Quantum Gravity from Conformal Field Theory

Hynek Paul*, based on work with F. Aprile, J. M. Drummond, P. Heslop

*H.Paul@soton.ac.uk

UNIVERSITY OF Southampton

Problem: missing theory of quantum gravity

As of now, there are two incredibly successful theories which describe different physics in completely different regimes of validity:

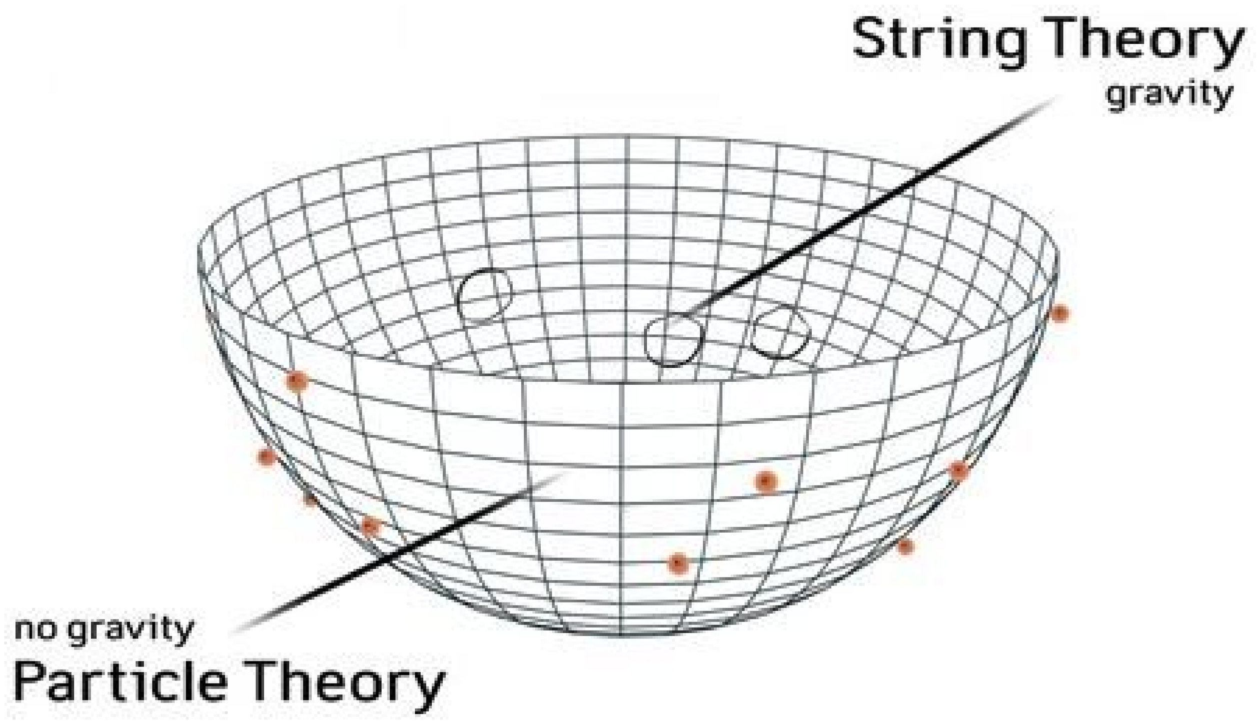
- **Standard Model of Particle Physics (SM)**
  A relativistic quantum theory unifying three out of the four fundamental forces (electromagnetism, strong and weak nuclear force) describing interactions of elementary particles at microscopic scales (e.g. particle collisions studied at CERN).
- **General Theory of Relativity (GR)**
  Describes the gravitational force on macroscopic scales (e.g. movement of satellites, planets, stars, black holes).

However, there is still an **unresolved problem**: we don't have a consistent theory of quantum gravity, i.e. it is not known how to unify gravity with quantum mechanics. At the moment, the most promising candidate for such a theory is **string theory**.

Holographic principle (AdS/CFT correspondence)

The AdS/CFT correspondence is a conjectured duality/equality between two very different theories:

- a d dimensional 'particle theory' with **no gravity**
- a $d+1$ dimensional string theory **with gravity**



[1]

Simplest example: $\mathcal{N}=4$ SYM

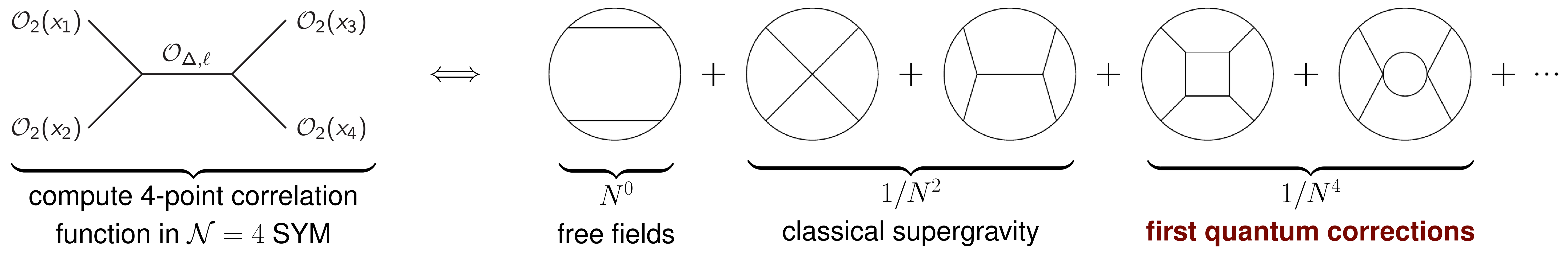
The best-studied example of the holographic principle is the following correspondence between two mathematical frameworks [2]:

- a 4 dimensional supersymmetric theory called **'$\mathcal{N}=4$ super Yang-Mills'** (SYM), which is a more symmetric version of the theory describing the strong nuclear force. Thanks to **maximal supersymmetry** (i.e. all of the particles in this theory are reorganised into one object called 'supermultiplet') this toy-model enjoys many special features which simplify the computations.
- its 5 dimensional holographic counterpart is type IIB **superstring theory** on $AdS_5 \times S^5$, containing supergravity in 5 dimensions as its low-energy theory.

In a special limit of this duality (at strong coupling and at large N), the superstring theory reduces to supergravity and one can compute quantum corrections to supergravity by studying SYM theory.

My research: computing 'masses' of bound supergravity states

Using the AdS/CFT correspondence we can make **predictions about quantum corrections to supergravity** on $AdS_5 \times S^5$ by mapping the problem to a computation in $\mathcal{N}=4$ SYM. Concretely, we study 4-point correlation functions (which can be thought of as the probability for a 2 $\to$ 2 particle scattering process) of gravitons and its Kaluza-Klein modes (denoted by $\mathcal{O}_p$, $p \geq 2$) in a $1/N$ expansion:



Making use of the superconformal symmetry of $\mathcal{N}=4$ SYM (one of the special features of this theory), we can apply the operator product expansion and analyse the spectrum of exchanged operators $\mathcal{O}_{\Delta,\ell}$, which turn out to be double-trace operators only. In the gravity theory, this corresponds to **gravitationally bound states** and we can compute their mass and binding energy.

Results and future applications

The exchanged operators can be unprotected and are of the form

$$\mathcal{O}_{pq} = \mathcal{O}_p \partial^\ell \Box^{\frac{1}{2}(\tau-p-q)} \mathcal{O}_q .$$

The **first correction to the mass** m_{pq} (or to be precise, the scaling dimensions Δ_{pq}) of these bound states can be computed to be [3]

$$m_{pq} \sim \Delta_{pq} = \Delta_0 - \frac{2}{N^2} \frac{2 M_t^{(4)} M_{t+\ell+1}^{(4)}}{\left(\ell + 2p - 2 - a - \frac{1+(-1)^{a+\ell}}{2}\right)_6} + \ldots ,$$

where

$$M_t^{(4)} \equiv (t-1)(t+a)(t+a+b+1)(t+2a+b+2).$$

As expected, the first correction is negative, justifying the interpretation of gravitationally bound states.

In the future, it would be interesting to apply our methods to different theories and maybe at some point make predictions for more realistic theories with less supersymmetry.

References

[1] https://www.learner.org/courses/physics/visual/img_lrg/AdSCFT.jpg. (2018). [image].

[2] J. M. Maldacena, Int. J. Theor. Phys. **38** (1999) 1113 [Adv. Theor. Math. Phys. **2** (1998) 231] [hep-th/9711200].

[3] see our papers: [arXiv] 1706.02822, 1706.08456, 1711.03903, 1802.06889

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