Understanding quantum gravity

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Introduction to quantum gravity

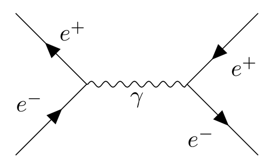
The two crowning achievements of theoretical physics in the 20th century were general relativity (GR) and quantum field theory (QFT). GR describes the force of gravity as the curvature of space, QFT describes the interaction of fundamental particles via the electromagnetic, weak nuclear and strong nuclear forces within that space. The combination of these two theories would be the crowning achievement of theoretical physics in the 21st century.

What is a quantum field?

Quantum field theory is the study of how 'particles' such as electrons and photons interact at the most fundamental level. Originally these were regarded as 'point particles'; objects which could no longer be further divided. With the discovery of quantum mechanics however it was realized that instead of a set position at a point these objects were more like 'probability clouds'. The greater the density of the cloud the more likely a particle is to be found there. When you consider this for multiple particles across all of space this cloud becomes a 'quantum field', imagine this field as a sea across all of space and the taller the wave the more likely the particle is going to be discovered there. We can understand a QFT by writing it's 'Lagrangian', this encodes all of the particles in a theory and how they interact with each other.

$$\mathcal{L}_{Higgs} = \frac{1}{2}\,\partial_\lambda\phi\,\partial^\lambda\phi - \frac{\mu^2}{2}\,\phi^2 + \frac{\lambda}{4!}\,\phi^4$$

These interactions can then be used to construct 'Feynman diagrams' which are used to aid calculations, the one below for example describes an electron and a positron annhiliating each other into a photon before the photon splits into an electron and positron pair.

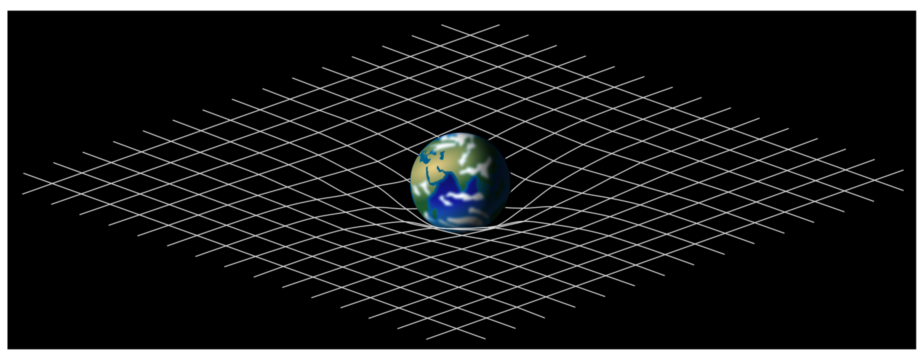


How do we describe gravity?

Einstein's greatest discovery was general relativity with the equation bearing his name,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

This equates the matter of the universe on the right with the curvature in space it creates on the left; we experience gravity as the path along this curved surface. GR was founded on the basis of the 'equivalence principle': that on small scales acceleration in a gravity free environment is indistinguishable from the effects of gravity. As a consequence we can treat the effects of gravity as a body moving on a curved surface. We use GR to understand processes such as black hole collisions, large scale structure and the extremely early times of the universe, as well as things closer to home such as communicating with satellites and making sure your GPS works properly.



How do we combine quantum field theory with gravity?

With great difficulty, but why is this? When we describe a QFT we are limited in the 'size' of our interaction terms and we can't exceed a size of 4 otherwise the theory breaks down. For example if we say a field ϕ has size 1, the largest interaction we can have is ϕ^4. The problem with gravity is that all of our interactions have a size of 5 or greater!

$$\mathcal{L}_{Einstein-Hilbert} = \frac{c^4}{16\pi G}\int\sqrt{-g}R\ \ d^4x = \frac{c^4}{16\pi G}\int(\partial_\lambda H_{\mu\nu})^2 + (H)^n\partial H\partial H\ \ d^4x$$

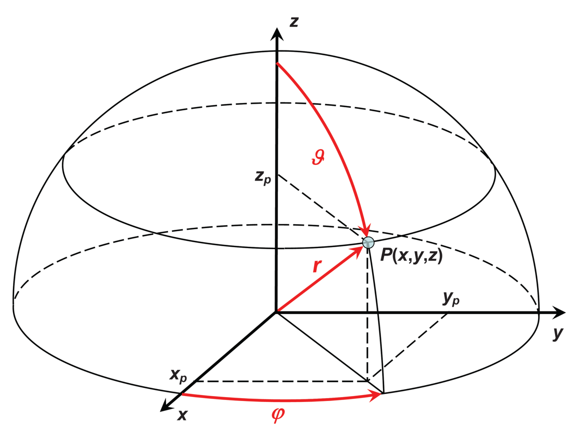
In the above equation the field H is our 'graviton', this is the force carrier for gravity like the photon is for electromagnetism.Because of the fundamental way we describe QFTs we can't use general relativity which so far is the best way to describe gravity. There are suggestions that general relativity is a low energy approximation of a more fundamental theory, one which is a QFT or goes beyong this framework. Such theories include string theory, loop quantum gravity and causal set theory.

A novel approach to combination

Since our difficulties stem from our interactions being too 'large', we can resolve this by introducing a factor to reduce the 'size' of our interactions. This factor has negative size and so ensures the overall interaction has size 4 which is what we need to have a well defined QFT.

$$\mathcal{L}_{EH} = \frac{c^4}{16\pi G}\int(\partial_\lambda H_{\mu\nu})^2 + f_n(\phi)\ \ (H)^n\partial H\partial H\ \ d^4x$$

The n in this new factor means that as the interaction term gets larger this factor will get correspondingly more negative, ensuring the correct size of these terms. In QFT we are also limited in the types of interactions we are allowed following something called 'symmetries'. A symmetry means that our Lagrangian must look the same after a transformation, for example in the Higgs Lagrangian above one symmetry could be $\phi \to -\phi$. Certain types of symmetries correspond to a fundamental force, the symmetry of gravity is 'diffeomorphism invariance' which essentially states that the laws of the universe are the same regardless of our choice in describing their position.



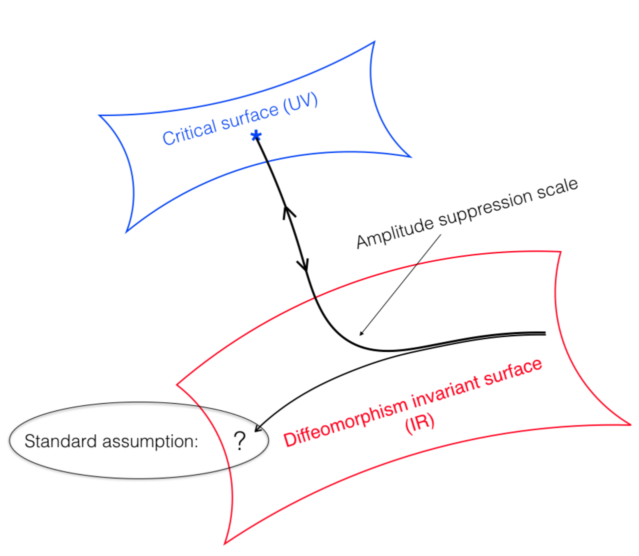
When we attempt to combine QFT with GR we have to respect these two demands, making sure the size of our interactions aren't too large as well as respecting the symmetry. These demands restrict what interactions are allowed but once we have the Lagrangian the world is our oyster!

1. Renormalization group properties in the conformal sector: towards perturbatively renormalizable quantum gravity - T.R. Morris 2. Quantum gravity, renormalizability and diffeomorphism invariance - T.R. Morris 3. BRST in the Exact RG - T.R. Morris, Yuji Igarashi & Katsumi Itoh 4. Polar co-ordinates diagram from SEOS project, https://seos-project.eu/laser-rs/laser-rs-c03-s01-p01.html, 5. Earth-GR diagram from BABW.com

WRG and the conformal factor instability

Here we discuss the research undertaken in a more complete manner. When describing a UV complete QFT in the language of the Wilsonian renormalization group (WRG) we describe such a theory as a CFT with (marginally) relevant perturbations only. The problem is that Einstein-Hilbert only has operators which are irrelevant. We resolve this problem in a natural way via the introduction of 'tower operators' which are a consequence of the conformal factor instability. This addresses this non-renormalizability directly. When expanding the metric in the Einstein-Hilbert action under the decompositions $g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$ and $H_{\mu\nu} = h_{\mu\nu} + \frac{1}{2}\delta_{\mu\nu}\phi$ we find the dilaton has a negative sign in the kinetic term.

$$\mathcal{L}_{EH} \supset \frac{1}{2}(\partial_\lambda H_{\mu\nu})^2 - \frac{1}{2}(\partial_\lambda\phi)^2$$



Often this minus sign is circumvented via $\phi \to i\phi$ however we choose to maintain it. We should define the space of our operators and demand that they are square integrable under the appropriate measure. In the typical case the kinetic term has a plus sign leading to a measure term with a negative exponent, resulting in polynomial operators. The dilaton sector however will have a measure with a positive exponent which does not permit polynomials.

$$\int_{-\infty}^{\infty}\mathcal{D}\phi\ \ e^{\frac{\phi^2}{\Lambda^2}}\ \ \mathcal{O}_n(\phi)\mathcal{O}_m(\phi) = \delta_{nm}.$$

This yields our tower operator $\delta^{(n)}_\Lambda(\phi) := \frac{\partial^n}{\partial\phi^n}\delta^{(0)}_\Lambda(\phi)$ with $\delta^{(0)}_\Lambda(\phi) := \frac{1}{\sqrt{2\pi\hbar\Lambda^2}}\ \ exp(-\frac{\phi^2}{2\hbar\Lambda^2})$. As a consequence of this we implement diffeomorphism invariance as a limit, not as a symmetry that was extant initially, see the above diagram.