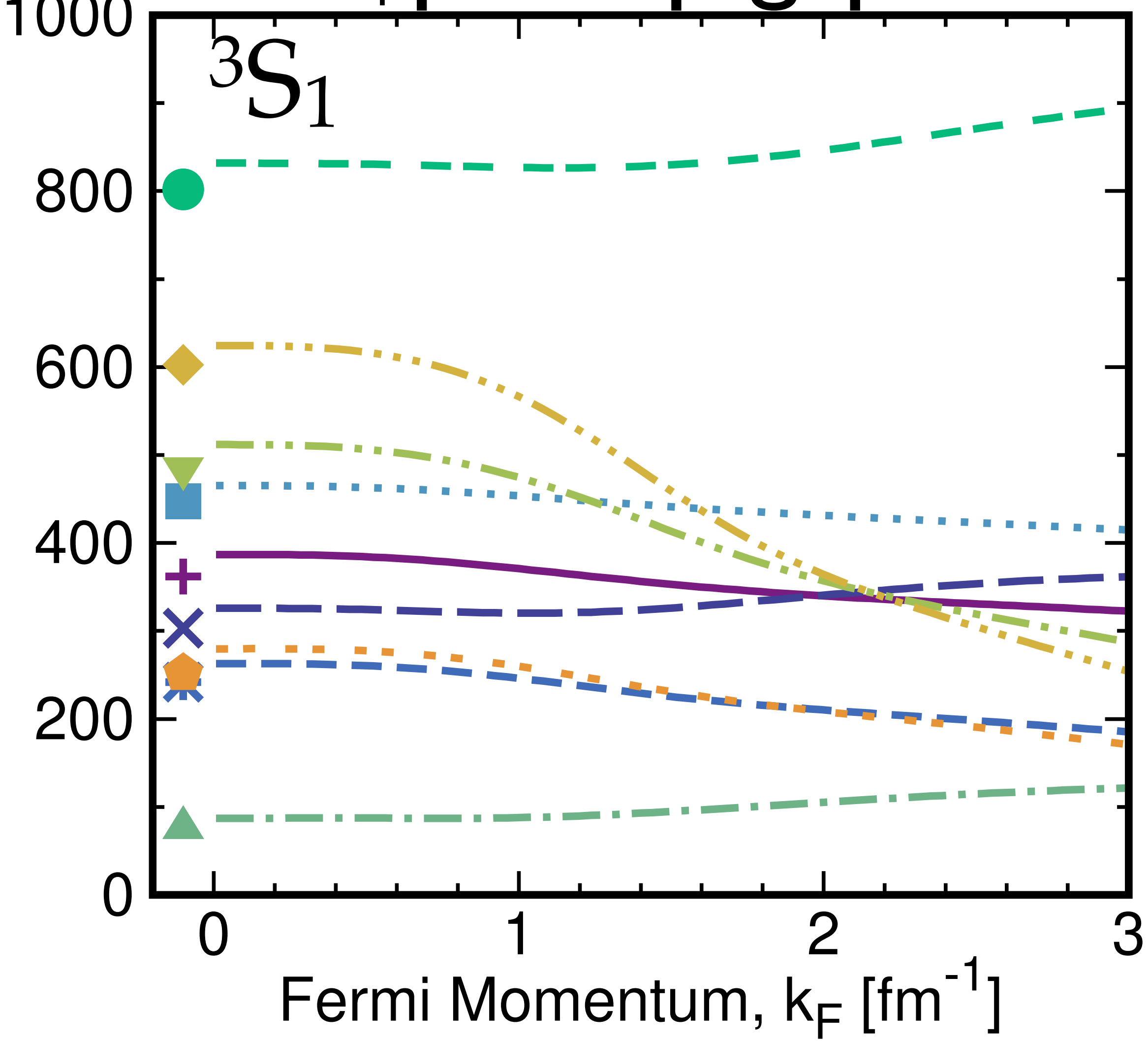
compstar
compstar-esf.org

R. Sellahewa & A. Rios, *PRC* **90**, 054327 (2014)

BCS-pairing with Gogny



Singlet nn gap



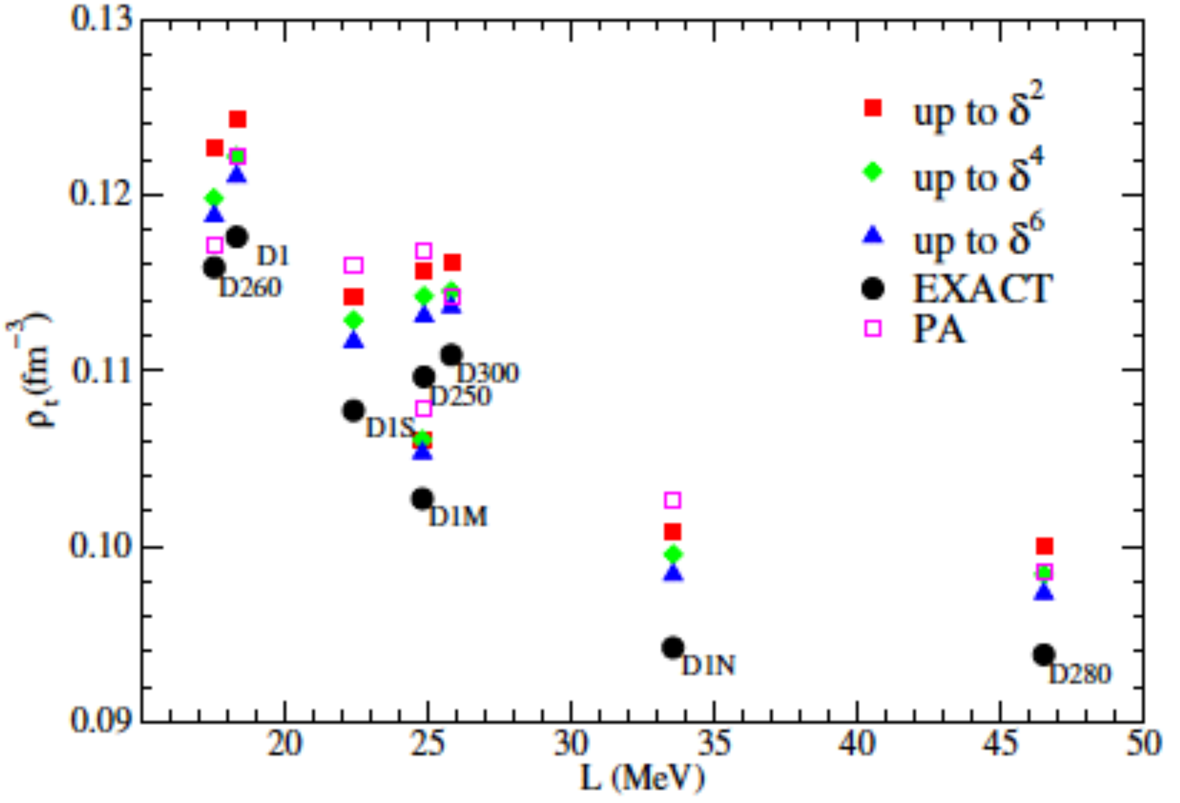
Triplet np gap

BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} + \qquad \chi_k = \varepsilon_k - \mu$$

R. Sellahewa & A. Rios, *in preparation* + R. Sellahewa, *PhD thesis*

Cooling



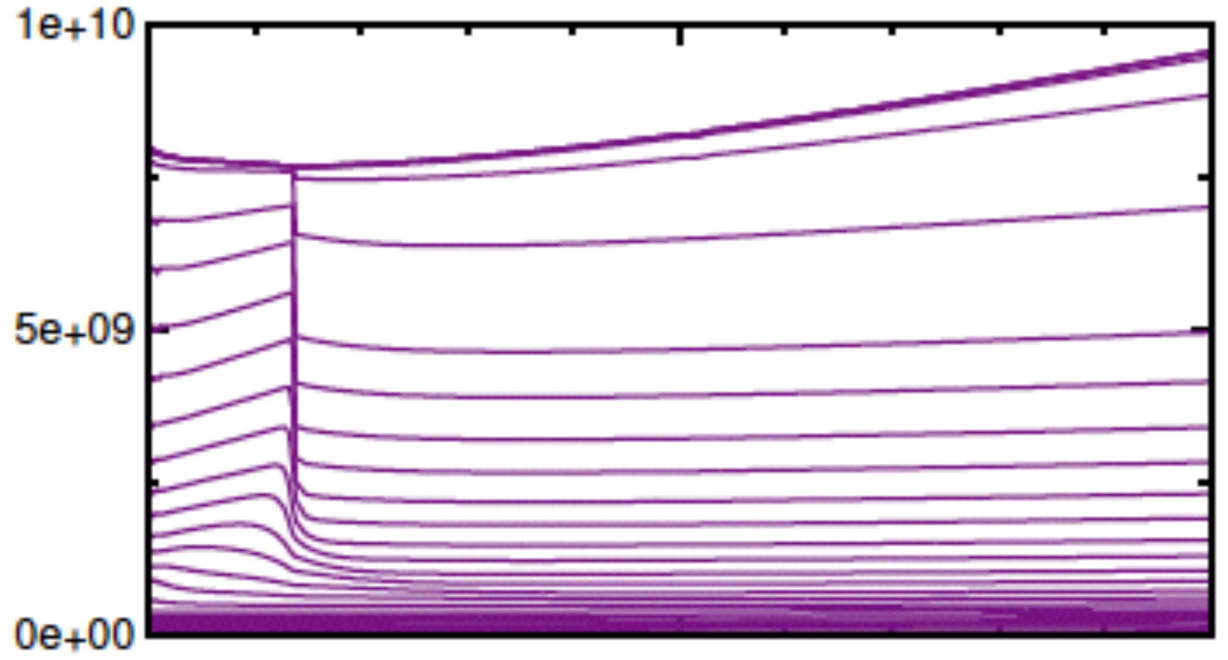
Gonzalez-Boquera, Centelles, Vinas
& A. Rios, *in preparation*

Ingredients (D1P)

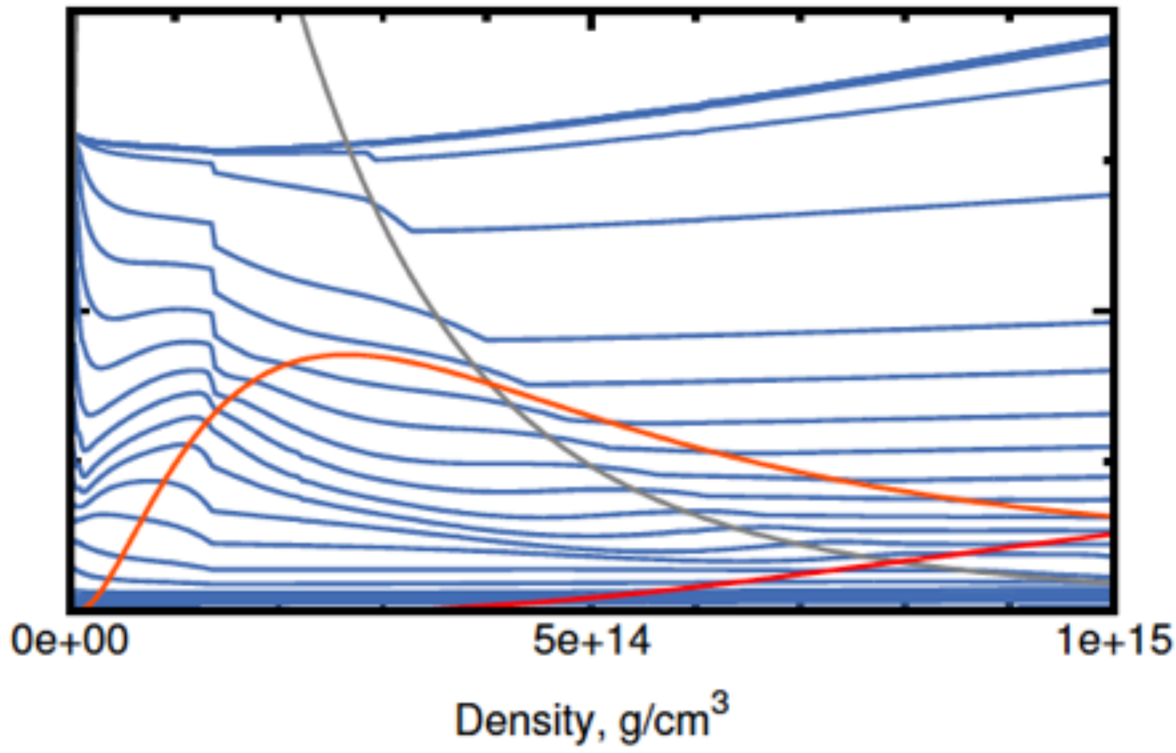
- **(a) Mass of pulsar**, 1.6M⊙ ✔
- **(b) EoS** (except crust) ✔
- **(c) Internal composition** ✔
- **(d) Pairing gaps (n $^1S_0$ & $^3P_2$)** ✔
- (e) Atmosphere composition

1.6M⊙, D1P

None

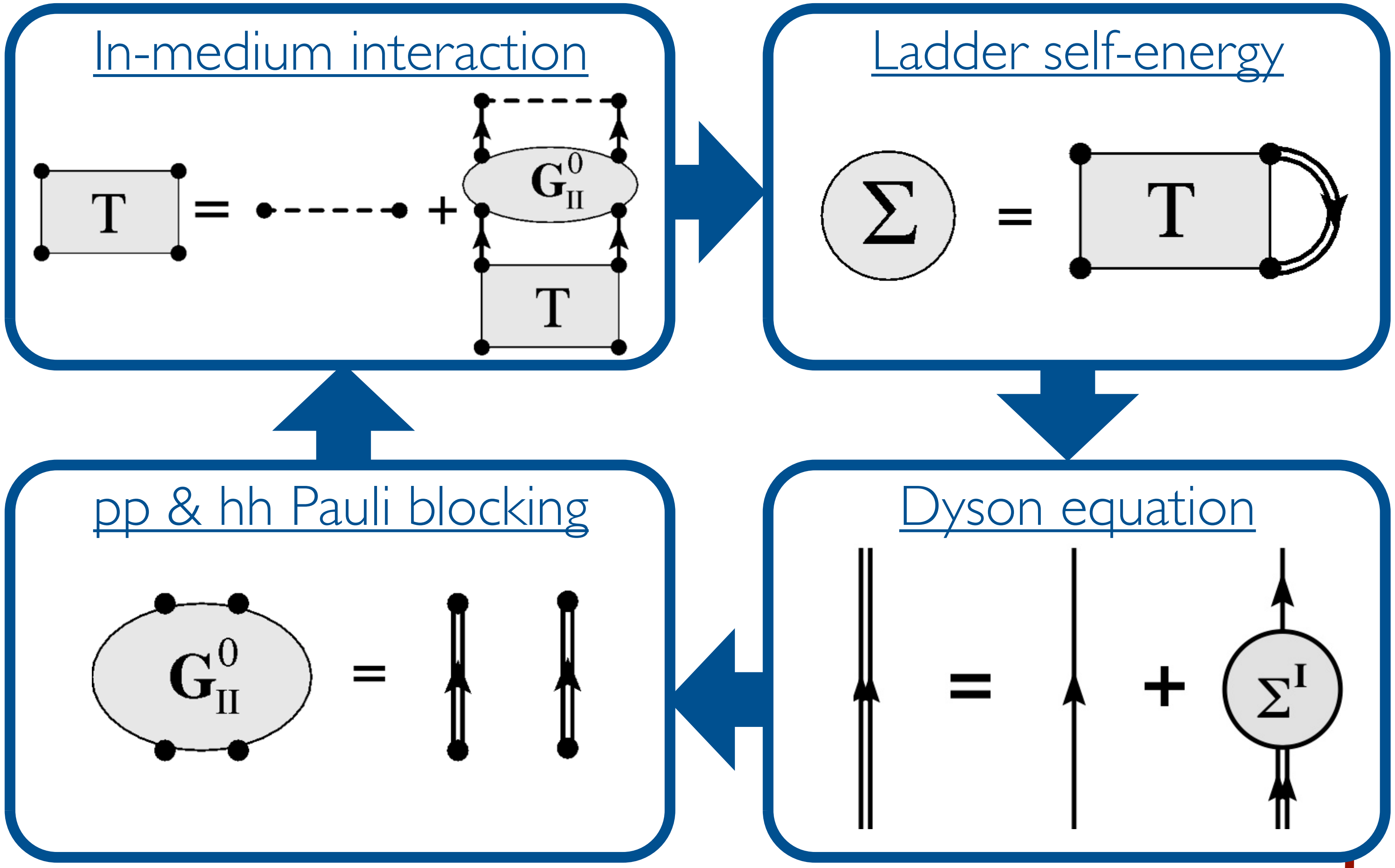


Full



R. Sellahewa, *PhD thesis*

SCGF Ladder approximation



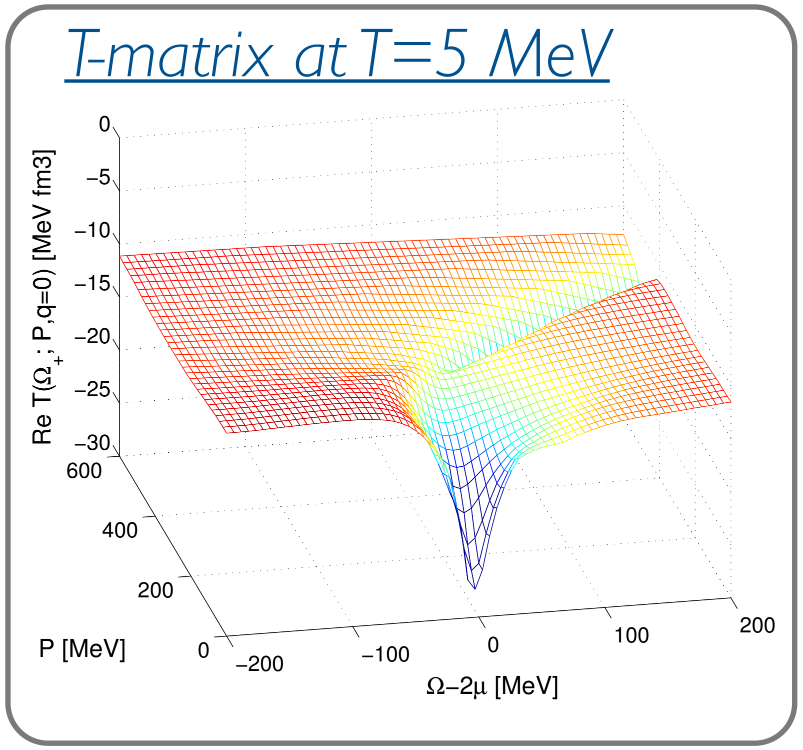
In-medium interaction

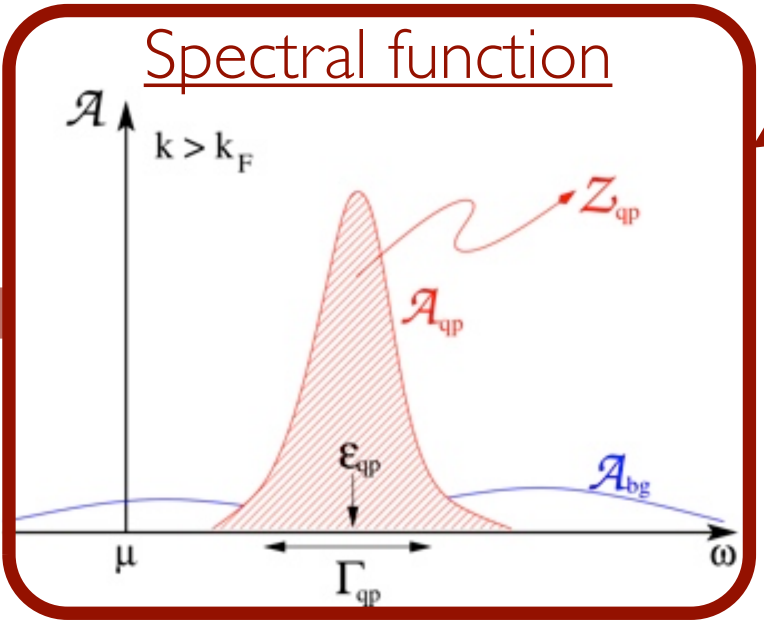
Ladder self-energy

pp & hh Pauli blocking

Dyson equation

- Self-consistent resummation
- Energy and momentum integral
- @Finite T (Matsubara)


T-matrix at T=5 MeV


Spectral function

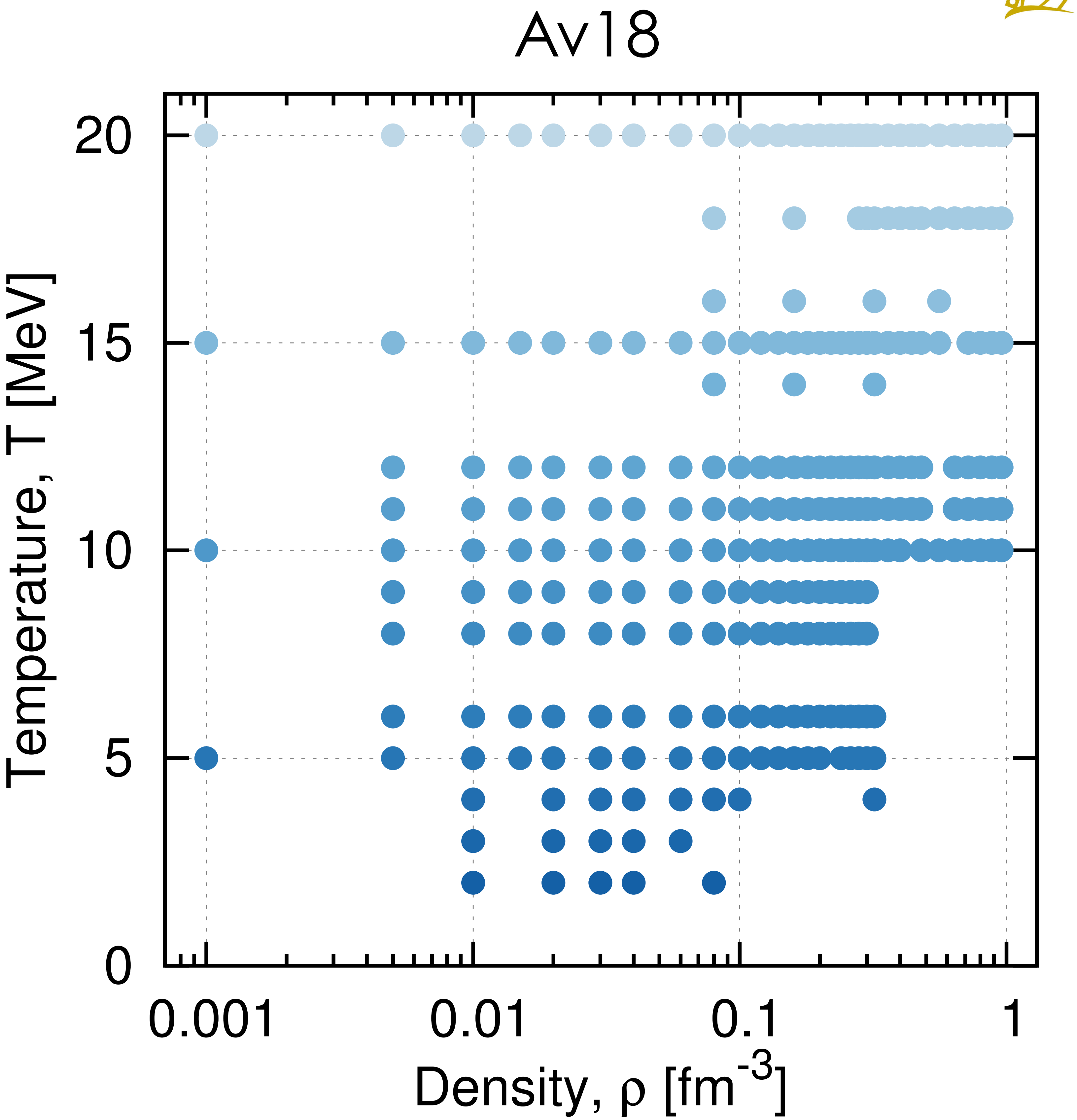
One-body properties
Momentum distribution
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

- **Advantages**
  - **All** kinds of NN interactions ✔
  - **3N** interactions ✔
  - Short-range & tensor **correlations** ✔
  - Density & **isospin** dependence ✔
  - Access to **off-shell** spectral function ✔
  - **Thermodynamically** consistent ✔

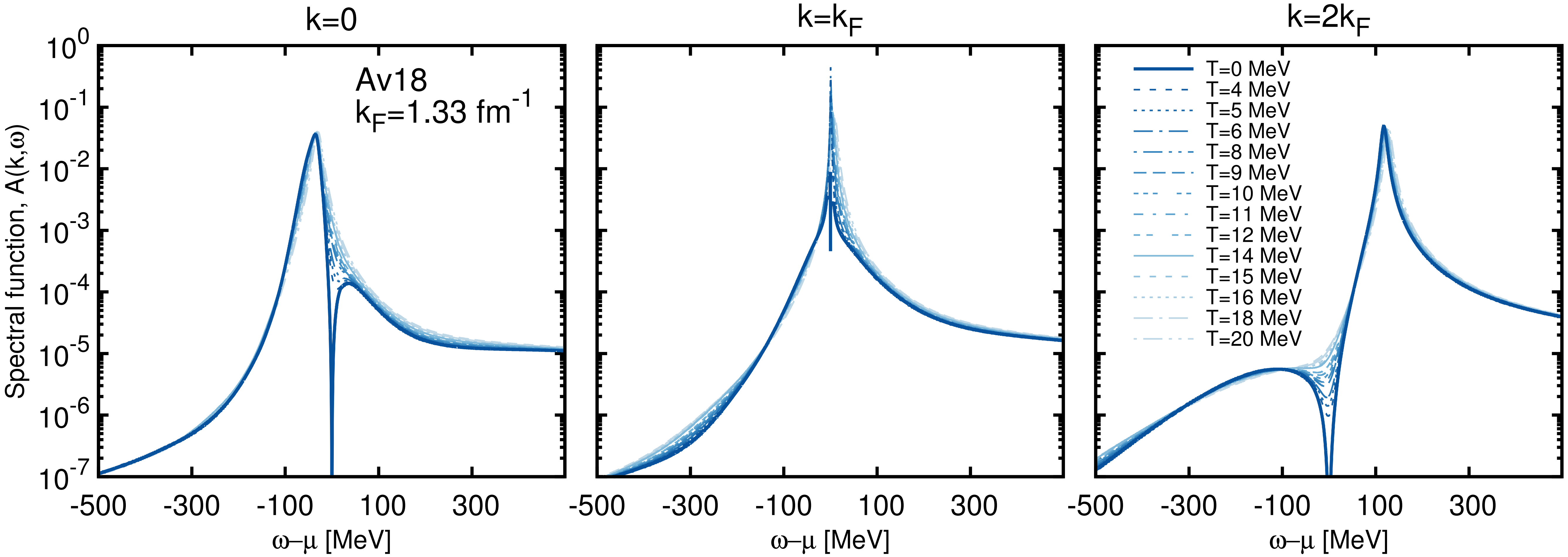
- **Limitations**
  - Non-relativistic ✘
  - Missing diagrams? ✘
  - No nuclear surface ✘

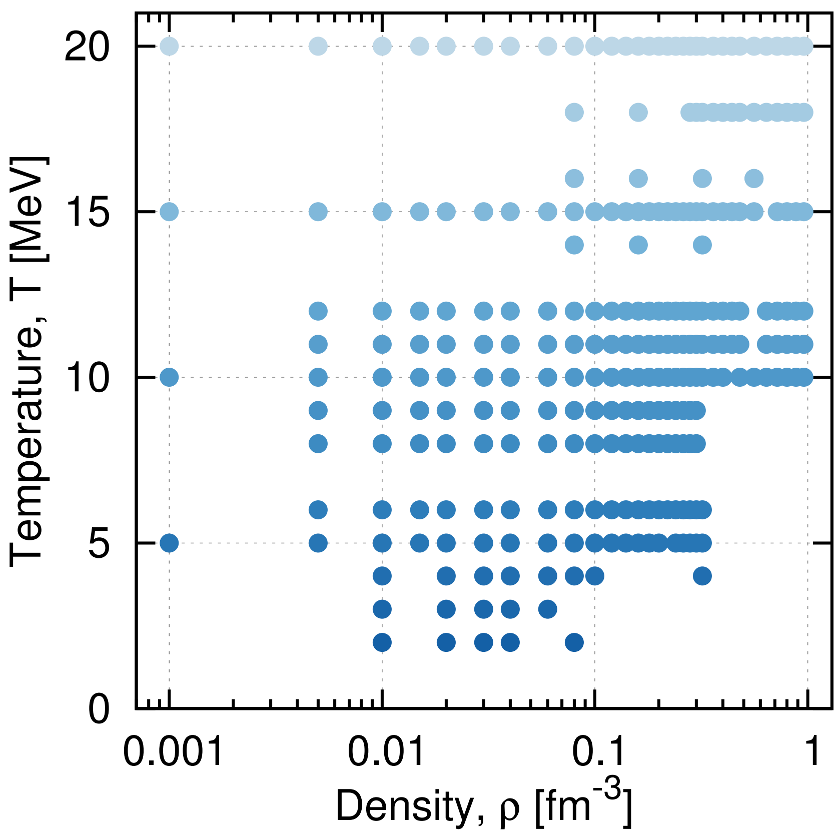
Available data

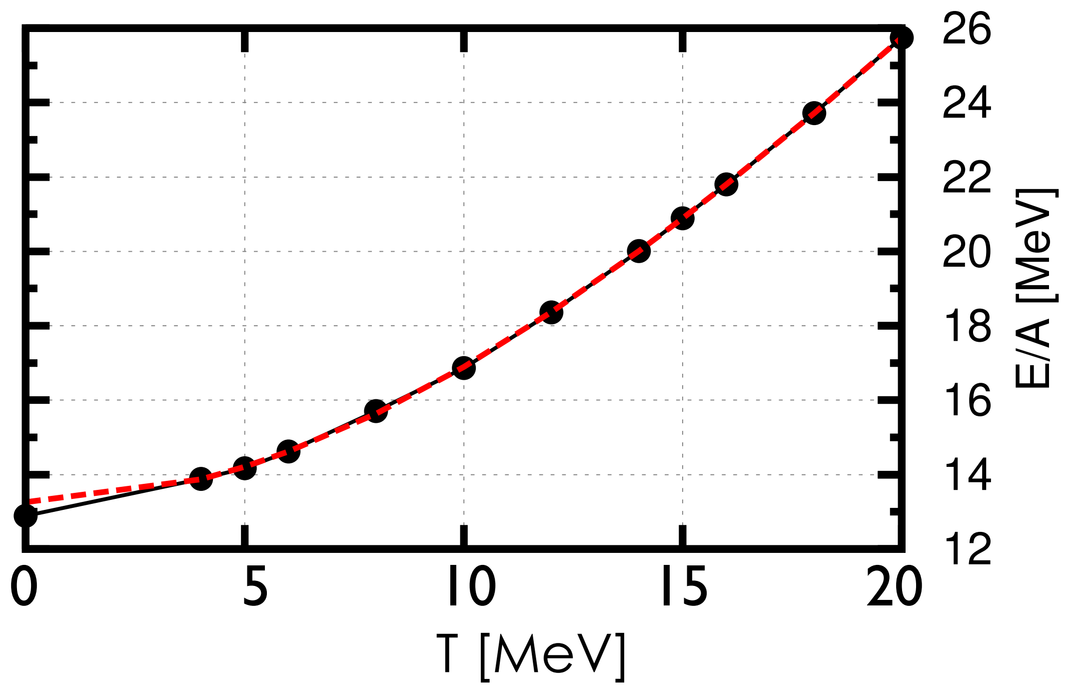


Self-energy, spectral function & thermodynamics

T=0 extrapolations

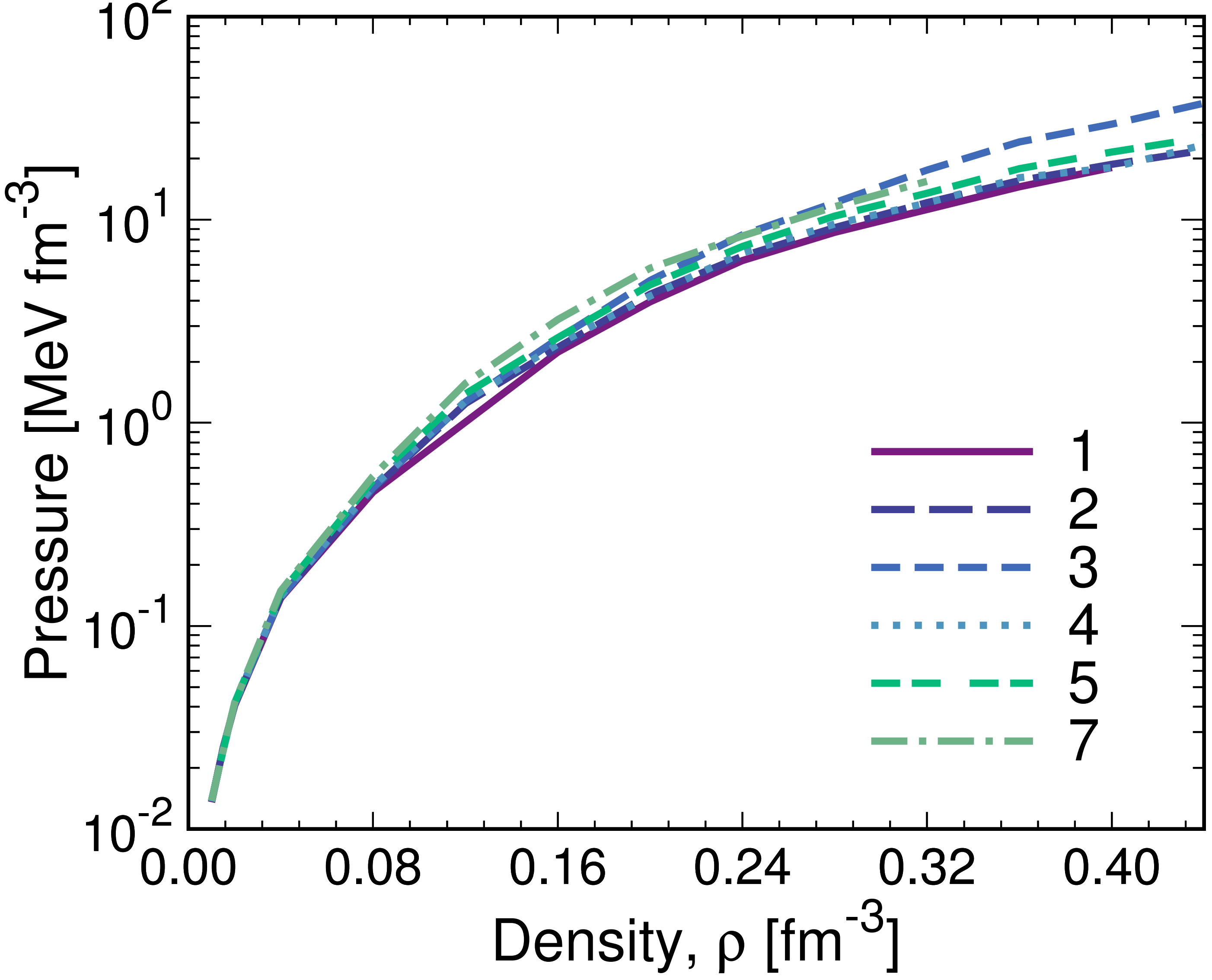


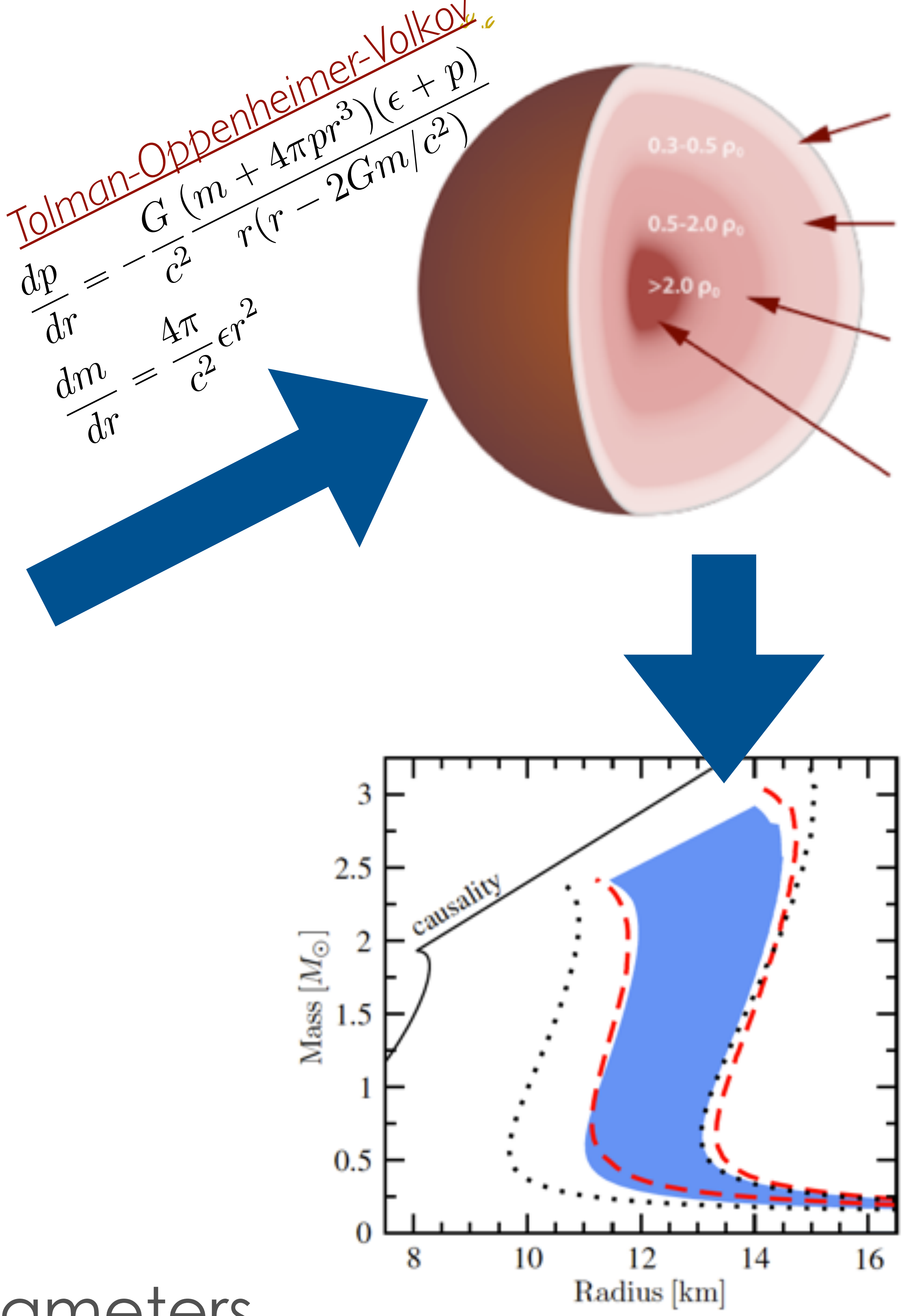




Neutron matter

EoS for neutron matter: SRG





$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi p r^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = \frac{4\pi}{c^2}\epsilon r^2$$

Hebeler, Lattimer, Pethick, Schwenk
ApJ **773** 11 (2013)

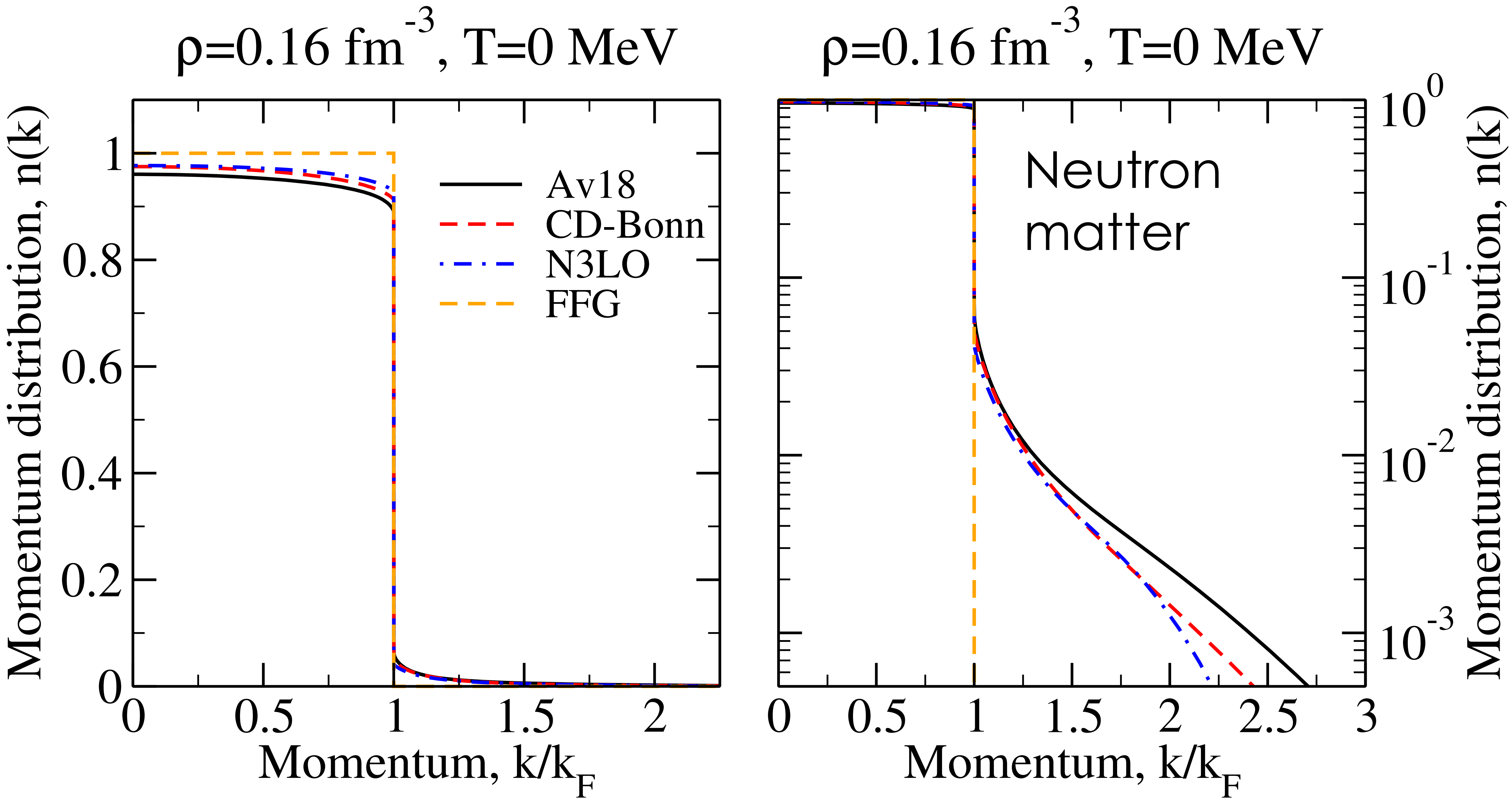
- Error band from fits in ChPT c_1, c_3 parameters
- Finite temperature & higher densities available

Carbone, Polls & Rios, *PRC* **90** 054322 (2014)
A. Carbone, Drischler, Hebeler, Schwenk, arXiv:1608.05615

Momentum distribution

Single-particle occupation

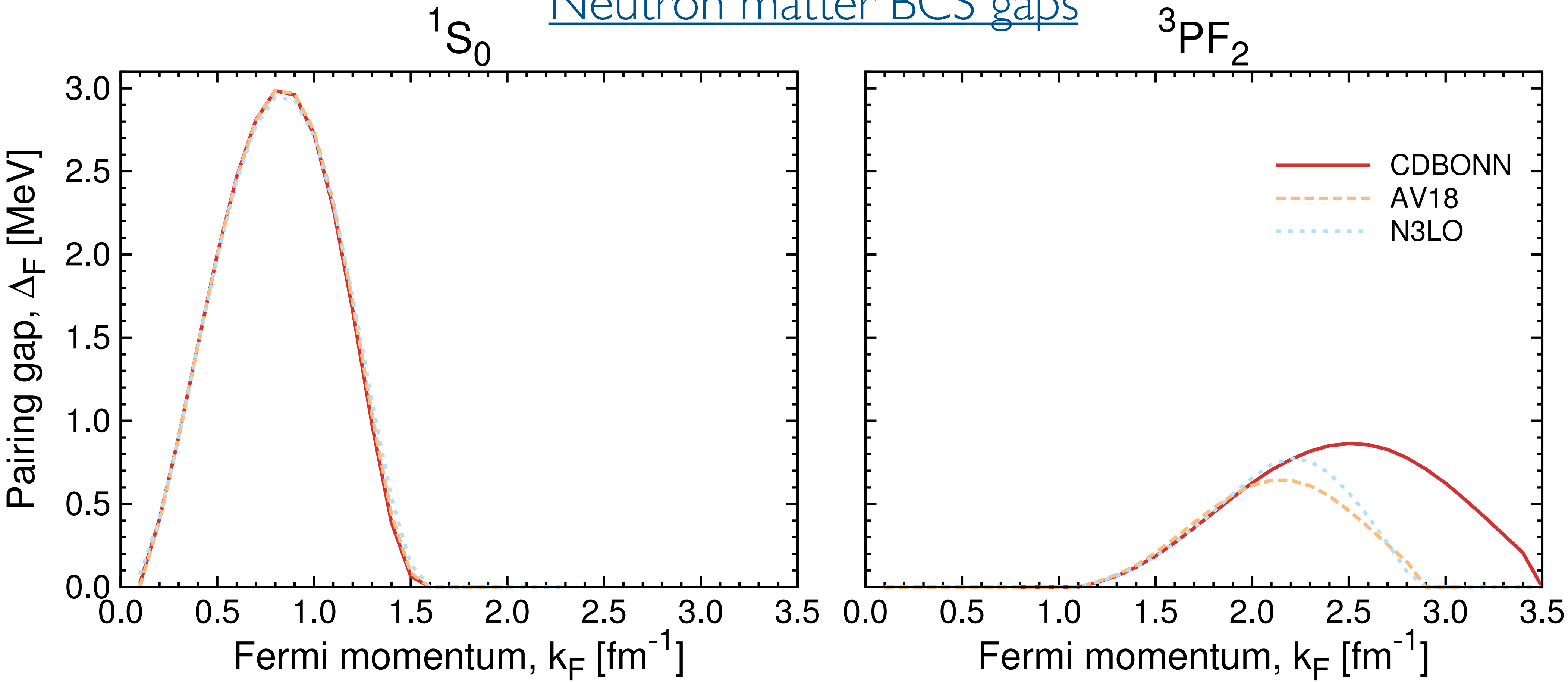
$$n(k) = \left\langle a_k^\dagger a_k \right\rangle \qquad \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$



- SNM: 11-13% depletion at low k, population at high k
- Dependence on NN interaction under control
- PNM: 4-5% depletion at low k

Bardeen-Cooper-Schrieffer pairing

Neutron matter BCS gaps



Dean & Hjorth-Jensen, *Rev. Mod. Phys.* **75** 607 (2003)

BCS equation

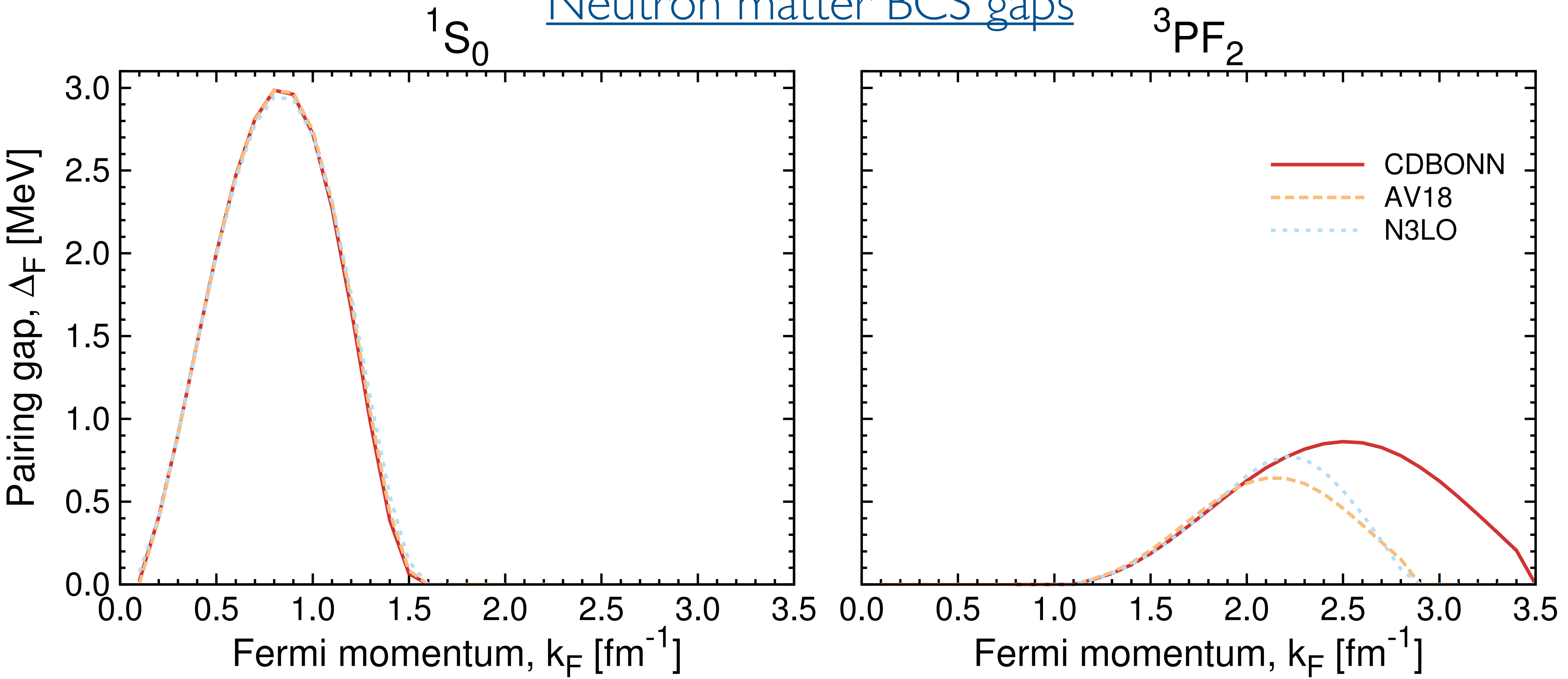
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad \textbf{+} \quad \chi_k = \varepsilon_k - \mu$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + U(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Maurizio, Holt & Finelli, *Phys. Rev. C* **90**, *044003* (2014)

Bardeen-Cooper-Schrieffer pairing

Neutron matter BCS gaps



Dean & Hjorth-Jensen, *Rev. Mod. Phys.* **75** 607 (2003)

BCS equation

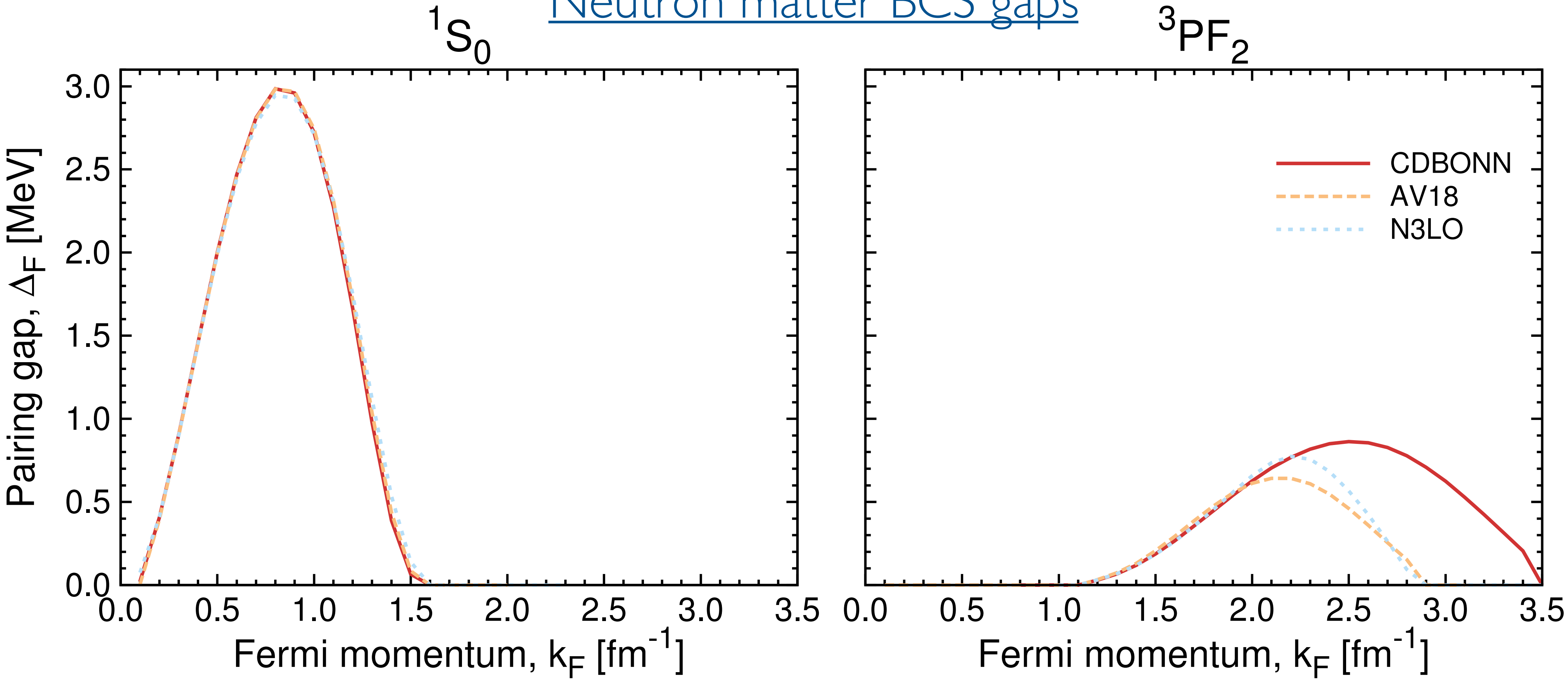
$$\boxed{\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}}\Delta_{k'}^{L'} \;\mathbf{+}\qquad \chi_k = \varepsilon_k - \mu}$$

- Single-particle spectrum choice: $\varepsilon_k = \dfrac{k^2}{2m} + \cancel{U}(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Maurizio, Holt & Finelli, *Phys. Rev. C* **90**, *044003* (2014)

Bardeen-Cooper-Schrieffer pairing

Neutron matter BCS gaps



Dean & Hjorth-Jensen, *Rev. Mod. Phys.* **75** 607 (2003)

BCS equation

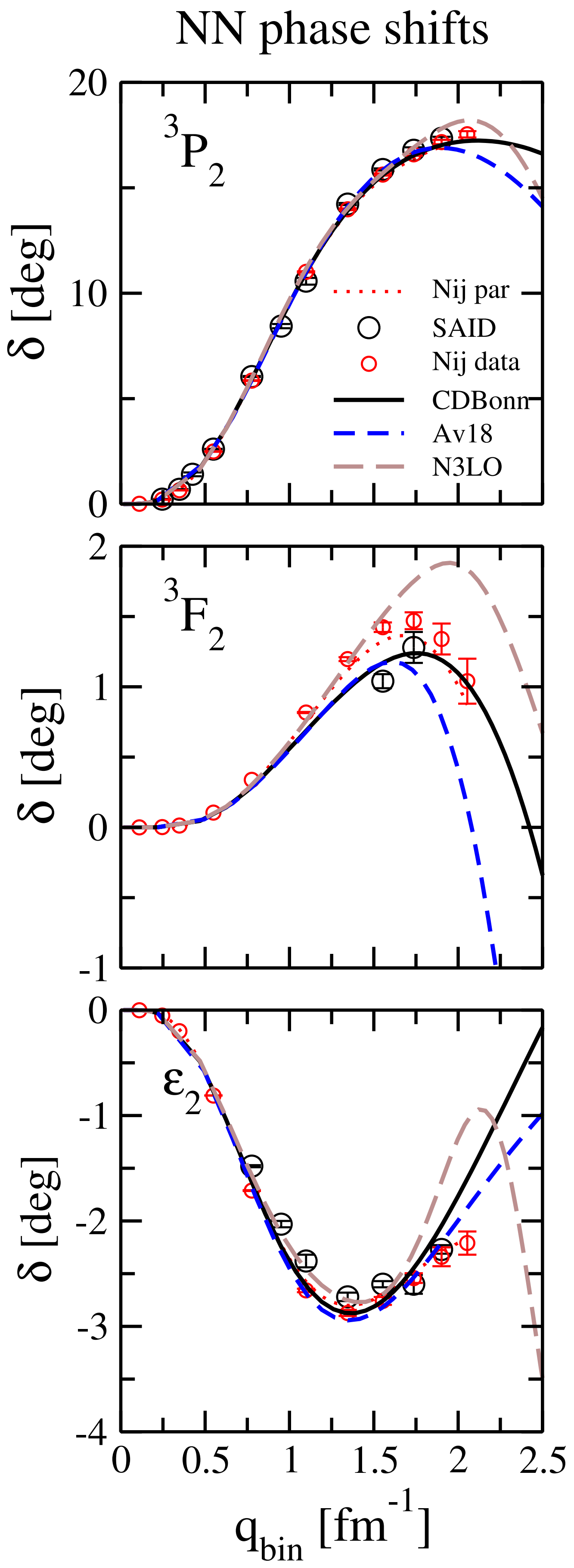
$$\boxed{\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad \mathbf{+} \qquad \chi_k = \varepsilon_k - \mu}$$

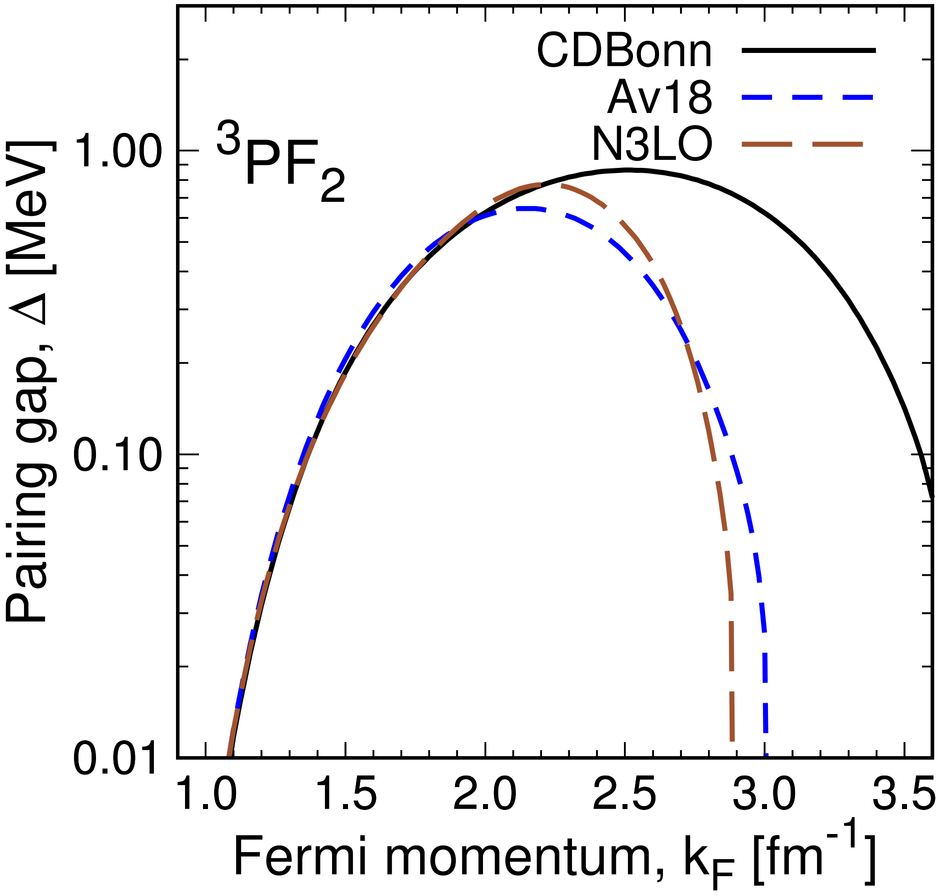
- Single-particle spectrum choice: $\varepsilon_k = \dfrac{k^2}{2m} + \xcancel{U}(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2 \approx |\Delta_k^L|^2$

Maurizio, Holt & Finelli, *Phys. Rev. C* ***90**, 044003* (2014)

Triplet pairing

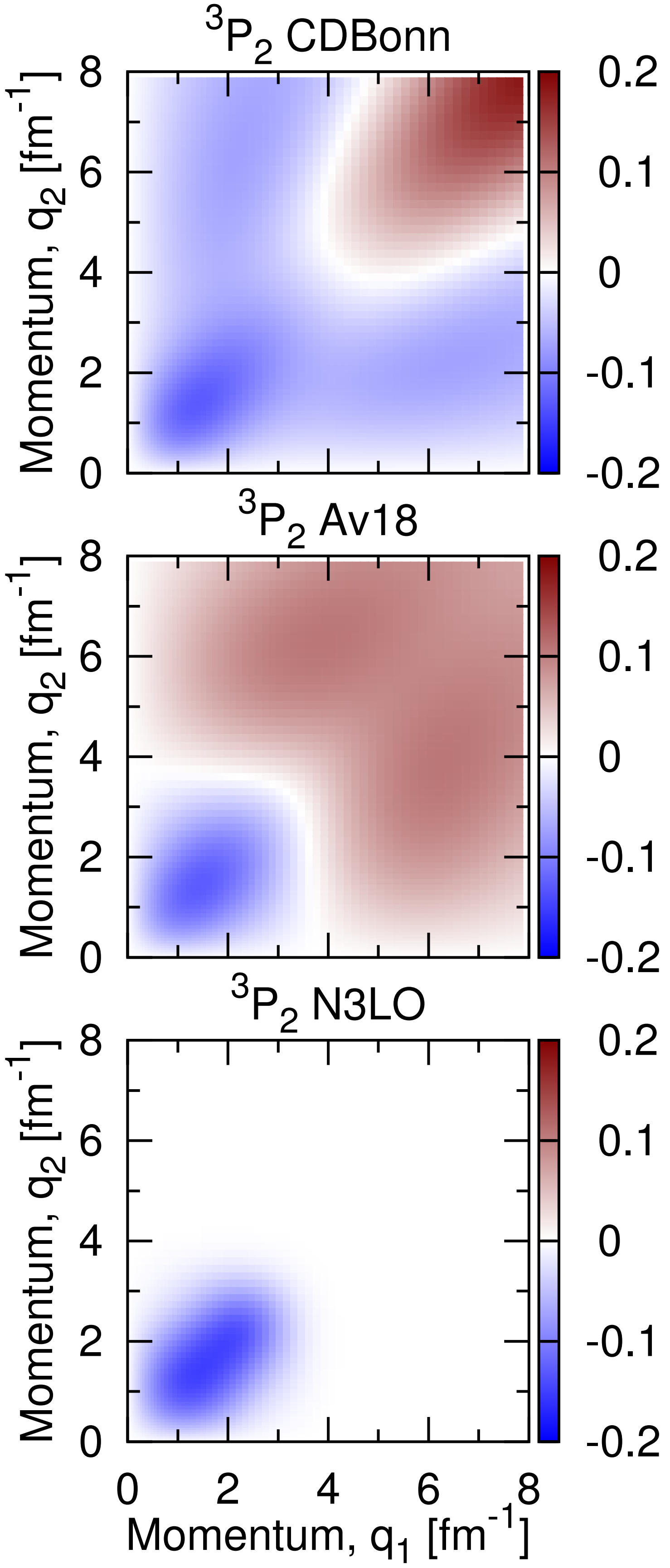
Phase-shift equivalent?

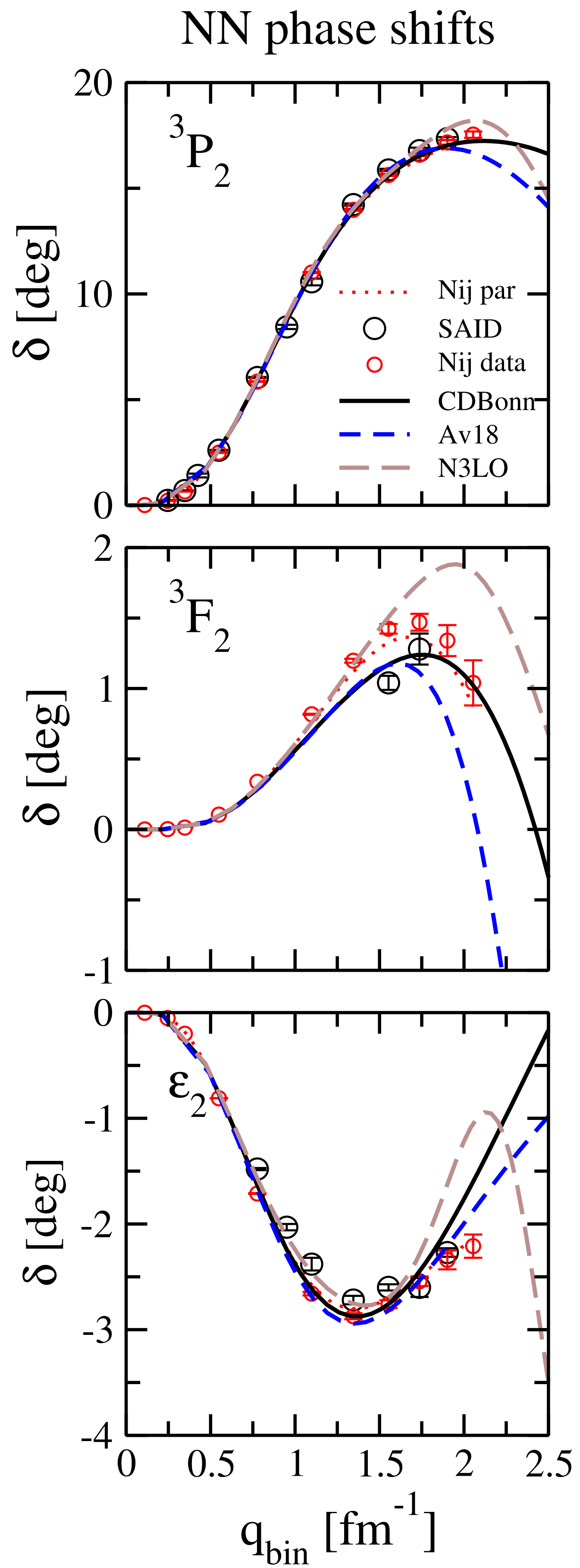


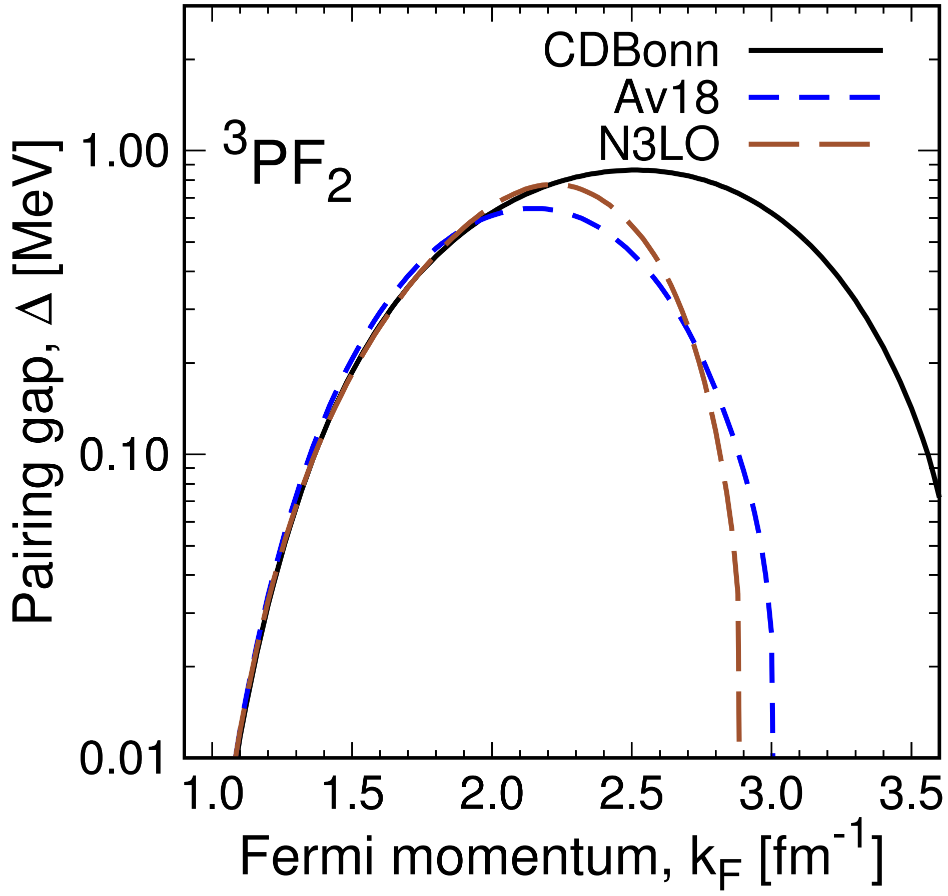


Triplet pairing

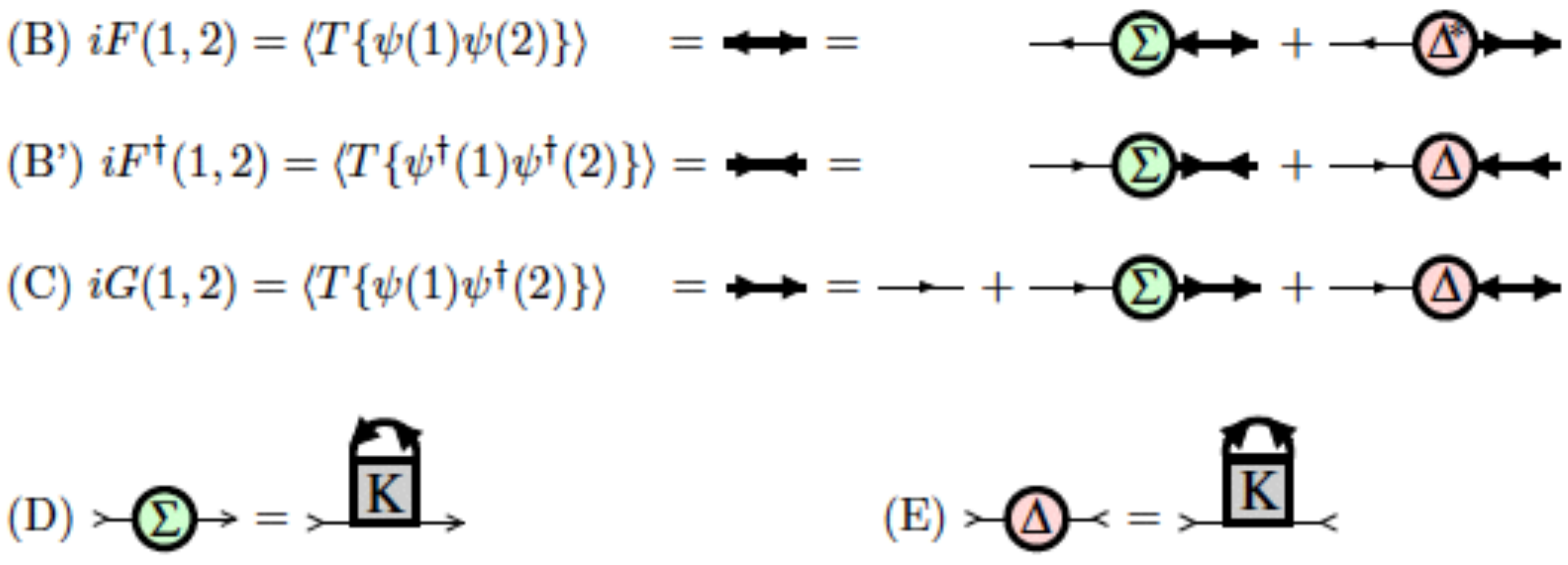
Phase-shift equivalent?







Beyond BCS 101: SRC



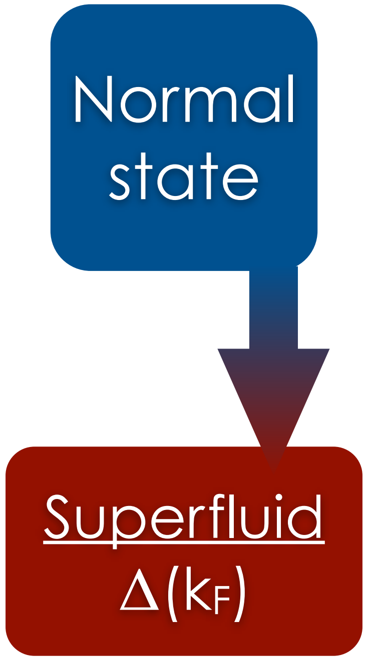
(B) $iF(1,2) = \langle T\{\psi(1)\psi(2)\}\rangle$

(B') $iF^{\dagger}(1,2) = \langle T\{\psi^{\dagger}(1)\psi^{\dagger}(2)\}\rangle$

(C) $iG(1,2) = \langle T\{\psi(1)\psi^{\dagger}(2)\}\rangle$

(D)

(E)



Normal state

<u>Superfluid</u>
Δ(k_F)

BCS+SRC equation

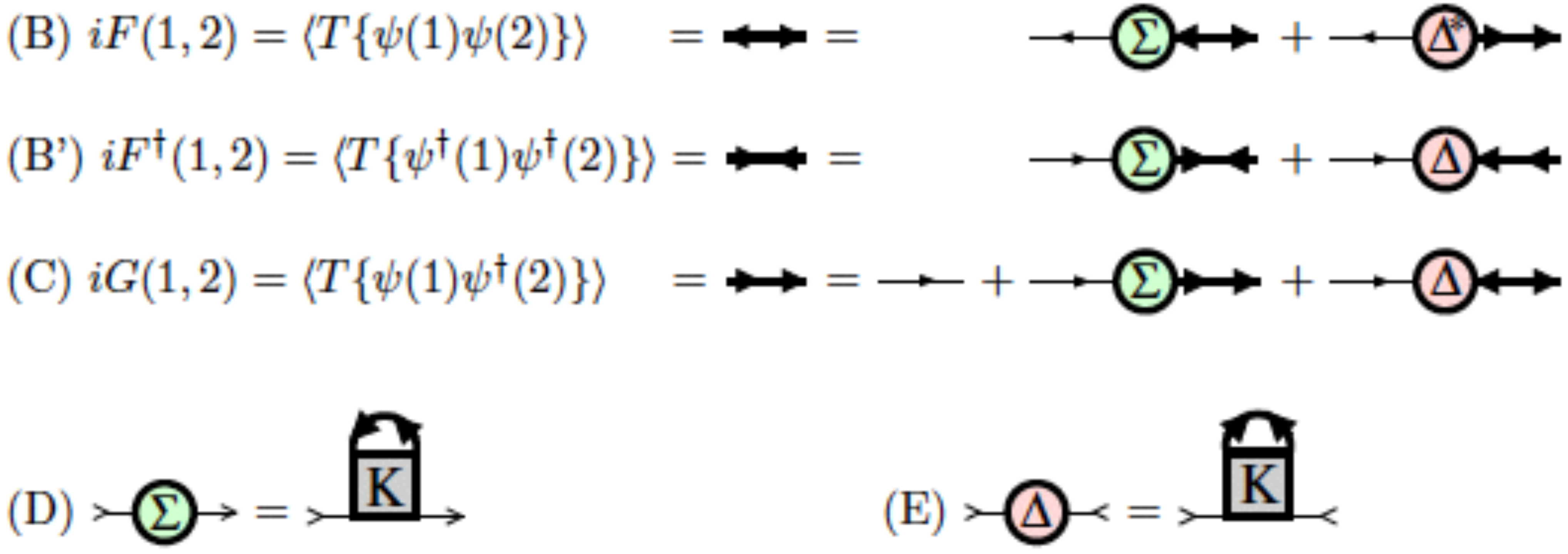
$$\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}}\Delta_{k'}^{L'} \; \mathbf{+}$$

$$\frac{1}{2\chi_k} = \int_{\omega}\int_{\omega'} \frac{1-f(\omega)-f(\omega')}{\omega+\omega'} A(k,\omega)A_s(k,\omega')$$

- **BCS** is *lowest order* in Gorkov Green's function expansion
- T-matrix can be extended to paired systems
- But full self-consistency is still missing

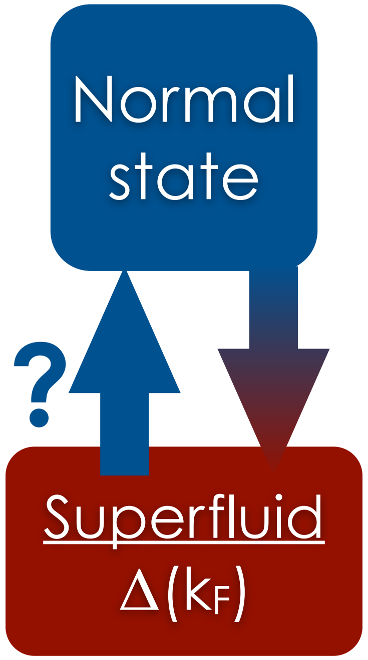
Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 101: SRC


(B) $iF(1,2) = \langle T\{\psi(1)\psi(2)\}\rangle$

(B') $iF^\dagger(1,2) = \langle T\{\psi^\dagger(1)\psi^\dagger(2)\}\rangle$

(C) $iG(1,2) = \langle T\{\psi(1)\psi^\dagger(2)\}\rangle$


Normal state

?

Superfluid

$\Delta(k_F)$

BCS+SRC equation

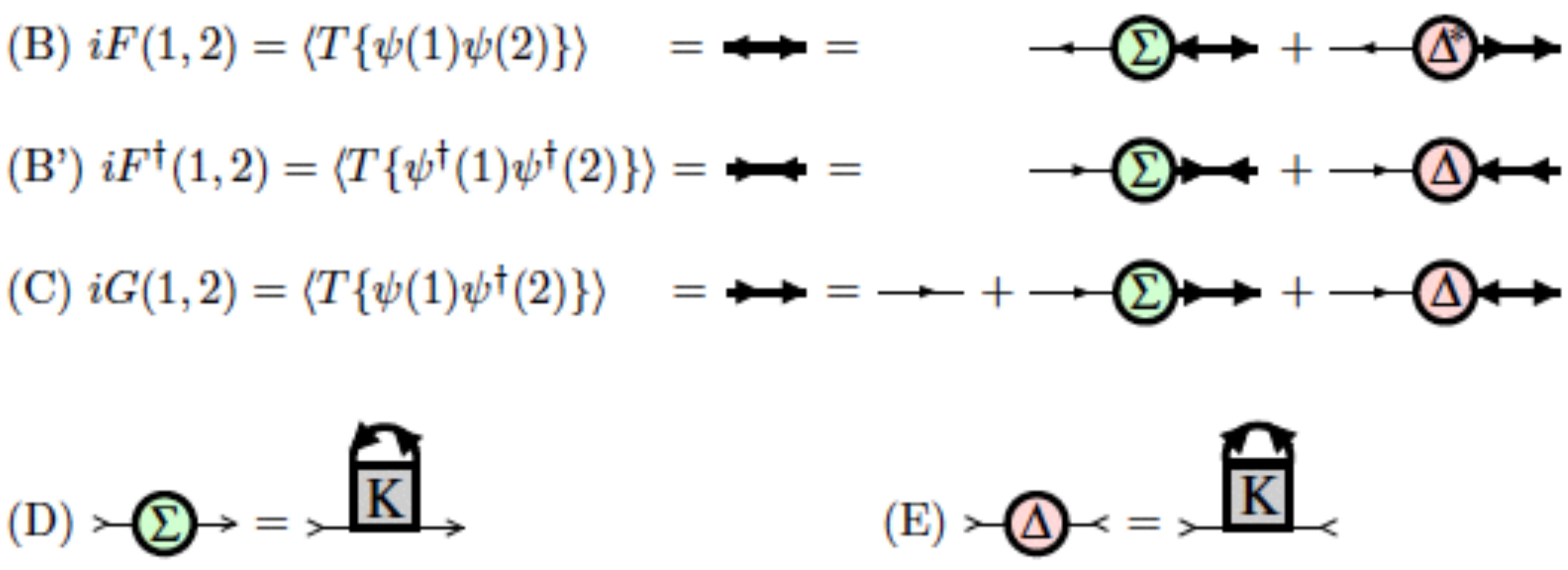
$$\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2+|\Delta_{k'}|^2}}\Delta_{k'}^{L'} \; \mathbf{+}$$

$$\frac{1}{2\chi_k} = \int_\omega\int_{\omega'} \frac{1-f(\omega)-f(\omega')}{\omega+\omega'}A(k,\omega)A_s(k,\omega')$$

- **BCS** is *lowest order* in Gorkov Green's function expansion
- T-matrix can be extended to paired systems
- But full self-consistency is still missing

Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 101: SRC



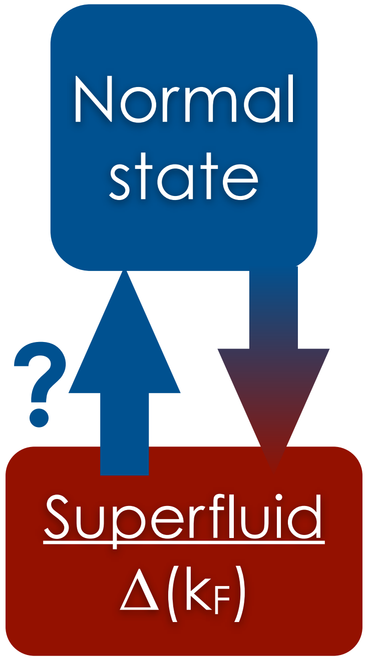
(B) $iF(1,2) = \langle T\{\psi(1)\psi(2)\}\rangle =$

(B') $iF^\dagger(1,2) = \langle T\{\psi^\dagger(1)\psi^\dagger(2)\}\rangle =$

(C) $iG(1,2) = \langle T\{\psi(1)\psi^\dagger(2)\}\rangle =$

(D)

(E)



BCS+SRC equation

$$\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2+|\Delta_{k'}|^2}}\Delta_{k'}^{L'} \; \mathbf{+}$$

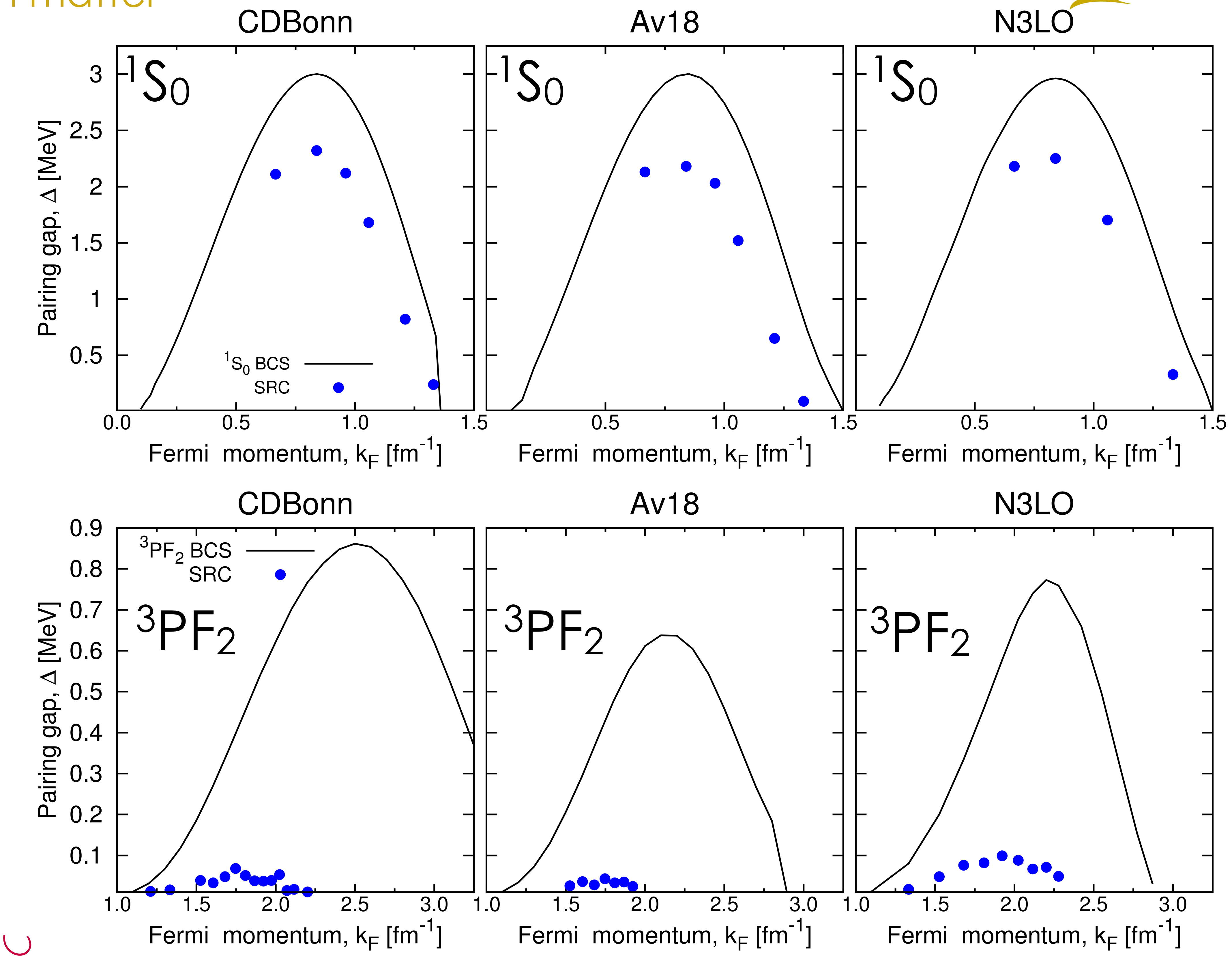
$$\frac{1}{2\chi_k} = \int_\omega\int_{\omega'} \frac{1-f(\omega)-f(\omega')}{\omega+\omega'}A(k,\omega)A_{\cancel{s}}(k,\omega')$$

- **BCS** is *lowest order* in Gorkov Green's function expansion
- T-matrix can be extended to paired systems
- But full self-consistency is still missing

Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 101: SRC

Neutron matter



BCS+SRC

$$\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\bar{\chi}_{k'}^2+|\Delta_{k'}|^2}}\Delta_{k'}^{L'} \quad \mathbf{+} \quad \frac{1}{2\chi_k} = \int_\omega\int_{\omega'} \frac{1-f(\omega)-f(\omega')}{\omega+\omega'}A(k,\omega)A_{\xcancel{s}}(k,\omega')$$

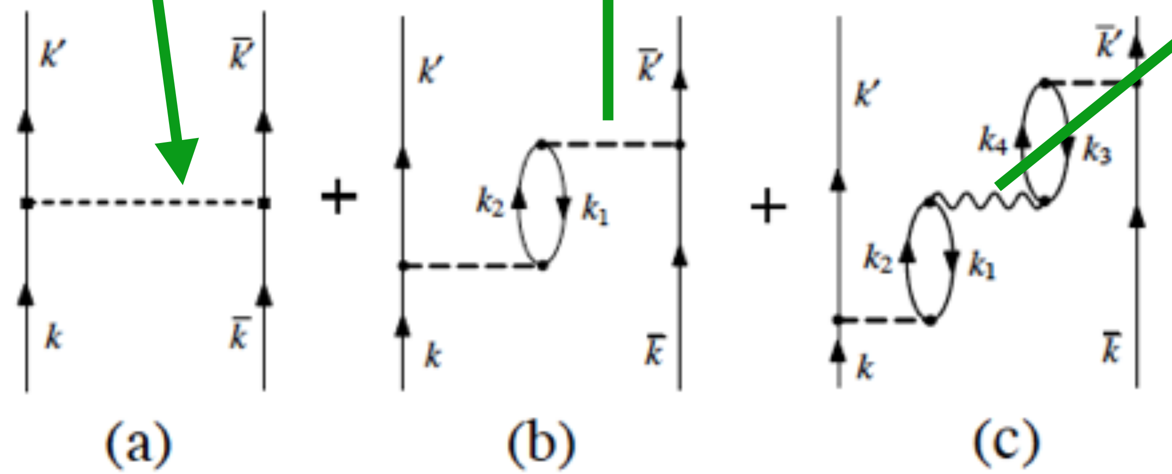
Beyond BCS 201: LRC

$$\Delta_k^L = -\sum_{L'}\int_{k'} \frac{\boxed{\langle k|V_{nn}^{LL'}|k'\rangle}\,?}{2\sqrt{\boxed{\chi_{k'}^2} + \checkmark |\Delta_{k'}|^2}}\Delta_{k'}^{L'}$$

ph recoupled G-matrix

Effective Landau parameters

$\mathcal{V}_{\text{pair}}=$



$$\langle 1\bar{1}|\mathcal{V}|1\bar{1}\rangle = \frac{1}{4}\sum_{2,2'}\sum_{S,T}(-)^S(2S+1)\langle 12|G_{ST}^{\text{ph}}|1'2'\rangle_A\langle 2'\bar{1}|G_{ST}^{\text{ph}}|2\bar{1}'\rangle_A\Lambda(22')$$

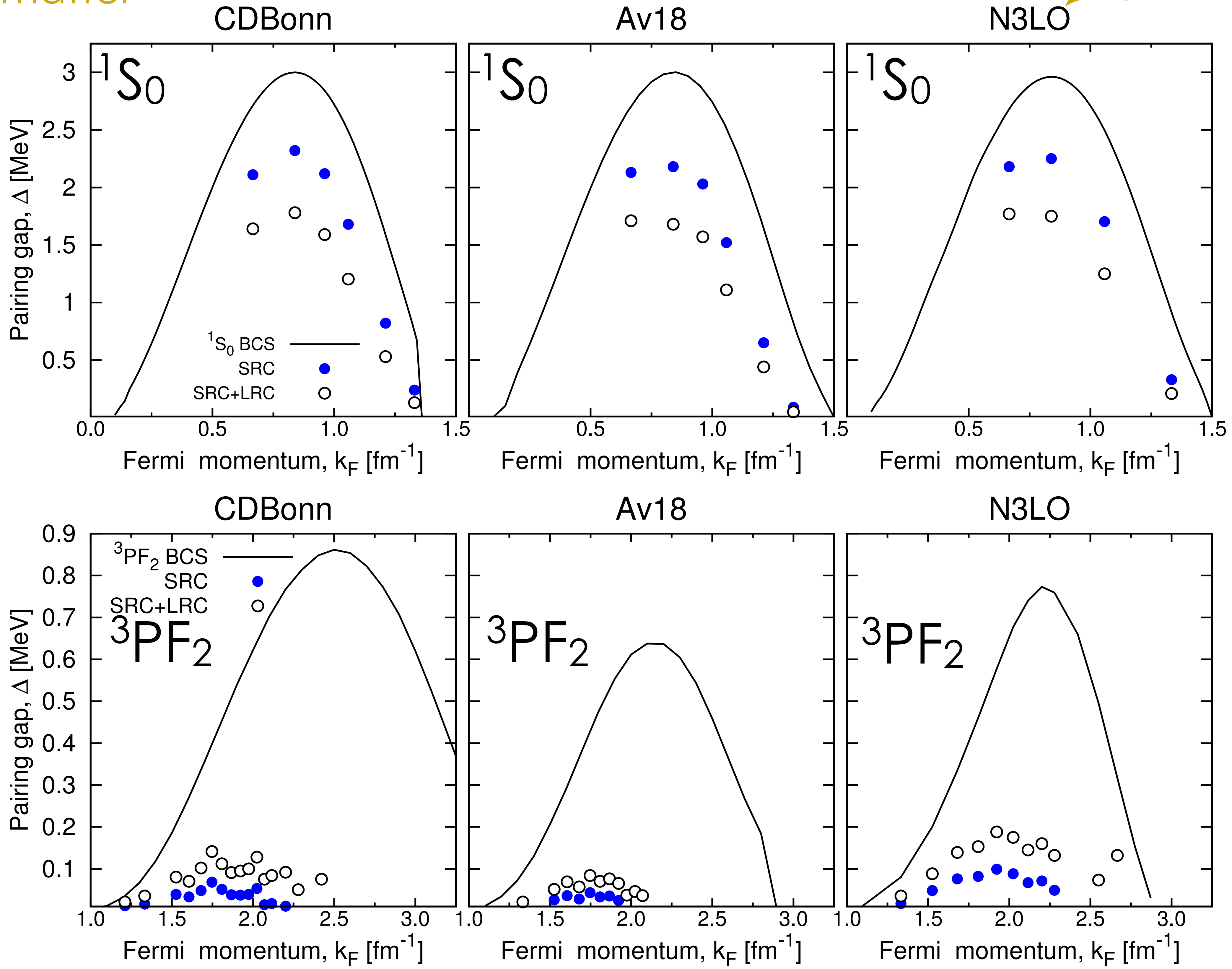
$$\Lambda_{ST}(q) = \frac{\Lambda_{ST}^0(q)}{1-\Lambda_{ST}^0(q)\times F_{ST}}$$

- Bare NN potential only is not the only possible interaction
- Diagram (a): nuclear interaction
- Diagram (b): in-medium interaction, density and spin fluctuations
- Diagram (c): included by Landau parameters

Cao, Lombardo & Schuck, *Phys Rev C* **74** 064301 (2006)

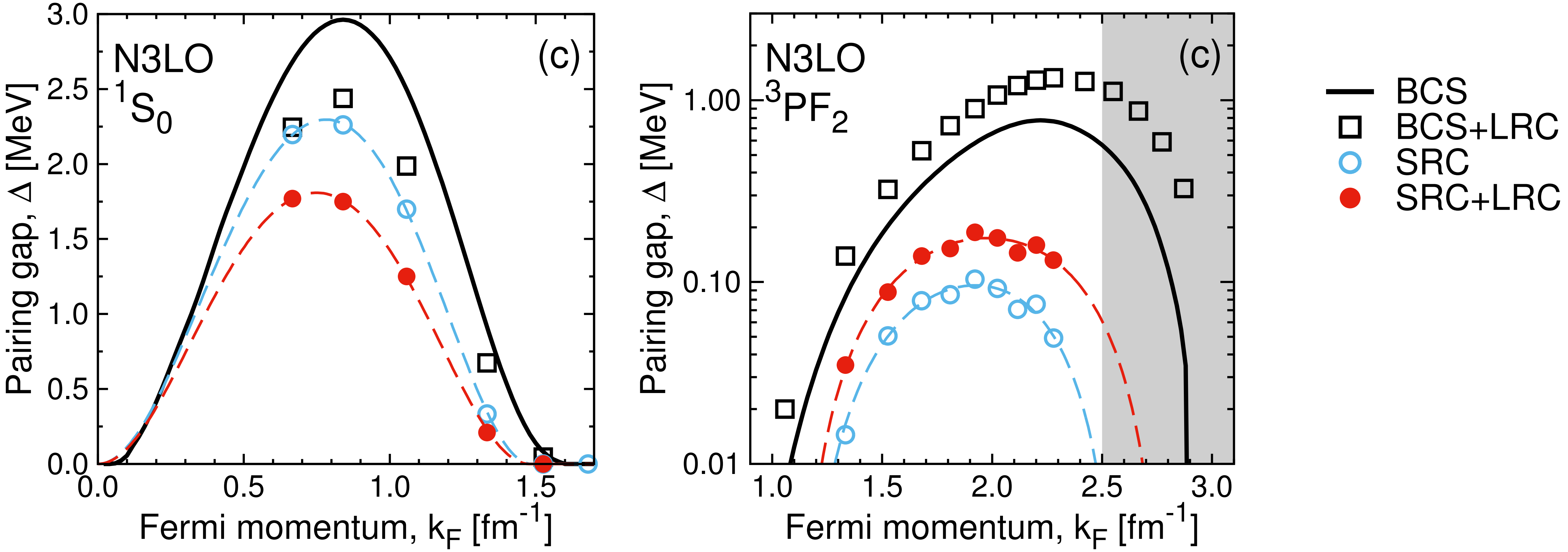
Beyond BCS 201: results $^1S_0$

Neutron matter



- LRC $^1S_0$ ($^3PF_2$) produces (anti-)screening

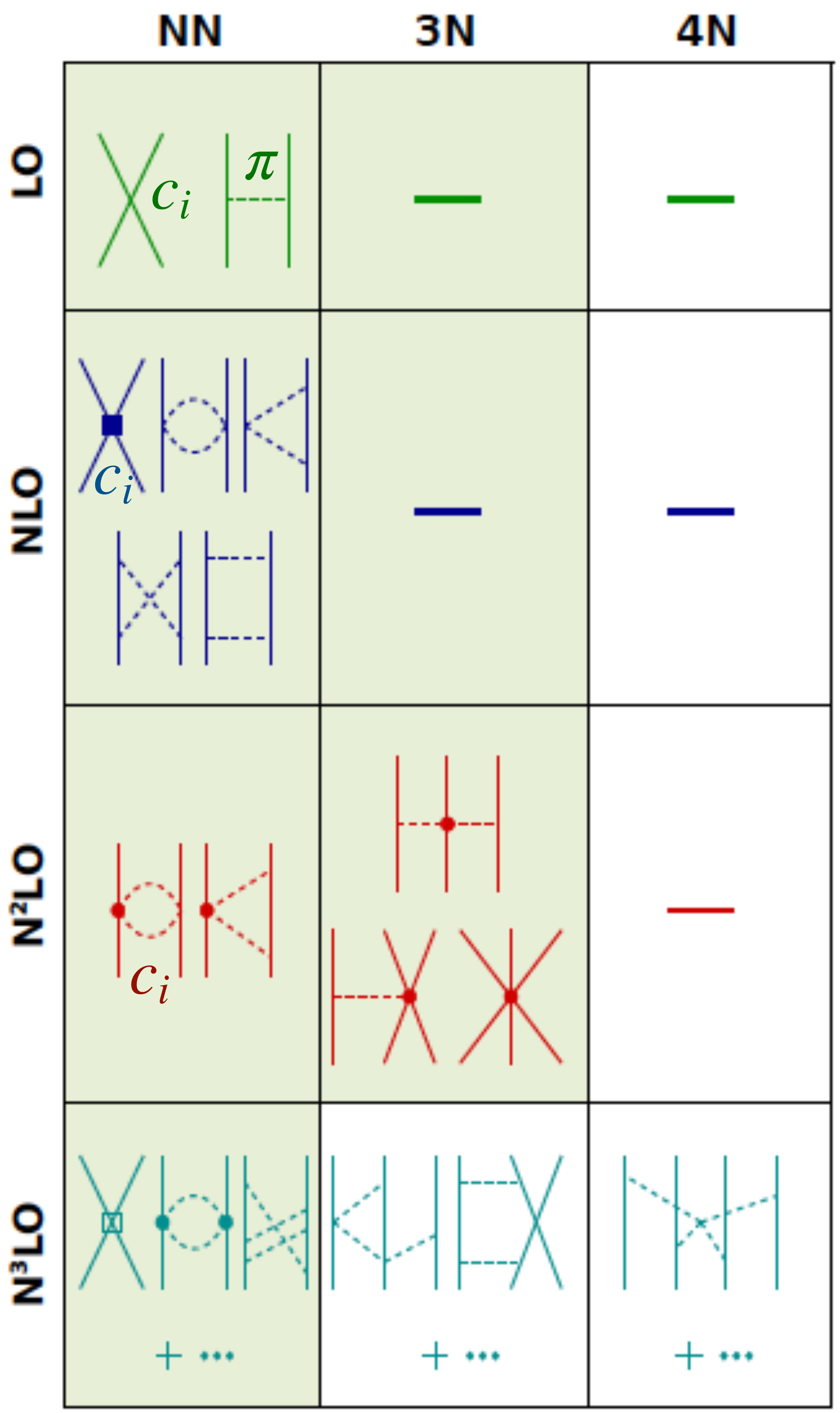
Summary



$$\Delta_L^{JST}(k_F) = \Delta_0 \frac{(k_F - k_0)^2}{(k_F - k_0)^2 + k_1} \frac{(k_F - k_2)^2}{(k_F - k_2)^2 + k_3}$$

		Δ_0	**k_0**	**k_1**	**k_2**	**k_3**
Singlet	CDBonn	18.18	0.05	1.39	1.45	0.81
	Av18	14.07	0.04	1.00	1.44	0.78
	N3LO	5.85	0	0.46	1.48	0.42
Triplet	CDBonn	0.41	1.03	0.56	2.81	1
	Av18	0.17	1.1	0.35	2.18	0.05
	N3LO	0.6	1.11	0.69	2.79	0.53

NN forces from EFTs of QCD



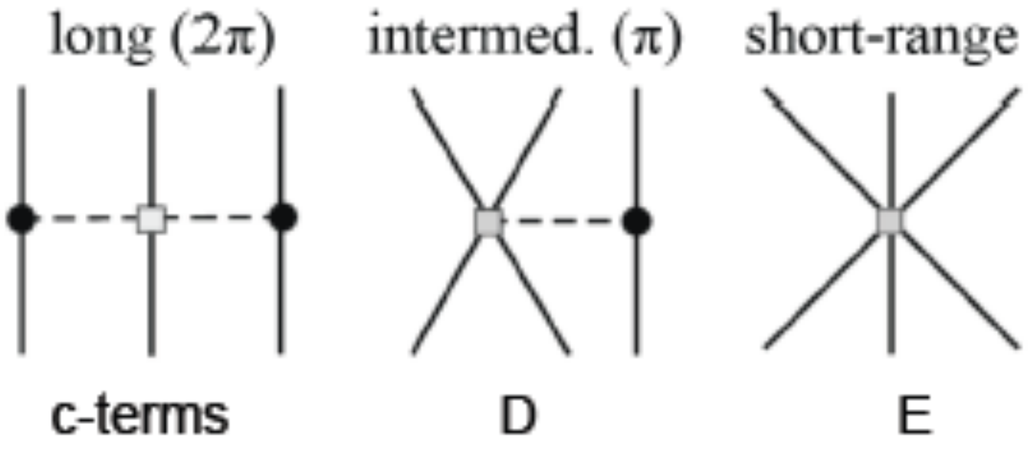
Robert Roth – TU Darmstadt – 04/2013

Chiral perturbation theory

- π and N as dof
- Systematic expansion
- 2N at N^{3}LO - LECs from πN, NN
- 3N at N^{2}LO - 2 more LECs
- (Often further renormalized)

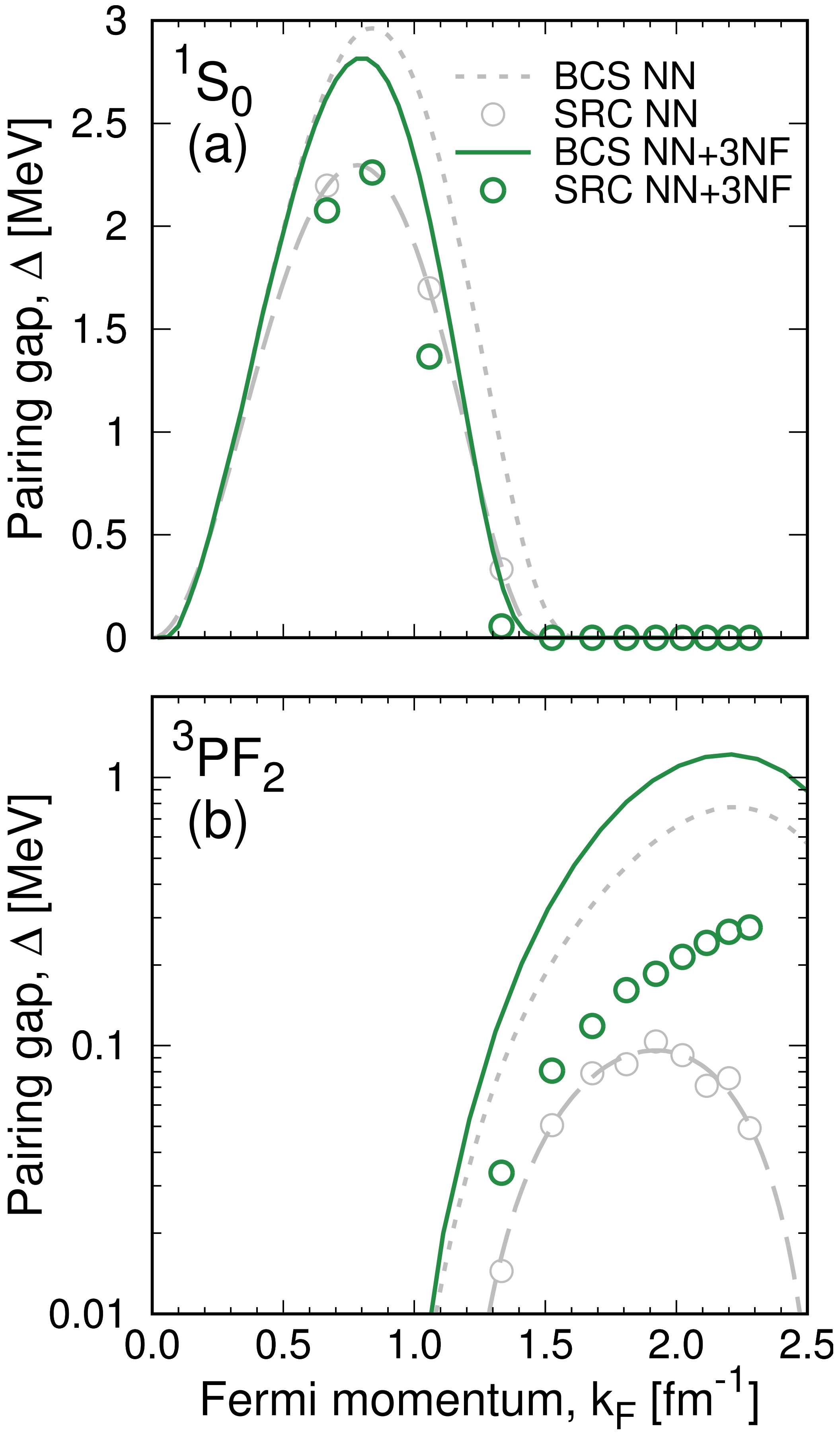
$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1\ \mathrm{GeV}$$

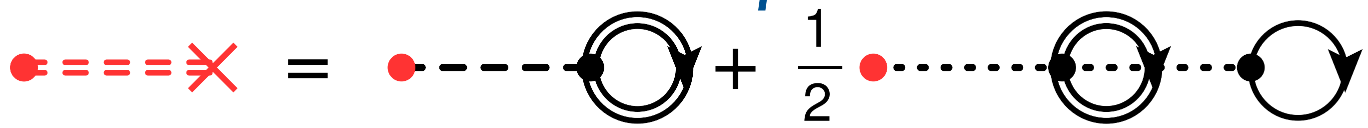


Weinberg, *Phys. Lett. B* **251** 288 (1990), *Nucl. Phys. B* **363** 3 (1991)
Entem & Machleidt, *Phys. Rev. C* **68**, 041001(R) (2003)
Tews, Schwenk *et al.*, *Phys. Rev. Lett.* **110**, 032504 (2013)

3BF effect: estimate



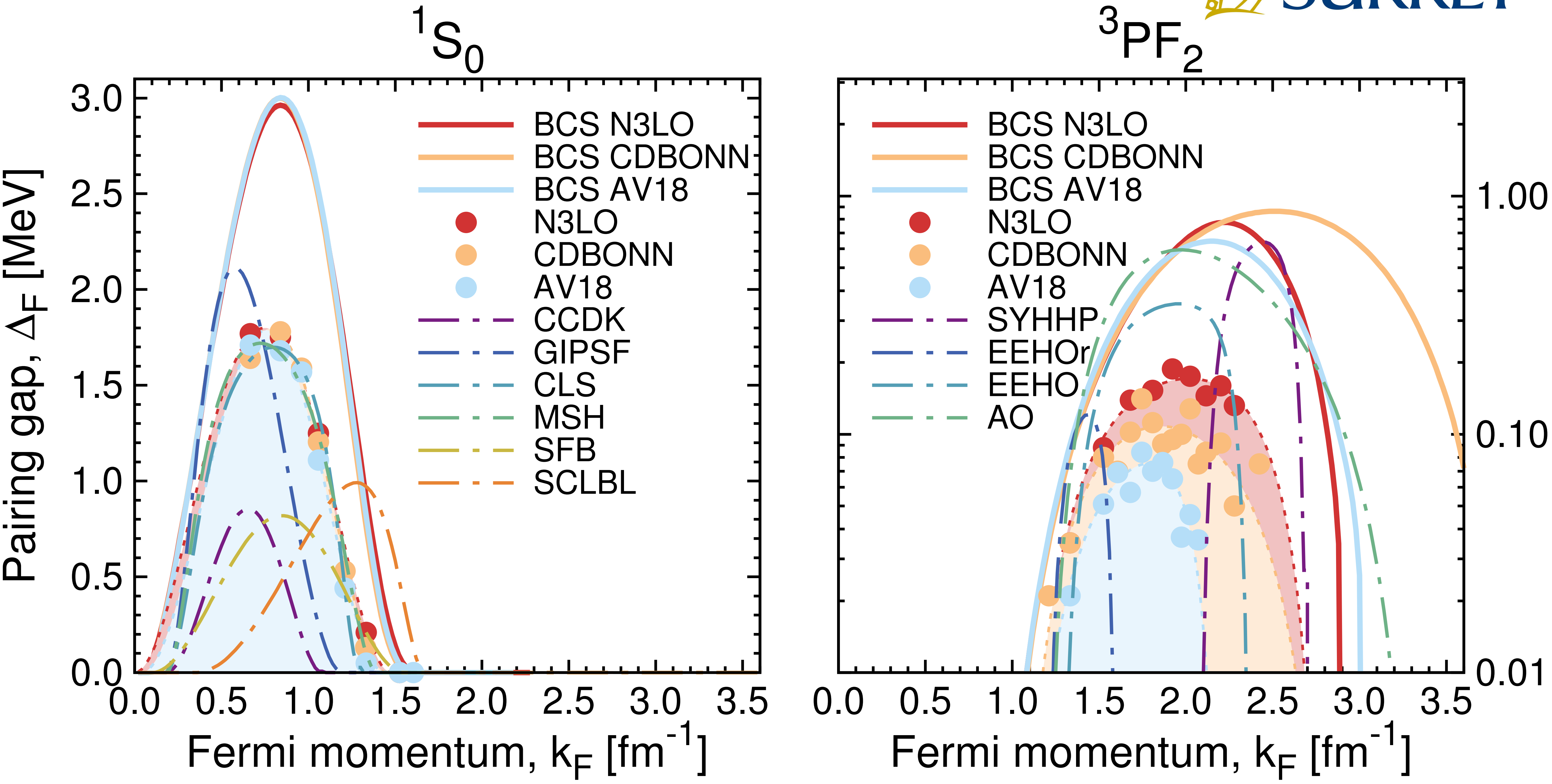
Effective one-body force ⇒*spectrum*



Effective two-body force ⇒*NN forces*

- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap
- Model dependence to be explored
- SRG dependence for systematics

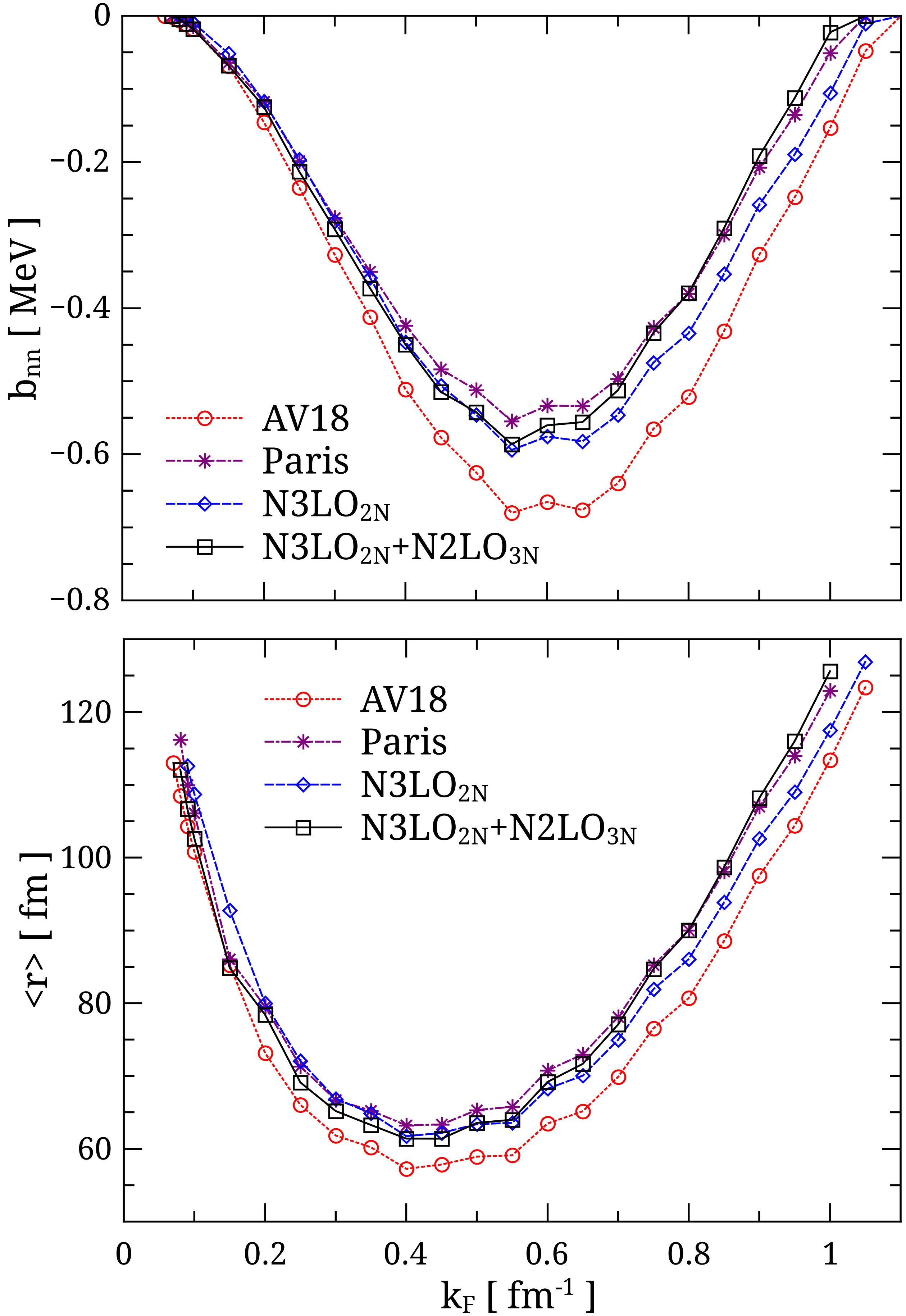
Beyond BCS pairing: overview



- Effect is **robust**: independent of NN potential
- 3NF effect **not** included in SRC, BCS indicates **small**
- **Singlet** channel under control in **astrophysical** situations

Dong, Rios, Dickhoff & Polls, *in preparation*

Dineutrons



BHF G-matrix

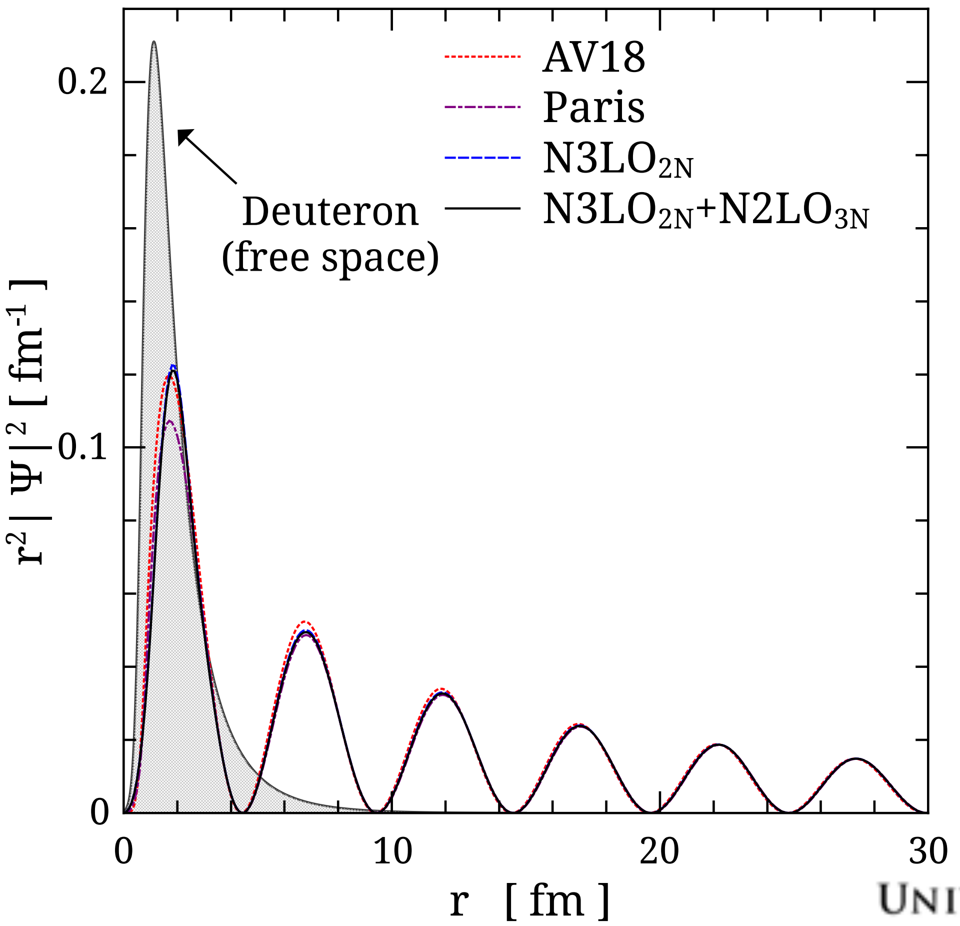
$$\mathcal{G}(\omega) = \mathcal{V} + \mathcal{V}Q(\omega)\mathcal{G}(\omega)$$

Poles

$$\det\left[1 - \mathcal{V}\mathcal{Q}(\omega_i)\right] = 0$$

$$b_{\mathrm{nn}} = \omega_i - \omega_{\mathrm{th}}$$

k_F=0.6 fm^{-1}



Isaule, Arellano, Rios, *in preparation*

Collaborators

- +A. Carbone
- +D. Ding, W. H. Dickhoff
- +A. Polls
- +C. Barbieri
- +V. Somà
- +H. Arellano, F. Isaule

TECHNISCHE
UNIVERSITÄT
DARMSTADT

Alexander von Humboldt
Stiftung/Foundation

WASHINGTON UNIVERSITY IN ST. LOUIS · 1853 ·

UNIVERSITAT DE BARCELONA

UNIVERSITY OF
SURREY

cea
Saclay

UNIVERSIDAD DE CHILE

Conclusions

- Ab initio nuclear **theory** to treat **correlations**
- **Talk** to us if you need **quantitative** predictions!

- **Different** NN forces give robust predictions

- Challenges ahead:
  - **Pairing** in isospin **asymmetric** matter
  - **Consistent** treatment of **cooling**, **glitch** & **EoS**
  - Improvements of **many-body theory**

a.rios@surrey.ac.uk
@riosarnau