Anomalous Supersymmetry

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1. Symmetries

A symmetry of a physical system is any set of transformations that leave some properties of that system invariant. A simple example is that of the reflection symmetry of a picture, where the left and right sides look like mirror images of each other.

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The symmetry principle plays a crucial and fundamental role in physics, since it puts constraints on the allowable dynamical laws we use to describe nature. Symmetries in nature imply the existence of conservation laws. The energy conservation for example, is a consequence of the time translation symmetry, i.e. the hypothesis that all physical laws remain the same throughout history.

2. Classical and Quantum Symmetries

In a classical theory, the equations that describe the behaviour of a physical system are derived from the Lagrangian, a mathematical function that contains all the information about the dynamics of the system. A symmetry of this theory (classical symmetry) is defined as any transformation that leaves the Lagrangian invariant.

At the quantum level the notion of symmetry is slightly more complicated. In quantum theories, we determine probabilities for the time evolution of physical systems by computing quantities known as correlation functions. It is quite often that the result of these quantum computations, turns out to be infinite! Then, we have to suitably obtain the quantum observables such that they are finite with a procedure called renormalization. In order to have a quantum symmetry, the original classical symmetry of the Lagrangian has to be preserved by the renormalization procedure.
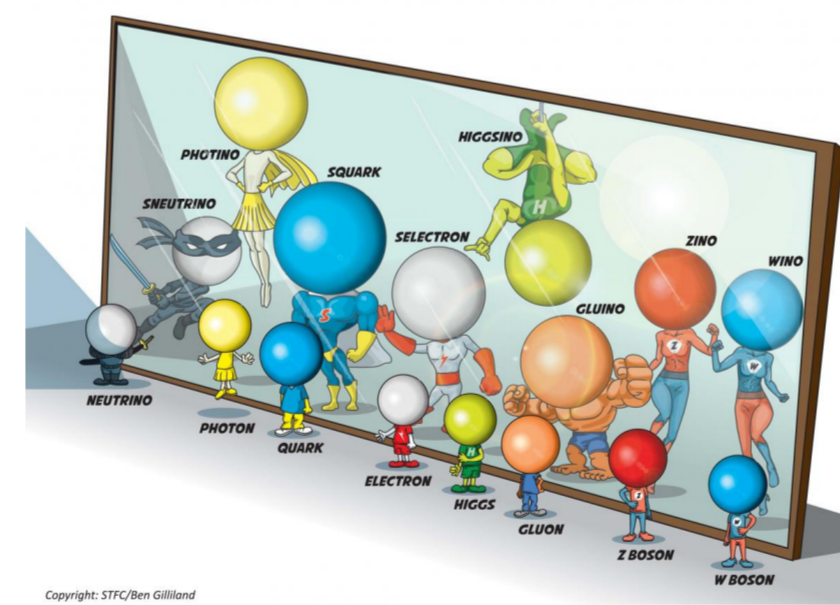
3. Anomalies

Sometimes classical symmetries fail to survive the quantization procedure. This means that the original symmetries of the classical theory are not actual symmetries of the full quantum theory. We call this phenomenon a quantum anomaly. Since the discovery of the axial anomaly by Adler, Bell and Jackiw in 1969, the anomalies played a central role in theoretical physics. In some cases the existence of (gauge) anomalies lead to inconsistencies, so their cancellation help us build viable physical models. In other cases (global) anomalies are linked to observable effects and explain experimental data, since they allow classically forbidden processes to occur. One such example is the pion decay into two photons.

4. Supersymmetry

In nature there are two classes of elementary particles. The bosons (e.g. photon) which have an integer-valued spin, and the fermions (e.g. electron) which have a half-integer spin. Fermions are usually associated with matter, whereas bosons are generally force carrier particles.
Supersymmetry is a symmetry that relates these two classes of particles. In a supersymmetric quantum theory every particle has a corresponding supersymmetric partner of the other type of spin.

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Scientists have not yet found evidence for supersymmetry in particle colliders, but physicists believe that supersymmetry may be important at higher energies than our colliders can reach.

5. Anomalies in Supersymmetric Theories (My research)

Recent arguments imply the existence of new global anomalies in supersymmetric quantum theories. In my research I use the simplest supersymmetric model that contains one bosonic and one fermionic particle and compute the appropriate quantities (correlation functions) to prove the validity of these new results. I also study the physical implications of these supersymmetry anomalies.

6. Results (for the experts)

We consider the free massless Wess-Zumino model

$$S = \int d^4x \left[-\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2}(\bar{\psi}\gamma^\mu \partial_\mu \psi) \right]$$

which is invariant under supersymmetry transformations of the form

$$\delta\psi \sim \phi \,, \qquad \delta\phi \sim \psi$$

and U(1) field transformations. The corresponding conserved currents are the supercurrent S^μ and the R-current R^μ.
We compute the 4-point correlation function of two supercurrents and two R-currents $\langle T(S^\mu(x_1)\bar{S}^\nu(x_2)R^\kappa(x_3)R^\lambda(x_4))\rangle$.
The Feynman diagrams for the connected part of this correlation function have the following form:
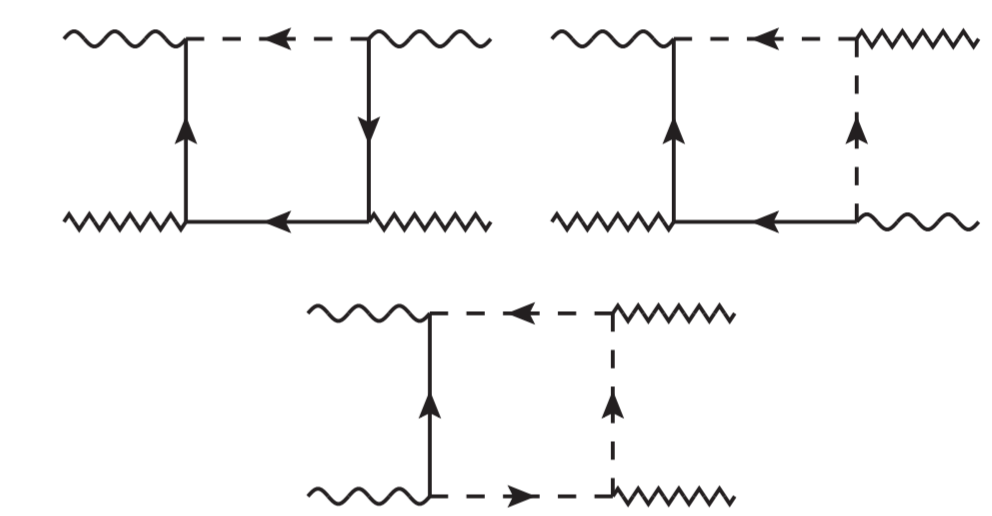
All the diagrams are linearly divergent. After regulating with either momentum cut-off or Pauli-Villars we find that the above 4-point correlation function is anomalous, i.e.

$$\partial_\mu \langle T(S^\mu(x_1)\bar{S}^\nu(x_2)R^\kappa(x_3)R^\lambda(x_4))\rangle = A_Q^{\nu\kappa\lambda}$$

where $A_Q^{\nu\kappa\lambda}$ is the supersymmetry anomaly. This new anomaly may have implications in phenomenology, early universe cosmology and localization computations.

References

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