Consumer default with complete markets: risk-based pricing and finite punishment*

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Abstract

This paper studies economies with complete markets where there is positive default on consumer debt. Households can default partially, at a finite punishment cost, and competitive intermediaries price loans' default risk individually. This environment yields only partial insurance. The risk-based pricing of debt makes it too costly for the borrower to achieve full insurance and there is too little trade in securities. Consumption, as well as debt, are positively correlated with idiosyncratic changes in income. This approach is in contrast with existing literature. Unlike the literature with default, there are no restrictions on the set of state contingent securities that are issued. Unlike the literature on lack of commitment, limited trade arises without debt constraints that rule default out. The present approach seems to imply more consumption inequality than the latter.

Keywords: consumer default, complete markets, endogenous incomplete markets, risk-based pricing, risk sharing

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1 Introduction

This paper studies consumer credit default in an economy with complete financial markets. In spite of the evidence of consumer credit delinquency and bankruptcy, the existence of positive default within this basic framework has received little attention in the literature on limited commitment. Works that consider complete markets rule out positive default; works that study equilibrium bankruptcy restrict the set of assets that can be traded. In this paper, positive default coexists with complete markets. This environment provides a novel explanation for financial frictions (or incomplete trade) and has implications for understanding the evidence on consumption inequality.

The first objective of this paper is to put forward a simple and tractable model that is suitable for the analysis of default with complete markets.[1] The approach rests on the existence of partial default and individualized risk-based pricing. The second objective is to identify, in that context, reasons why default matters for the allocation of resources, even in a frictionless competitive environment. It will study the consequences of default for risk sharing (i.e., insurance) and intertemporal consumption smoothing. The third objective is to assess the practical significance of this risk-pricing approach. We will compare its implications for consumption inequality with those from a model which – in the vein of much literature on endogenous incomplete markets – has debt constraints that rule out default.

Some motivation for entertaining partial default is in order. In the U.S., formal bankruptcy filings have attracted much analysis and discussion, yet default outside formal bankruptcy procedures is substantial. [2] Informal bankruptcy might account for the bulk of loan write-offs, a point already made by Dawsey and Ausubel (2004). Unlike the full discharge of unsecured debts in formal procedures like Chapter 7, informal bankruptcy is best seen as involving debtors failing to repay a chosen part of their liabilities. The significance of informal bankruptcy supports the present focus on partial default. Similarly, in the context of sovereign debt, partial default seems to be the norm.[3]

The simple general equilibrium model consists of a two-period endowment economy populated by two types of households who are subject to idiosyncratic endowment risk in the

[1] An early word on terminology is in order. Here, as in much literature on limited commitment (e.g., Kehoe and Levine (1993), Alvarez and Jermann (2000)) we speak of complete markets as the availability of securities spanning the space of promised deliveries. This is a narrower notion than in Dubey, Geanakoplos, and Shubik (2005) where tradeable assets are defined also over the level of default punishments, including infinite penalties. See below for further discussion.

[2] This is the theme of recent empirical and theoretical studies in Dawsey and Ausubel (2004), Dawsey, Hynes, and Ausubel (2008) and Chatterjee (2010). In the Survey of Consumer Finances 2007, about 1% of the U.S. population had filed under Chapter 7, whereas over 5% held delinquent loans. See Díaz-Giménez, Glover, and Ríos-Rull (2011).

[3] See, for example, Sturzenegger and Zettelmeyer (2007). It is then to be noted that the analysis of sovereign debt since Eaton and Gersovitz (1981), including more recently Arellano (2008), Benjamin and Wright (2009), and Yue (2010), appears to consider default only as a binary choice.

second period. Regarding financial markets, agents can borrow and lend freely through perfectly competitive financial intermediaries, and have access to a full set of securities. Borrowers can default fully or partially on their promised deliveries. Default carries a utility penalty to the household which depends on the scale of default. The same penalties apply in all contracts. Because different levels of debt are associated with a different default rate, loans of different size command a different price. In order to characterize the behavior of intermediaries and households, one needs to account for the price schedule associated with all possible contracts, including those that are not traded in equilibrium. Using specific functional assumptions, we characterise the equilibrium and draw analytical and numerical results.

A preliminary question begs some discussion. How can positive default arise with complete assets? The common view is that default happens as the result of a contingency against which a contract cannot be written. Lenders are still willing to trade since ex-ante they can write off the bankruptcy losses against gains in other states. With one security available for each contingency, however, there is no room for such compensation across states on any one asset. If default is on the totality of debt then the asset will not be traded. That default can be partial is therefore essential for bankruptcy to arise in this paper. It involves an ex-ante known proportion in each particular state – rather than a probability distribution over states – of debt going unpaid. Contracts will thus be ex-post incentive compatible.[4]

The key result is that bankruptcy on its own has real consequences for the allocation of consumption. The reason is that prices reflect the risk of different contract sizes in a way that affects the borrowing and lending decisions in equilibrium. Specifically, the price of securities declines with the value of the promised deliveries. Since a borrower's marginal gain in terms of current consumption from issuing debt is then less than its price – a price wedge – an economy with endogenous bankruptcy will feature less trade in assets. Specifically, the possibility of bankruptcy implies imperfect consumption risk sharing and suboptimal intertemporal consumption smoothing. Across states of nature in the second period, individual consumption varies positively with income. If, as it is arguably the case, penalty levels vary positively with household income, the model can also account for default decreasing with income.

It is critical that the default decisions are endogenous and that contracts are priced individually. If default rates for each individual type and contingency were exogenous then the price of claims would be given to households and allocations would be optimal, featuring full risk sharing. If contracts for each contingent state were anonymous pools, with the same price for asset sales of different size, then the price faced by an individual borrower would not vary with the amount of securities sold and there would be no price wedge, yielding full risk sharing.[5]

[4]Notice that, in this form, bankruptcy can happen in a deterministic economy.
[5]With assets are treated as pools, partial default would emerge similarly though.

In order to assess the implications for consumption inequality in this risk-pricing model, we consider a debt-constrained version of the economy à la Kehoe and Levine (1993) as a comparison benchmark. This obtains naturally if the penalty for defaulting is fixed regardless of its scale. Default, which becomes in effect an all-or-nothing choice, is ruled out and liabilities cannot exceed a certain endogenous debt limit. The source of imperfect risk sharing is the same lack of commitment in the two models, yet the mechanisms are quite different. For a class of symmetric economies, the risk-pricing model encompasses less risk sharing and more consumption inequality than the debt-constrained model. Illustrative numerical examples support this result more generally. Interestingly, this suggests that default makes markets more, rather than less, incomplete.

In considering complete markets, this paper is related to the large and influential literature on limited enforcement with complete markets. This includes Kehoe and Levine (1993), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri (2006), and Kehoe and Perri (2002). Like those works, we also derive endogenous incomplete insurance given a full set of securities, but this is done without imposing an exogenous participation constraint or not-too-tight borrowing limits. Furthermore, in the present paper there is positive default in equilibrium, whereas these other papers rule out positive bankruptcy as an equilibrium outcome.[6] The existing literature has used that framework to address empirical facts on consumption and wealth inequality. As said, the approach here seems to have novel implications and should be relevant for understanding that evidence.

This paper is also related to the recent quantitative literature on consumer credit and bankruptcy. The works that undertake a quantitative general equilibrium analysis of incomplete markets include Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), and Livshits, MacGee, and Tertilt (2007), Mateos-Planas (2009), Mateos-Planas and Ríos-Rull (2010), Benjamin and Mateos-Planas (2011), and Athreya, Tam, and Young (2009). The present paper deals in a similar way with the pricing of default risk but does not rule out trade opportunities arbitrarily by restricting the set of assets available. Unlike our paper, these works also consider all-or-nothing default decisions.[7]

Dubey, Geanakoplos, and Shubik (2005) present a model of default accommodating various specifications of the tradeable set of assets. In the present paper, the set of tradeable assets corresponds to a specification with Arrow securities and finite uniform default punishment across assets.[8] In contrast with the individualized risk-based pricing of assets of the present

[6]There is also a literature on endogenous limits with incomplete markets that similarly rules out positive default in equilibrium. This includes Zhang (1997), Mateos-Planas and Seccia (2006), Ábrahám and Cárceles-Poveda (2010), and Andolfatto and Gervais (2008).

[7]Kehoe and Levine (2006) brings together the two streams by studying an economy with incomplete markets and collateral constraints that reconciles the outcomes of the debt-constrained model with complete markets.

[8]See their example 1, sec.$\dot{6}$. In this general sense, this stops short of being complete markets as it fails

paper, that paper treats assets as pools. The asset holder does not know the identity or asset position of the assets seller, only the price and average default rate. As already indicated, with pooling, default would have no impact on risk sharing.[9] Compared with the present paper, further arbitrary exogenous restrictions on the tradeable set would have to be assumed in order to account for incomplete risk sharing under price pooling.

In the theoretical literature, default has been typically studied in models with incomplete financial markets. The emphasis has been on the role of default as providing some degree of insurance against individual risk. This is the case, for example, in Zame (1993) and, again, Dubey, Geanakoplos, and Shubik (2005). This paper studies instead situations where there is a complete set of securities and insurance considerations are abstracted from. The role of default in this context must be a distinct one.

2 A basic model

The economy lasts for two periods. The possible states are $s = 0, 1, , , , S$. In the first period the state is 0; in the second period the state of nature can be $s = 1, ..., S$. The probability of a state s in the second period is denoted as π_s. There are two types $i = A, B$ of individual households. The two groups are of the same size. These agents receive endowments of goods that depend on their type i and the state of nature $s = 0, 1, ..., S$, and are denoted by y_s^i. In the first period, households can borrow or save against second-period states $s = 1, ..., S$ using securities traded through banks. Let a_s^i denote the risk-free delivery values of the assets that pay in state s held by an individual of type i. Let l_s^i denote the promised delivery values of the loans held by an individual of type i that pay in state s. Let p_s denote the price of assets at 0. As for debts, since different levels of debt may carry a different risk of failure, the price at 0 of debt held by agent i with promised delivery value l in state s is a function of its face value $Q_s^i(l)$. A borrower can default on a fraction of their debt. Let d_s^i denote the fraction of promised repayments l_s^i that borrower i fails to deliver in state s. Individuals consume in each period their net available resources, and c_s^i denotes consumption in state s by an individual of type i.

Individual preferences are represented by a function of consumption and default. A borrower who defaults experiences a utility stigma loss z that depends on the rate of default incurred d_s^i. More specifically, the utility of an individual of type i is represented by

$$u(c_0^i) + \beta \sum_{s=1,...,S} \pi_s [u(c_s^i) - z(d_s^i)],$$

where $u(.)$ is the period utility function and $z(.)$ describes the penalty for default, and β is

to span the space of all possible penalties. We have ruled out penalties as a dimension of assets and, importantly, assets with infinite punishment. Allowing for such assets, the first best would always obtain.

[9] As long as default is not 100% causing trade in Arrow securities to collapse.

a discount parameter. We will often use specific functional form for preferences, with

$$\begin{aligned} u(c) &= \log c \\ z(d) &= \eta d^{\gamma} \end{aligned} \tag{1}$$

where η and γ are positive parameters accounting for the level and curvature of the penalty, respectively.

Banks intermediate the trade in assets and debts. Their lending activity carries a risk of default. Let $D_s^i(l)$ denote the default rate associated with a value l of promised deliveries in state s by agent i. The revenues to the bank for lending l claims against state s to agent i are then $(1 - D_s^i(l))l$. The costs to the bank consist of the delivery value of the deposits taken a and an intermediation cost, a proportion ν of the debt value. The bank's book balancing requires the value of deposits taken to equalise the value of loans made, that is $Q_s^i(l)l = p_s a$. Therefore the bank's cash flow is:

$$\left(1 - D_s^i(l) - \frac{Q_s^i(l)}{p_s} - \nu\right) l.$$

There is a market open for loans of every possible size. All markets clear under perfect competition and free entry in intermediation.

3 Equilibrium

The equilibrium determines the price of assets p_s and, for all tradeable debt sizes l, the price schedule $Q_s^i(l)$ and default risk risk schedule $D_s^i(l)$. There will be a single type of borrower i under each state s. All traded loans are of a particular size l_s^i and carry a single effective default rate $d_s^i = D_s^i(l_s^i)$ and price $q_s^i = Q_s^i(l_s^i)$, if $l_s^i > 0$.

In order to determine these specific realizations, one will need to understand the values associated with off equilibrium allocations for contracts that are not traded, as described by the mappings $Q_s^i(.)$ and $D_s^i(.)$. On one hand, consumers make their borrowing and default plans bearing in mind how the interest charged changes with the liabilities through $Q_s^i(.)$. On the other hand, banks bidding entry into the industry take into account the variation in the risk of default associated with loans of different size through $D_s^i(.)$. In this model, these off equilibrium expectations will be consistent with the assumption made that markets for non traded contracts are open.[10] We turn now to defining the equilibrium more precisely.

[10]This is not unlike Dubey, Geanakoplos, and Shubik (2005)'s refinement to prevent the existence of markets being ruled out by excessively pessimistic expectations.

One remark about the equilibrium is that borrowing or savings will only be positive for one agent in any state $s = 1, ..., S$. That is, if $l_s^i > 0$ then $a_s^i = 0$, $l_s^j = 0$, and $a_s^j > 0$. This allows us to use a sparse notation. The no-shortselling property is an equilibrium result which will be further discussed below.

3.1 Definition

An equilibrium for the above economy consists of the following objects for each state s, and for all agents i and $j \in \{A, B\}$: Traded prices q_s^i and p_s, price menus for debt $Q_s^i(.)$, default functions $D_s^i(.)$, portfolios a_s^j and l_s^i, and consumption allocations c_s^i and c_s^j, and default rates d_s^i. They satisfy the following conditions:

(i) Consumer maximisation: For each i, given p_s and $Q_s^i(.)$, the choices $a_s^i \geq 0$, $l_s^i \geq 0$ and d_s^i maximise utility for i subject to

$$c_0^i = y_0^i - \sum_{s\geq 1} p_s a_s^i + \sum_{s \geq 1} Q_s^i(l_s^i) l_s^i$$

and, if $l_s^i > 0$,

$$c_s^i = y_s^i - (1 - d_s^i) l_s^i, \;\; s = 1, ..., S$$

or, if $a_s^i > 0$,

$$c_s^i = y_s^i + a_s^i, \;\; s = 1, ..., S$$

(ii) Off-the-equilibrium default: For a given loan size $l > 0$, the value of the default function schedule $D_s^i(l)$ is the $\tilde{d}$ that maximises agent i's utility in state s, $u(c_s^i) - \eta(\tilde{d})$, subject to

$$c_s^i = \;\; y_s^i - (1 - \tilde{d}) l, \;\; s = 1, ..., S \;\; .$$

(iii) Bank competition: Given the default schedule $D_s^i(.)$, the price menu $Q_s^i(.)$ satisfies zero-profit on condition any potential credit contract size l.

(iv) Consistency: the traded price corresponds to the traded contracts

$$q_s^i = Q_s^i(l_s^i), \;\; l_s^i > 0$$

and the default rate corresponds to the traded contract

$$d_s^i = D_s^i(l_s^i), \;\; l_s^i > 0.$$

(v) Market clearing:

$$\sum_i c_0^i = \sum_i y_0^i,$$

$$\sum_i c_s^i + \sum_i \nu l_s^i = \sum_i y_s^i,$$
$$\sum_i q_s^i l_s^i = \sum_i p_s a_s^i.$$

In point(i) the budget constraint shows that the borrower i bears in mind the effect of the level of debt on the price through $Q_s^i(.)$. More specifically, if $l_s^i > 0$, for the borrower the optimality condition for debt is

$$u'(c_0^i)\left[q_s^i + Q_s^{i\prime}(l_s^i)l_s^i\right] = \beta u'(c_s^i)(1-d_s^i)\pi_s, \;\; l_s^i > 0 \tag{2}$$

where use has been made of the condition in (iv). Regarding the default choice,

$$u'(c_s^i)l_s^i = z'(d_s^i), \;\; l_s^i > 0 \tag{3}$$

For the lender with $l_s^i = 0$, the standard conditions for optimal savings holds, so

$$u'(c_0^i)p_s = \beta u'(c_s^i)\pi_s, \;\; l_s^i = 0 \tag{4}$$

Point (ii) describes the default behavior for arbitrary levels of debt. The optimality condition in this case is analogous to the condition for the equilibrium allocation (3), so $D_s^i(l)$ solves

$$u'(y_s^i - (1 - D_s^i(l))l)l = z'(D_s^i(l)) \quad \text{all} \quad l > 0. \tag{5}$$

Note that, because of separability of the utility function, the default menu has the convenient property that it does depend on the price schedule $Q_s^i(.)$. Point (iii) describes the zero-profit condition for all loan sizes. Given the expression for the bank's cash-flow presented earlier, this can be written more explicitly as:

$$1 - D_s^i(l) - \frac{Q_s^i(l)}{p_s} - \nu = 0 \quad \text{all} \quad l > 0. \tag{6}$$

The definition of the price of debt traded in equilibrium in point (iv) along with the fact just discussed that the realised default $d_s^i = D_s^i(l_s^i)$ if $l_s^i > 0$, permits writing an analogous expression specifically for the active intermediaries:[11]

$$1 - d_s^i - \frac{q_s^i}{p_s} - \nu = 0 \tag{7}$$

Point (v) describes clearing in the goods markets and financial markets, the latter implying $q_s^i l_s^i = p_s a_s^j$ when $l_s^i > 0$. Combining these market clearing conditions with the budget constraints in point (i) and the zero-profit condition (7), one can write the consumption

[11] Portfolios for an individual could not have both l_s^i and a_s^i positive. For the two (interior) conditions (2) and (4) to hold, it is required that $q_s^i/p_s > 1 - d_s^i$, a contradiction with (7).

allocation as follows:

$$\begin{array}{rcl} c_0^i & = & y_0^i + \sum_s p_s(y_s^i - \nu l_s^i - c_s^i) \\ c_s^i & = & y_s^i - (1-d_s^i)l_s^i, \quad l_s^i > 0 \\ c_s^j & = & y_s^j + (q_s^i/p_s)l_s^i, \quad l_s^j = 0 \end{array} \tag{8}$$

Note that (7) implies that the last equality can be written as $c_s^j = y_s^j + (1 - d_s^i - \nu)l_s^i$.

3.2 Characterisation

The set of equations (2), (3), (4), (7) and (8) forms a system in the endogenous variables consisting of prices p_s and q_s^i, debt promised deliveries l_s^i, default d_s^i, and consumption allocations c_0^i and c_s^i, for $i = A, B$ and $s = 1, ..., S$. However this system does not fully determine the outcomes because it cannot deal with the sensitivity of the price of debt to changes in the borrowing decision expressed in the derivative of the price schedule $Q_s^{i\prime}$ in (2). The two remaining equations (5) and (6) precisely describe the pattern for default and pricing which is needed to pin down this derivative. Therefore, in order to characterise the equilibrium, we will first study the properties of the price menu implied by (5) and (6).

To be specific, we will consider the functional forms for utility and stigma costs in (1). The default mapping $D_s^i(l)$ is determined from (4) as the solution to

$$\frac{l}{y_s^i - (1 - D_s^i(l)l)} = \gamma\eta D_s^i(l)^{\gamma-1}.$$

Computing its derivative $D_s^{i\prime}(l)$ and, using condition (2) with (7), evaluating it at the equilibrium l_s^i, one obtains:

$$D_s^{i\prime}(l_s^i) = \frac{1 - d_s^i + \frac{1}{\gamma\eta}{d_s^i}^{1-\gamma}}{\left(\frac{1}{\gamma\eta}{d_s^i}^{-\gamma} - 1\right)(\gamma - 1) + \gamma}\frac{1}{l_s^i}$$

Now the zero-profit condition (5), with (6), shows that the change in the price of debt with liabilities has the opposite sign, $Q_s^{i\prime}(l) = -q_s^i(1 - d_s^i - \nu)^{-1}D_s^{i\prime}(l)$. With this information, one can find an explicit expression for the change in the amount that one can borrow by incurring extra debt:

$$q_s^i + Q_s^{i\prime}(l_s^i)l_s^i = q_s^i(1 - d_s^i - \nu)^{-1}\left[\frac{{d_s^i}^{-\gamma}}{(\gamma-1){d_s^i}^{-\gamma} + \gamma\eta}((1 - d_s^i)(\gamma - 1) - d_s^i) - \nu\right] \tag{9}$$

This term (9) has to be positive in an equilibrium with positive borrowing and lending where (2) holds. By incurring more liabilities for tomorrow, the borrower must be able to raise more consumption today. Note that the default penalty being steep enough, or $\gamma > 1$, is a necessary condition for this to happen. On the other hand, this condition guarantees that, off the equilibrium, the default risk increases and hence the debt price decreases with the value of debt promised repayments.

Proposition 1 *Assume the functional forms in* (1). *Then* $q_s^i + Q_s^{i\prime}(l_s^i)l_s^i$ *is given by* (9). *In an equilibrium with positive borrowing its sign must be positive and so* $\gamma > 1$. *In such an equilibrium,* $D_s^{i\prime}(l_s^i) > 0$ *and* $Q_s^{i\prime}(l_s^i) < 0$.

Note $\gamma > 1$ is a weak condition for existence. We will often invoke from (9) the stronger condition that $\gamma - 1 - d_s^i/(1 - d_s^i) > 0$, which involves the endogenous default rates.

4 Risk sharing and smoothing

In this section, we discuss the properties of the equilibrium regarding risk sharing and the consequences of default for the intertemporal allocations of consumption. For each of the two issues, we proceed in two steps. We first consider the case where default is exogenous as the standard benchmark where there is full risk sharing and default does not matter. Then we study the model with endogenous default in order to establish and explain that these properties do not hold. This section will conclude with a discussion of the pattern of default and a comparison with an economy with assets' pooling.

To facilitate the discussion we will assume away intermediation costs and set $\nu = 0$. Also, we assume there is no aggregate risk so $y_s^i + y_s^j$ is a constant y for all s.

4.1 Risk sharing

Consider first the situation where the default rates d_s^i are exogenous. An equilibrium is still characterized by the above conditions except (3), (5) and (6), and with the property that risk does not depend on loan size or $Q_s^{i\prime}(l) = 0$.

In this setting with exogenous default, the first-order conditions (2) and (4), alongside the no-arbitrage condition (7) that $p_s = q_s^i/(1 - d_s^i)$, imply the equalization of the intertemporal marginal rates of substitution of the two agents in each state, that is $u'(c_s^i)/u'(c_0^i) = u'(c_s^j)/u'(c_0^j)$. On the other hand, the market clearing condition (8) implies that $c_s^i + c_s^j = y$. Combining these two properties,

$$\frac{u'(c_s^i)}{u'(y - c_s^i)} = \frac{u'(c_0^i)}{u'(c_0^j)}.$$

It follows that consumption of any agent is invariant to the state s. There is, in other words, complete risk sharing. The intuition for this result is standard. With default rates given exogenously, no-arbitrage prices fully account for the incomplete repayment of promised deliveries. No-arbitrage prices lead to the equalization of marginal rates of substitution across agents.

We turn now to the case with endogenous default. The equilibrium is described by equations (2), (3), (4), (7) and (8) with (9). Complete risk sharing fails to hold in this case. Although the same no-arbitrage condition stands, there is no equalization of marginal rates of substitution across agents. The reason is that the borrower household takes into account the endogenous response of the price to their loan choice. More formally, let i the agent that borrows against a particular state s. First, from intertemporal optimality in (2) and (4) we obtain, respectively,

$$\frac{u'(c_s^i)}{u'(c_0^i)} = \frac{q_s^i + Q_s^{i\prime}(l_s^i)l_s^i}{\beta(1-d_s^i)\pi_s},$$

and

$$\frac{u'(c_s^j)}{u'(c_0^j)} = \frac{p_s}{\beta\pi_s}.$$

Now the no-arbitrage condition (7) that $p_s = q_s^i/(1-d_s^i)$ does not imply equalization of marginal rates of substitution as long as $Q_s^{i\prime} \neq 0$. Combining these expressions with market clearing (8), one obtains:

$$\frac{u'(c_s^i)}{u'(y-c_s^i)} = \frac{q_s^i + Q_s^{i\prime}(l_s^i)l_s^i}{q_s^i}\frac{u'(c_0^i)}{u'(c_0^j)}.$$

The gap between the marginal rate of substitution of the borrower and the lender in a particular state is given by the term $(q_s^i + Q_s^{i\prime}(l_s^i)l_s^i)/q_s^i < 1$, to be compared with a value of 1 in the risk sharing case above. This price ratio is the current marginal gain to borrowing relative to the marginal cost to saving. It turns out that it is a negative function of the default rate d_s^i: higher default is associated with steeper borrowing costs. Therefore in general full risk sharing will fail to hold as long as default varies across states s. To see this more explicitly, using (9), we can write this expression as

$$\frac{u'(c_s^i)}{u'(y-c_s^i)} = \frac{1}{(\gamma-1)+\gamma\eta {d_s^i}^{\gamma}}\left[\gamma - 1 - \frac{d_s^i}{1-d_s^i}\right]\frac{u'(c_0^i)}{u'(c_0^j)}. \qquad (10)$$

So individual consumption levels vary with the default rate d_s^i in a given state s. More specifically, the borrower's consumption c_s^i increases with the default rate. With higher default and the resulting increased costs of borrowing, this individual will have borrowed less against s and can consume more in that state. The implications of (10) can be summarized graphically by the positively sloped curve in Figure 1 associated with intertemporal optimality.

The optimal choice of default provides another key relation between the default rate and consumption of an individual who is in debt. The optimality condition for default (3), using

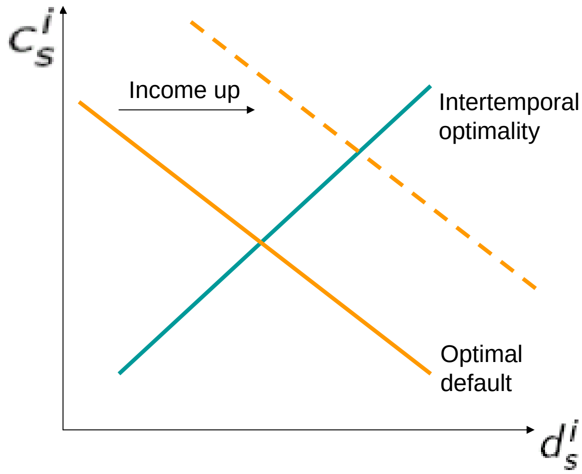

Figure 1: Equilibrium d_s^i and c_s^i when type i is a debtor in s.

the functional assumptions in (1) and market clearing (8), can be written as

$$\frac{y_s^i - c_s^i}{c_s^i} = \gamma\eta(1 - d_s^i){d_s^i}^{\gamma-1}. \tag{11}$$

This expression relates consumption, default and the endowment in a particular state. It will be important to know how the right-hand side of this expression changes with the default rate. Its derivative has the same sign as $(\gamma - 1)/\gamma - d_s^i$ so the RHS of (11) is hump shaped. Now, the fact discussed that the RHS of (9) is positive implies that, given $\gamma > 1$, the sign of the expression $\gamma - 1 - d_s^i/(1 - d_s^i)$ must be positive. It follows that $(\gamma - 1)/\gamma - d_s^i > 0$ so the RHS of (11) is increasing in the default rate d_s^i. This implies that more consumption leads to a lower default rate. With a lower marginal utility, the household is in less need to prop up consumption by means of shunning promised deliveries. Graphically, this relationship (11) is represented as the negatively sloped optimal default curve in Figure 1.

We discuss now how individual consumption changes across states s. We consider first changes where the household i remains the debtor so (10) and (11) apply throughout. Suppose that the income endowment y_s^i increases across two states s. Condition (11) can be rewritten $y_s^i = c_s^i + c_s^i\gamma\eta(1 - d_s^i){d_s^i}^{\gamma-1}$. Given the positive relationship between consumption and default from (10), the state with higher income will have a higher consumption if the RHS of (11) is increasing in d_s^i, a property we have just established. More intuitively, for given consumption, a larger endowment means the household must have higher debts and defaults more (i.e., (11)); markets read steeper interest rates into this and households borrow less in the first place and consume more in the second period (i.e., (10)). Graphically in Figure 1, consider the equilibrium allocation in state s as the intersection of the two solid curves. For a state with a larger endowment, the optimal default curve lies further to the right, implying a higher level of consumption.

Proposition 2 *(Imperfect income risk sharing with constant roles) Across states s where type i is the debtor, its consumption c_s^i and income endowment y_s^i are positively correlated.*

Note that the response of consumption to the rise in idiosyncratic income is mediated by a rise in default which, by increasing the cost of further borrowing in equilibrium, deters the accumulation of debt. However, this positive association between income and default is not a necessary implication of the theory if there is some other factors affecting the cost of borrowing that also changes with individual income. This will be discussed below.

Consider now comparisons across states where a particular household changes role. That is, suppose an equilibrium where agent i holds debt in state s and holds assets in another state s'. Again, optimal borrowing against state s is characterized using (2) by

$$\frac{u'(c_s^i)}{u'(c_0^i)} = \frac{q_s^i + Q_s^{i\prime}(l_s^i)l_s^i}{\beta(1-d_s^i)\pi_s},$$

and optimal saving against state s' reads from (4) as

$$\frac{u'(c_{s'}^i)}{u'(c_0^i)} = \frac{p_{s'}}{\beta\pi_{s'}} = \frac{q_{s'}^j}{\beta(1-d_{s'}^j)\pi_{s'}},$$

where the last equality follows from the zero-profit condition (7). Now perfect risk sharing amounts to having an equality between the two RHS terms in these two expressions. We show this cannot be true by way of contradiction. Assuming this equality, the property that $Q_s^{i\prime}(.) < 0$ implies that $q_s^i/(\beta(1-d_s^i)\pi_s) > q_{s'}^j/(\beta(1-d_{s'}^j)\pi_{s'})$. From market clearing in (8), there must also be risk sharing for agent j, which, by an analogous argument, means that the contrary inequality must hold. The same argument can be used to establish that risk-sharing fails to hold in the sense that

$$\frac{q_s^i + Q_s^{i\prime}(l_s^i)l_s^i}{\beta(1-d_s^i)\pi_s} < \frac{q_{s'}^j s}{\beta(1-d_{s'}^j)\pi_{s'}}. \tag{12}$$

The marginal cost to debt exceeds the marginal return to savings. This property together with the preceding two expressions implies that $c_s^i > c_{s'}^i$. That is, a household's consumption is higher in the states where they hold debt than in the states where they hold savings. Given this we can also establish how income correlates with consumption. Consumption levels for household i are determined from (8) as $c_s^i = y_s^i - (1-d_s^i)l_s^i$ and $c_{s'}^i = y_{s'}^i + (1-d_{s'}^j)l_{s'}^j$. The fact that $c_s^i > c_{s'}^i$ clearly requires that $y_s^i > y_{s'}^i$ so consumption and income are positively related.

Proposition 3 *(Default, debt/saving and consumption with changing roles) Across states s where household i switches between debtor and saver, its consumption c_s^i, debt level l_s^i and income endowment y_s^i are positively correlated.*

Having established that risk sharing fails, some further intuition might be helpful. One way to gain this intuition is by comparison the the outcomes for an economy where default rates are exogenous but coincide with the equilibrium default rates of our economy. Consider the constraints that determine consumption in (8). In both economies, consumption changes less than income as the household holds more debt liabilities in states where income is higher. In the exogenous-default case the portfolio composition varies across states to an extent that neutralizes the effect of income on consumption. In our endogenous-default economy, portfolios also adjust but to a lesser extent thus making consumption responsive to income. So the key point behind imperfect risk sharing is that households stop short of arranging their portfolios as needed to achieve complete consumption smoothing. Why is that? The interest rate faced by the household rises with the level of the debt issued. This makes them more cautious about taking the large debt positions needed to fully insure consumption. This is a reflection that financial markets price the fact that risk depends on individual (borrowing) decisions.

Interestingly, we have found that, with a complete set of assets, default is not insurance but rather the opposite. This is in contrast with the role generally ascribed to default in the incomplete markets literature.[12]

4.2 Consumption smoothing

We also would like to discuss how, for the same default rates, the economy with endogenous default and the economy with exogenous default differ in terms of allocations and prices. By construction, the risk premium is the same in the two economies. Therefore, one needs to figure out how the level of interest rates and hence consumption differ between them.

The simplest way to make this point is to consider a deterministic version of the model, with only one state in the second period. One can now compare this economy with the analogous case where default is exogenous, and the default rate is the same. The main difference between these two economies is found in the intertemporal optimality condition for the borrower in equation (2). For given prices of debt, with endogenous default the borrower faces a steeper interest rate, or a lower marginal benefit to issuing debt. She will issue less debt and consume less in the first period compared to the exogenous case. In equilibrium, a higher price of debt brings about the corresponding reduction in lending from the lender according to (4). Therefore, there is less consumption smoothing in the case where default is endogenous.

[12] See also Athreya, Tam, and Young (2009).

4.3 The relationship between income and default

In the previous analysis of Figure 1, higher income leads to more default. This might be a problematic implication but is not a fundamental consequence of the model. It comes about only because of the simplifying maintained assumption that the punishment parameter η is the same for all individuals in all states. The punishment could plausibly be increasing in the level of individual income. Creditors can collect more from higher earnings; legal bankruptcy protection is often means tested.[13] If this is the case, higher income can cause the default rate to decrease and still have consumption increasing.

To see this define η_s^i as the default penalty for an individual of type i in state s, and substitute it for η in (10) and (11). Consider a change in the state s such that income y_s^i goes up. Suppose this also implies a rise in η_s^i. The graphic analysis in Figure 2 will suffice to make the point. As seen before, the increase in the endowment of income y_s^i shifts the default optimality curve to the right. The associated increase in the parameter for the cost of default η_s^i reduces default and shifts this curve back to the left. On the other hand, the higher cost of default reduces borrowing and increases consumption thus shifting the intertemporal optimality curve upwards. Note that the change in the punishment η_s^i can always be chosen to overturn the effect of income and leave the optimal default curve unchanged, in which case higher consumption and lower default follow as income rises. The same type of outcome still obtains when, as depicted in Figure2, the reaction of the default cost is smaller.

4.4 Pooling

Dubey, Geanakoplos, and Shubik (2005) have studied situations where contracts of different size are pooled together. In the context of our model, pooling means that there is a single price q_s^i, rather than a menu of prices, for debt sold by agent i with delivery in state s, irrespective of the amount sold l_s^i. In other words, the menu of prices $Q_s^i(.)$ is a flat function. Formally, the equilibrium is characterized as before except that in the intertemporal optimality condition for the borrower (2) the term $Q_s^{i\prime}(l_s^i) = 0$. A consequence is that this economy achieves the first best allocation with full risk sharing. Although, by virtue of (3), default rates are endogenous, market prices account for the event of default efficiently. This observation underscores the conclusion that risk-based pricing of loans is an essential ingredient in our explanation of endogenous financial frictions. Risk pooling is not only an exogenous assumption but also one that necessitates further exogenous degrees of market incompleteness in order to account for imperfect insurance.

[13] Another argument is that the punishment could be a function of the absolute value of the debt reneged upon rather than, as we have postulated, its proportion. We do not analyze this explicitly in order to preserve the sought tractability.

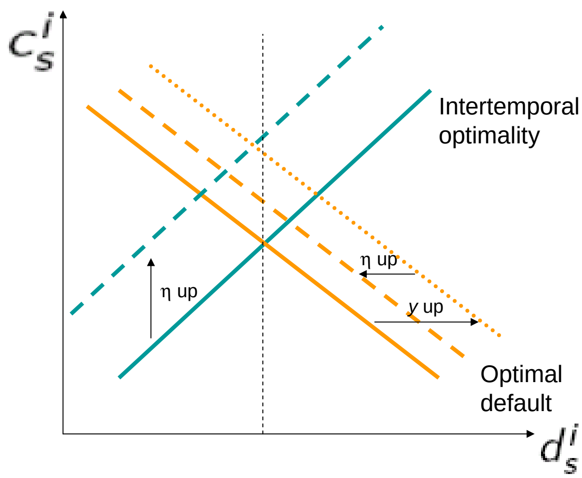

Figure 2: Equilibrium d_s^i and c_s^i when type i is a debtor in s. A rise in income y_s^i and η_s^i increases consumption and lowers default.

5 Consumption inequality: risk-pricing or debt-constraints

This section discusses the differences between the present risk-pricing model and the debt-constrained model regarding the implications for consumption inequality. The ability of different models for understanding the evidence on risk-sharing has been the subject of an important body of quantitative literature (e.g., Krueger and Perri (2006), Kaplan and Violante (2010)). The conclusion that seems to emerge is that the debt-constrained model in the Kehoe and Levine (1993) tradition underestimates inequality. It is thus important to understand how the risk-pricing model could comparatively perform on this dimension.

The debt-constrained economy obtains by changing the specification of the punishment technology z in (1). It suffices to assume that the utility penalty is a fixed amount, equivalent to the punishment for fully defaulting η, independently of the scale of default. This assumption would naturally lead defaulters to default on all their debts. This being so will cause intermediaries not to offer loans that induce any default (i.e., to command a price of zero). This results in a situation where loans traded carry no default risk, so lending rates are risk free so (7) implies $q_s^i = p_s$, but the size of loans is constrained by the maximum debt that would not trigger default. Formally, individuals of type i face a debt limit against state s that we denote $\bar{l}_s^i$. This is determined as the value that makes the individual indifferent between defaulting or not:

$$u(y_s^i) - \eta = u(y_s^i - \bar{l}_s^i). \tag{13}$$

Faced with such a limit to the value of debt that can be sold, the consumer optimality condition that replaces (2) and (4) is

$$p_s u'(c_0^i) \geq \beta \pi_s u'(c_s^i) \tag{14}$$

with strict equality if the debt constraint does not bind, and generically an inequality when the constraint binds $l_s^i = \bar{l}_s^i$. This economy features imperfect risk sharing if in some states and for some households the debt constraint binds.

One could argue that, at least for some level of the default penalty, the risk-pricing model brings about more inequality than the debt-constrained economy. We have discussed that there is imperfect risk sharing in the risk-pricing economy as long as there is positive default in equilibrium. For any finite penalty, there will always be some default in that setting. On the other hand, in the debt-constrained economy one can surely find a large enough punishment that makes consumption inequality zero. By a continuity argument, there will also be a region where the debt-constrained economy has binding debt limits and incomplete insurance but still displays less inequality than the risk-pricing economy. While some especial assumption on the punishment technology made in this paper might play a part, it betrays the sense that the risk-pricing economy might have less risk-sharing more generally. If default can be partial, there will be more states where default and hence the distortion that follows is positive.

5.1 A symmetric case

We use a specific version of the model to establish analytically the lower degree of risk sharing in the risk-pricing economy compared to the debt-constrained economy. The two types of households have the same initial endowment y_0, with the aggregate income $y = 2y_0$. There are only two states s in the second period. In one state, one type of household receives a high income y_l and the other type receives a low realization y_a (the choice of subscripts will become clear very soon); in the other state, the allocation of endowments across household types is the reverse. The total endowment is as in the initial period so $y_l + y_a = y$. The two states occur with the same probability $\pi = 1/2$. The two types of households are therefore ex-ante identical.

In an equilibrium with trade, the allocations of debt l, assets a and default d, and the prices of debt and assets q and p, are all independent of the state. Consumption in the first period c_0 is the same for the two types of households. In the second period, the household who receives the high endowment y_l owes promised deliveries of debt l and consumes c_l; the households with the low endowment y_a holds claims a and consumes c_a. These values of consumption are independent of the contingent state s. It is simple to define a measure of risk sharing, θ, as the difference between the high income endowment y_l and consumption in that state c_l:

$$\theta \equiv y_l - c_l \in [0, 1/2(y_l - y_a)]. \tag{15}$$

Zero risk sharing obtains when $\theta = 0$ and consumption matches the endowment; full risk sharing occurs when the total endowment is evenly split and $c_l = 1/2(y_l + y_a)$. A lower θ is associated with more consumption inequality.

Given the symmetry of outcomes, the equilibrium of the risk-pricing economy can be written explicitly in terms of two relationships involving the default rate d and the degree of risk sharing θ. The first relation comes from the intertemporal optimal borrowing/lending conditions for the household (2) and (4). Having used market clearing $y = c_a + c_l$ from (8), no-arbitrage $p(1-d) = q$ from (7), the risk-based price term from (9), and the definition in (15), we find the *cost-of-borrowing* condition:

$$\frac{u'(y_l - \theta)}{u'(y - y_l + \theta)} = \frac{1}{(\gamma - 1) + \gamma\eta d^\gamma}\left(\gamma - 1 - \frac{d}{1-d}\right) \tag{16}$$

This traces a negative relation between the default rate d and the extent of risk sharing θ.[14] The RHS term is the price term from (9) that captures the distortion or wedge that risk-pricing brings about. As discussed earlier, this is a negative function of the default rate d as it makes it more costly for the household to borrow. The LHS is the relative marginal utility of the consumer in the good state which is positively related to risk sharing θ. A higher default rate reduces debt and hence increases consumption in the good state thus decreasing risk sharing. The second relationship between d and θ derives from the condition for optimal default (3). Using the budget relation $l(1-d) = y_l - c_l$ from (8), the *optimal-default* condition reads:

$$u'(y_l - \theta)\frac{\theta}{1-d} = \eta\gamma d^{\gamma-1} \tag{17}$$

It describes a positive relation between risk sharing and the default rate. More risk sharing means higher debt liabilities in the good state and consequently a bigger incentive to default. An equilibrium is values θ and d solving (16) and (17).

Consider now the equilibrium in the debt-constrained economy. In the case where the penalty η is large enough that the debt constraint is never binding the conclusion is straightforward. This economy has perfect insurance and, consequently, less inequality than the risk-based pricing economy. We turn to the more interesting case where the debt limit binds. In the present symmetric environment, the value of consumer liabilities l coincides with the debt limit $\bar{l}$ which, by the consumer budget in (8), is related to the household's consumption in his good state by $\bar{l} = y_l - c_l$. The participation condition (13), with the definition in (15), allows us to characterize the degree of risk sharing in the debt-constrained economy as the value θ solving

$$u(y_l - \theta) - [u(y_l) - \eta] = 0 \tag{18}$$

[14] Naturally, we will be supposing that the condition for existence discussed earlier $\gamma - 1 - d/(1-d)$ holds.

The LHS is the gain to not defaulting, a decreasing function of θ. For a higher degree of risk sharing, the high-income household would be better off by choosing to default.

We now compare the value of θ that solves (16) and (17) for the risk-pricing economy, with the one that solves (18) for the debt-constrained economy. We will now use the log utility assumption from (1). In the risk-pricing model, we know the default rate cannot exceed the maximum consistent with existence, that is $d < \overline{d} \equiv (\gamma - 1)/\gamma$. Then the default optimality condition (17) implies that the degree of risk sharing under risk-pricing is bounded from above by

$$\frac{\eta y_l}{1/\overline{d}^{\gamma-1} + \eta}.$$

We now evaluate the participation condition on the LHS of (18) for the debt-constrained model at this level of risk sharing. This yields

$$-\log(1 + \eta\overline{d}^{\gamma-1}) + \eta > 0,$$

implying that this level of risk sharing is too low to be an equilibrium for the debt-constrained economy. We have thus established that there is more consumption inequality in our risk-based pricing economy.

Proposition 4 *Consider the symmetric case. Given the same penalty parameter η, the risk-pricing model generates less risk sharing and more consumption inequality than the debt-constrained model.*

5.2 Numerical illustration

At this point, we resort to numerical examples to illustrate this result as well as the fact that equilibrium existence is not a degenerate fiction. Only two random states s are assumed. The first example has symmetric consumers where the only difference is the state when they get the bad shock, but the probability is the same. This corresponds to the setting studied in Proposition 4. The parameters are $\beta = 1$, $y_0^i = 0.50$ all i, equal probabilities $\pi_s = 0.50$, symmetric income processes y_s^i, $(0.60, 0.40)$, and punishment $\eta = 0.12$ and $\gamma = 3.0$. It suffices the consider outcomes for just one agent. Example 1 shows the results for the two economies and, as a benchmark reference, those under full risk-sharing for an economy with commitment. The risk-pricing model delivers far more consumption volatility and less trade in assets. This is reflected in a lower risk-free lending rate and higher borrowing rates. This is a conservative example in that it sets a considerably low default penalty. Mild punishment parameters, needed in order to generate some inequality in the debt-constrained economy, are associated with very large default rates in the risk-pricing economy.

Example 1: symmetric case

	RISK-SHARING	RISK-PRICING	DEBT-CONSTRAINTS
c_0^i	0.500	0.500	0.500
$\{c_s^i\}$	(0.500, 0.500)	(0.583, 0.417)	(0.532, 0.468)
l_s^i	0.100	0.0268	0.0678
p_s	0.500	0.599	0.534
d_s	–	0.357	0.000
q_s	0.500	0.385	0.534
$\bar{l}$	–	–	0.0678

The previous example focuses on within period insurance only. The second example involves effects on smoothing over time as well. This is an asymmetric economy in that household A, having a back-loaded endowment profile, borrows against all future states. The parameters are as follows: $\beta = 1$, $\pi_s = 0.50$ all s, $y_0^A = 0.40$, $\{y_s^A\} = (0.54, 0.48)$, $y_0^B = 0.60$, $\{y_s^B\} = (0.46, 0.52)$. The punishment parameters are the curvature $\gamma = 3.0$, and the penalty level $\eta = 0.20$ for the risk-pricing model. For the debt-constrained model, we assume a milder fixed punishment of 0.10. Note this different choice works against the contention that the debt-constrained model features less inequality and should strengthen the point.[15] The results are displayed in the next table as Example 2. The debt-constrained model has the limit binding only in one of the states and individuals can achieve perfect intertemporal smoothing in the other state. We see that the risk-pricing economy generates less smoothing both over time and across states since there is less trade.

Example 2: asymmetric case

	RISK-SHARING	RISK-PRICING	DEBT-CONSTRAINTS
c_0^A	0.4550	0.4227	0.4452
$\{c_s^A\}$	(0.4550, 0.4550)	(0.5133, 0.4673)	(0.4887, 0.4452)
c_0^B	0.5450	0.5773	0.5548
$\{c_s^B\}$	(0.5450, 0.5450)	(0.4867, 0.5327)	(0.5113, 0.5548)
l_s^A	(0.085, 0.025)	(0.0424, 0.0168)	(0.0513, 0.0348)
p_s	(0.500, 0.500)	(0.5931, 0.5419)	(0.5420, 0.5000)
q_s^A	(0.500, 0.500)	(0.3729, 0.4094)	(0.5420, 0.5000)
d_s	–	(0.3712, 0.2445)	–
$\bar{l}_s^A$	–	–	(0.0513, 0.0457)
$\bar{l}_s^B$	–	–	(0.0438, 0.0494)

6 Concluding remarks

Risk-based pricing and partial default in consumer credit can help to account for limited trade even when there is a complete set securities. The specific mechanism differs from the existing literature on debt-contrained economies. The individualized pricing of default in this paper brings about price wedges that distort the borrowing decisions. This model

[15] Furthermore, with same $\eta = 0.20$, the debt-constrained model produces full risk sharing.

seems to imply more consumption inequality than the debt-constrained model typical of much applied literature on the subject. This property might prove significant for understanding the evidence.

This analysis can be extended fruitfully in various directions. First, one can investigate with this novel approach the ability of different factors to account for the observation of rising bankruptcy and the expansion of credit and levels of debt. This type of questions have only being pursued in incomplete markets economies. Second, bankruptcy matters even without the frictions introduced by market incompleteness. This insight might prove important for a better understanding of financial frictions and welfare policy in actual economies.

The model in this paper is deliberately simple and, in the interest on analytical tractability, exploits some specific functional assumption. In order to draw firm practical conclusions, a quantitative model of risk-pricing with complete markets is called for. This is being pursued in ongoing research.

References

Ábrahám, Á., and E. Cárceles-Poveda (2010): "Endogenous trading constraints with incomplete asset markets," *Journal of Economic Theory*, 145(3), 974–1004.

Alvarez, F., and U. Jermann (2000): "Efficiency, equilibrium, and asset pricing with risk of default," *Econometrica*, 68(4), 775–797.

Andolfatto, D., and M. Gervais (2008): "Endogenous debt constraints in a life-cycle model with an application to social security," *Journal of Economic Dynamics and Control*, 32(12), 3745–3759.

Arellano, C. (2008): "Default risk and income fluctuations in emerging economies," *The American Economic Review*, 98(3), 690–712.

Athreya, K., X. Tam, and E. Young (2009): "Unsecured credit markets are not insurance markets," *Journal of Monetary Economics*, 56(1), 83–103.

Benjamin, D., and M. Wright (2009): "Recovery Before Redemption: A Theory of Delays in Sovereign Debt Renegotiations," .

Chatterjee, S. (2010): "An Equilibrium Model of the Timing of Bankruptcy Filings," Presented at Credit, defaul and bankruptcy conference, LAEF, UCSB.

Chatterjee, S., D. Corbae, M. Nakajima, and J. Ríos-Rull (2007): "A quantitative theory of unsecured consumer credit with risk of default," *Econometrica*, 75(6), 1525–1589.

Dawsey, A., and L. Ausubel (2004): "Informal bankruptcy," unpublished.

Dawsey, A., R. Hynes, and L. Ausubel (2008): "The Regulation of Non-Judicial Debt Collection and the Consumer's Choice Among Repayment, Bankruptcy and Informal Bankruptcy," *University of Virginia Legal Working Paper Series*, p. 42.

Díaz-Giménez, J., A. Glover, and J. Ríos-Rull (2011): "Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update," *Federal Reserve Bank of Minneapolis Quarterly Review*, 34(1), 2–31.

Dubey, P., J. Geanakoplos, and M. Shubik (2005): "Default and Punishment in General Equilibrium1," *Econometrica*, 73(1), 1–37.

Eaton, J., and M. Gersovitz (1981): "Debt with potential repudiation: Theoretical and empirical analysis," *The Review of Economic Studies*, 48(2), 289–309.

Kaplan, G., and G. Violante (2010): "How Much Consumption Insurance Beyond Self Insurance? ," *American Economic Journal: Macroeconomics*, 38(2), 53–87.

KEHOE, P., AND F. PERRI (2002): "International business cycles with endogenous incomplete markets," *Econometrica*, 70(3), 907–928.

KEHOE, T., AND D. LEVINE (1993): "Debt-constrained asset markets," *The Review of Economic Studies*, 60(4), 865–888.

——— (2001): "Liquidity constrained markets versus debt constrained markets," *Econometrica*, 69(3), 575–598.

——— (2006): "Bankruptcy and collateral in debt constrained markets," *NBER Working Paper*.

KRUEGER, D., AND F. PERRI (2006): "Does income inequality lead to consumption inequality? evidence and theory1," *Review of Economic Studies*, 73(1), 163–193.

LIVSHITS, I., J. MACGEE, AND M. TERTILT (2007): "Consumer bankruptcy: A fresh start," *The American Economic Review*, 97(1), 402–418.

MATEOS-PLANAS, X. (2009): "A model of credit limits and bankruptcy," *Manuscript*.

MATEOS-PLANAS, X., AND J. V. RÍOS-RULL (2010): "Credit lines," *Manuscript*.

MATEOS-PLANAS, X., AND G. SECCIA (2006): "Welfare implications of endogenous credit limits with bankruptcy," *Journal of Economic Dynamics and Control*, 30(11), 2081–2115.

STURZENEGGER, F., AND J. ZETTELMEYER (2007): *Debt defaults and lessons from a decade of crises*. The MIT press.

YUE, V. (2010): "Sovereign default and debt renegotiation," *Journal of International Economics*, 80(2), 176–187.

ZAME, W. (1993): "Efficiency and the role of default when security markets are incomplete," *The American Economic Review*, 83(5), 1142–1164.

ZHANG, H. (1997): "Endogenous borrowing constraints with incomplete markets," *Journal of Finance*, 52(5), 2187–2209.