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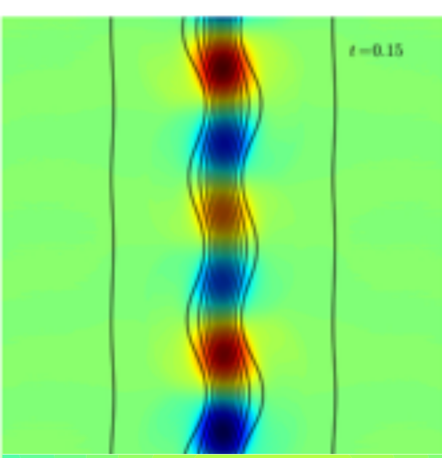
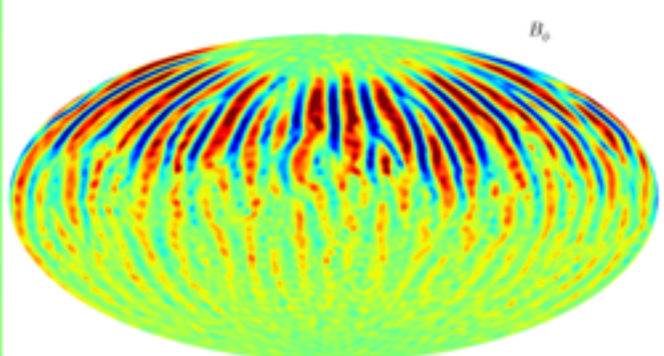
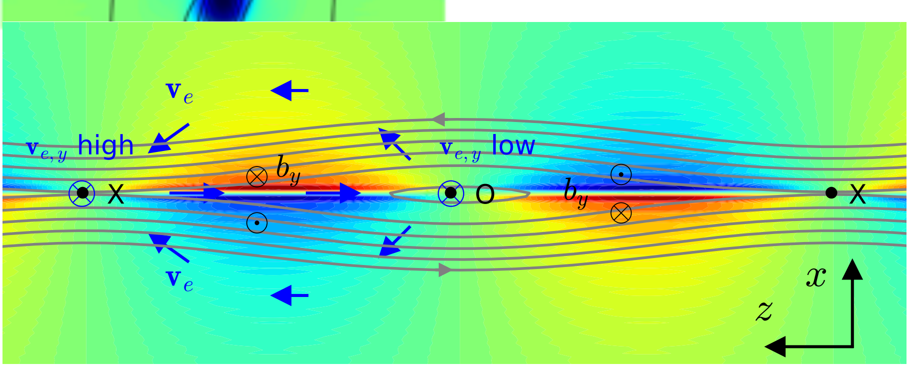
Resistive and Ideal Instabilities in Hall-MHD







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NewCompStar WG Meeting, Southampton, 13-14/9/2016

Outline

- Hall-MHD, motivation and mathematical formulation.
- Evolution: turbulence - instabilities.
- Ideal Density-Shear Instability.
- Resistive Tearing Instability.

Gourgouliatos et al. 2015 MNRAS
Gourgouliatos, Hollerbach & Wood, 2016 PNAS
Gourgouliatos & Hollerbach 2016, MNRAS in press
Wood et al. 2014, Physics of Plasmas

Role of Hall-MHD

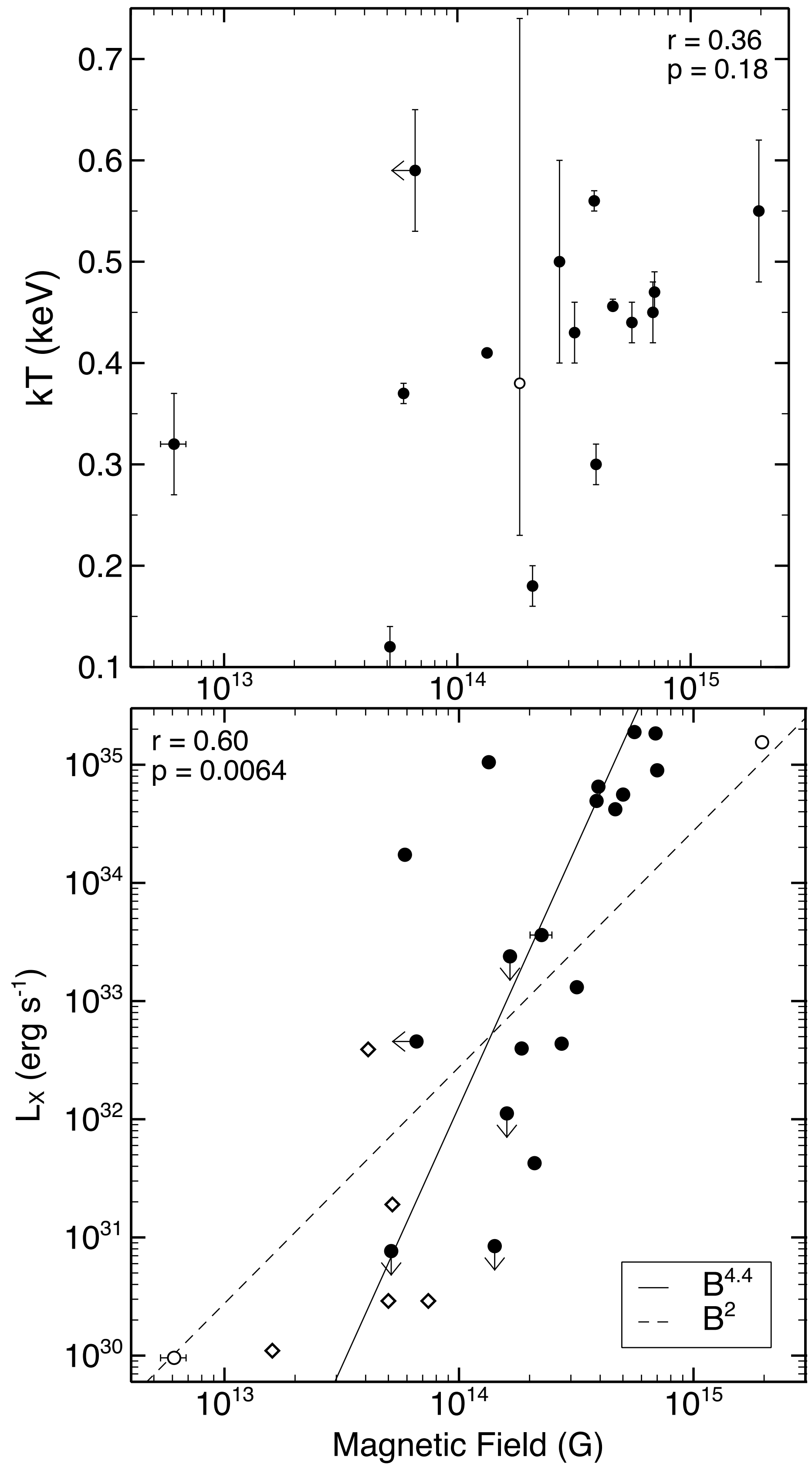
L_X vs kT: thermal radiation originates from hotspots (~ km).

Decay must be more efficient than Ohmic.

Bursts: strong localised magnetic field (especially in magnetars with weaker dipole magnetic field).

Can the Hall effect speed up magnetic field decay and create strong localised magnetic fields?

- Secular Evolution
- Turbulent Cascade
- Instabilities

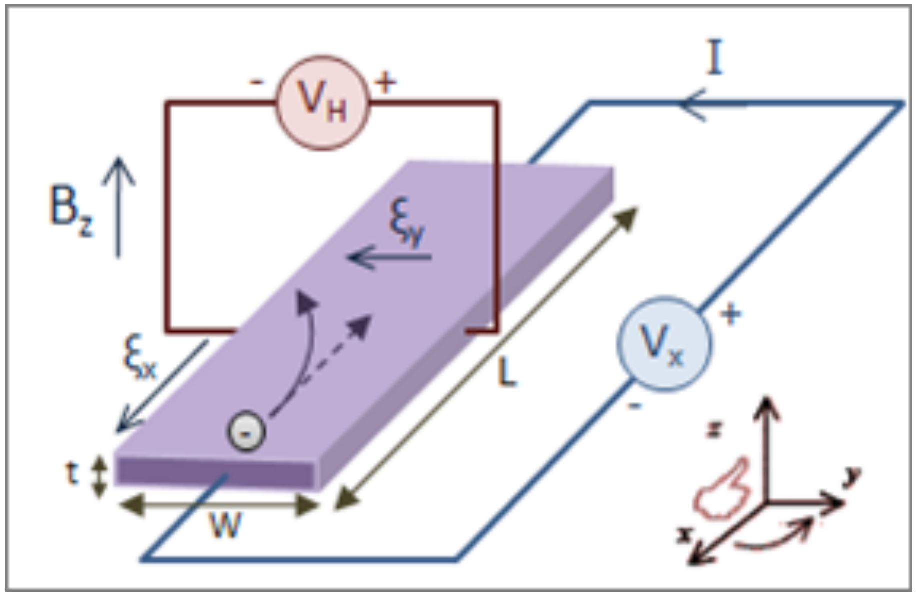


Olausen & Kaspi

Hall Drift - Electron MHD

A magnetic field perpendicular to a conductor, deflects electrons towards one edge of the conductor, leading to the appearance of a voltage.

(Hall 1880)



- Only electrons can move.
- The field is advected by the electron fluid and gets dissipated due to resistivity.
- The conductor is strong enough to balance Lorentz forces.

$$\mathbf{j} = -n_{\mathrm{e}} \mathrm{e} \mathbf{v}_{\mathrm{e}}$$

$$\mathbf{E} = -(\mathbf{v}_{\mathrm{e}}/c) \times \mathbf{B} + \mathbf{j}/\sigma$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{4\pi \mathrm{e}} \nabla \times \left(\frac{\nabla \times \mathbf{B}}{n_{\mathrm{e}}} \times \mathbf{B} \right) - \frac{c^2}{4\pi} \nabla \times \left(\frac{\nabla \times \mathbf{B}}{\sigma} \right)$$

Goldreich & Reisenegger 1992

Hall-MHD in Neutron Star Crusts

$$\frac{c}{4\pi e}\nabla\times\left(\frac{\nabla\times\mathbf{B}}{n_e}\times\mathbf{B}\right)$$ Hall

$$\frac{c^2}{4\pi}\nabla\times\left(\frac{\nabla\times\mathbf{B}}{\sigma}\right)$$ Ohmic

Hall term dominates over Ohmic if

$$R_H = \frac{\sigma|B|}{cen_e} \gg 1$$

Appropriate combination of B, σ, n_e

n_e: 10^{31}-10^{36} cm^{-3}

σ: 10^{21}-10^{24} s^{-1}

For B>10^{12}G a significant part of the crust has R_H>>1.

Potekhin et al. 1999, Cumming et al. 2004

Does stronger magnetic field mean that the crust will be dominated by Hall-MHD?

Yes, up to a point. If the magnetic field is too strong the crust yields:

$$\frac{B_{cr}^2}{8\pi} \approx 10^{-3} P_e$$

B_{cr}~10^{15}G (intermediate depth)

Horowitz & Kadau 2009, Chugunov & Horowitz 2010, Beloborodov & Levin 2014, Gourgouliatos & Cumming 2015, Lander et al. 2015, Lander 2016, Li et al. 2016

Hall Instability

Condition for instability:

Energy from the background equilibrium state $\mathbf{B_0}$ is transferred to the perturbation **b.**

$$\nabla \times \left(\frac{\nabla \times \mathbf{B_0}}{n_e} \times \mathbf{B_0} \right) = 0$$

$$\frac{dE_{\mathrm{B}}}{dt} = \frac{1}{4\pi} \int \mathbf{B} \cdot \frac{d\mathbf{B}}{dt} dV = -\frac{1}{4\pi} \int (\mathbf{j} \cdot \mathbf{E}) dV = 0$$

$$\int (\mathbf{B_0} + \mathbf{b})^{\mathbf{2}} \mathbf{dV} = \mathrm{const.}$$

$$\frac{d}{dt} \int \mathbf{b^2 dV} > \mathbf{0}$$

- This exchange of energy may lead to severe deformation of the background field.
- Formation of smaller structures will speed up Ohmic decay.

Rheinhard & Geppert 2002
Rheinhardt et al. 2004,
Pons & Geppert 2010

Density-Shear Instability (Ideal)

Plane parallel background field and electron density

$$\frac{\partial \mathbf{B}}{\partial y} = 0 \qquad \mathbf{B} = B_z(x)\hat{\mathbf{z}}$$

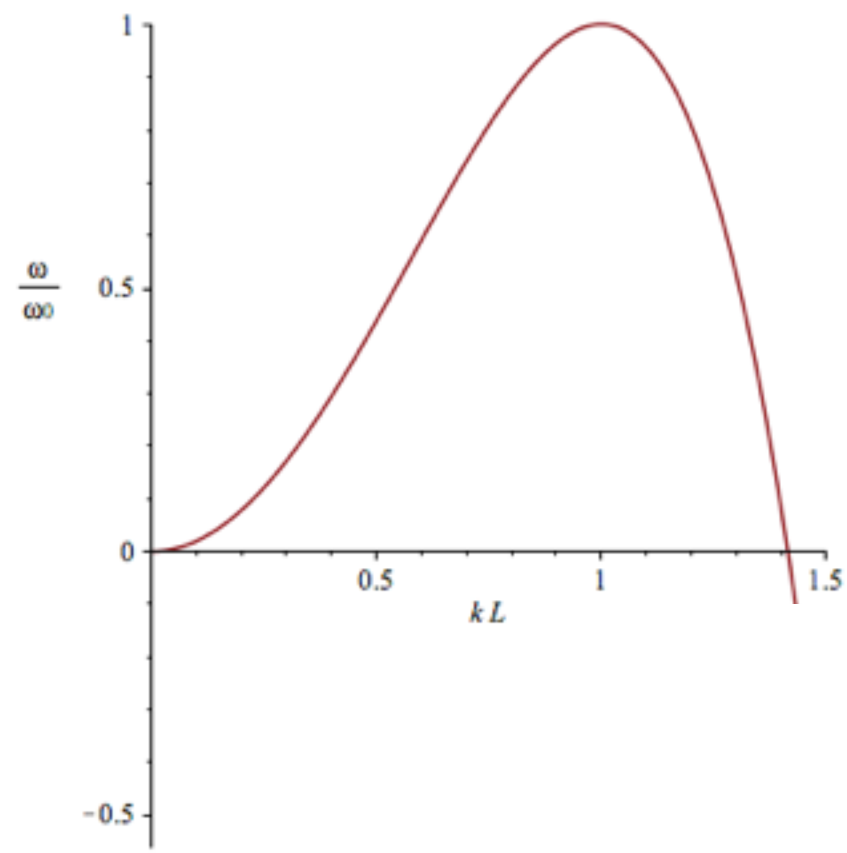
By construction this is a state of Hall equilibrium

$$\frac{\partial n_e}{\partial y} = 0 \qquad n_e = n_e(x)$$

$$\frac{b_x''}{b_x} = \frac{(B_z'/n_e')'}{B_z/n_e} + k_z^2 + \frac{\omega^2 n_e^2}{B_z^2 k_z^2}$$

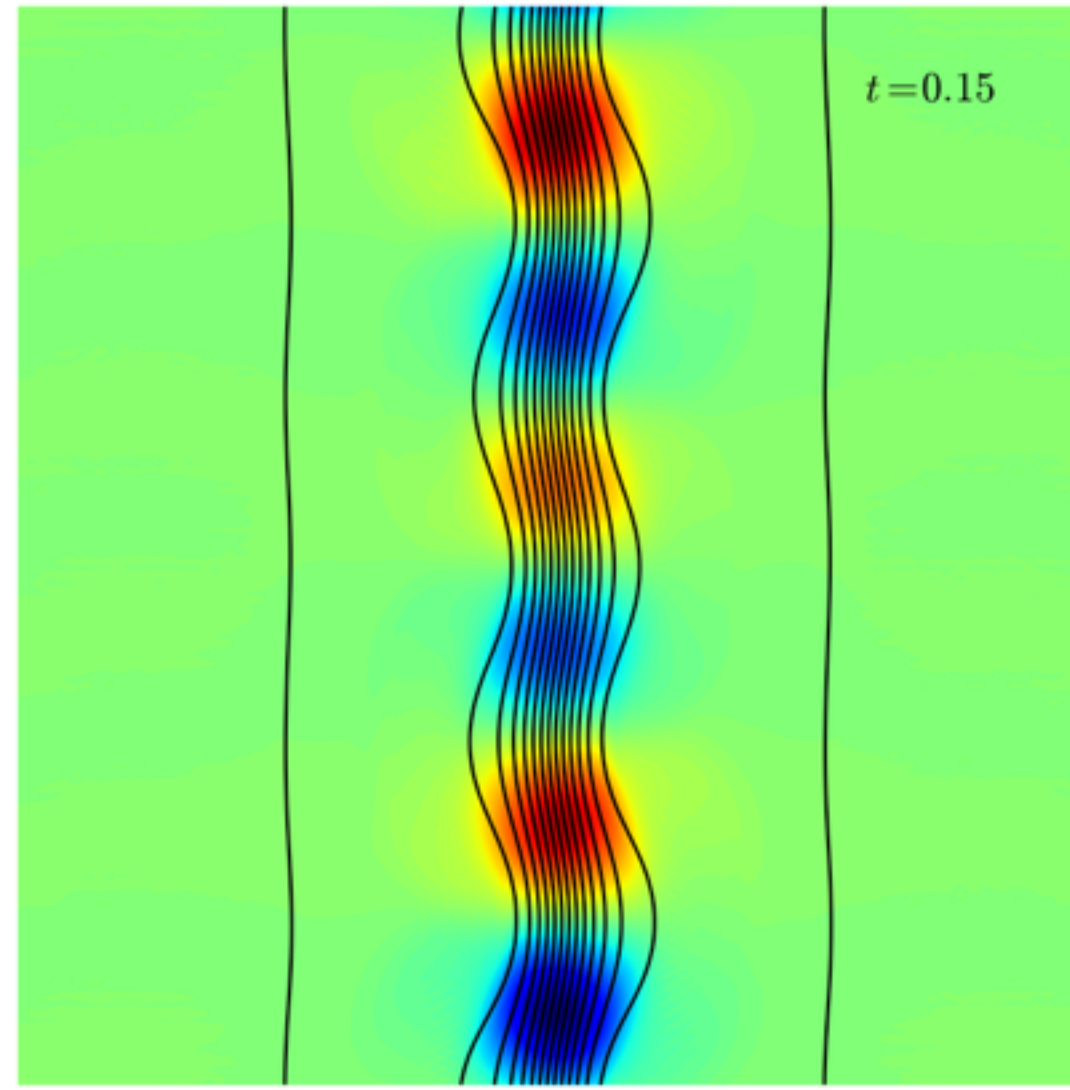
$$B_z \propto \exp(-x^2/L^2)$$

$$n_e \propto \exp(-x^2/L^2)$$



Wood et al. 2014

Simulation



Slight modification:

1. Superposition of a weak uniform magnetic field (10^{-2}).
2. Small base electron density (10^{-2}).
3. Additional weak perturbation (10^{-4}).

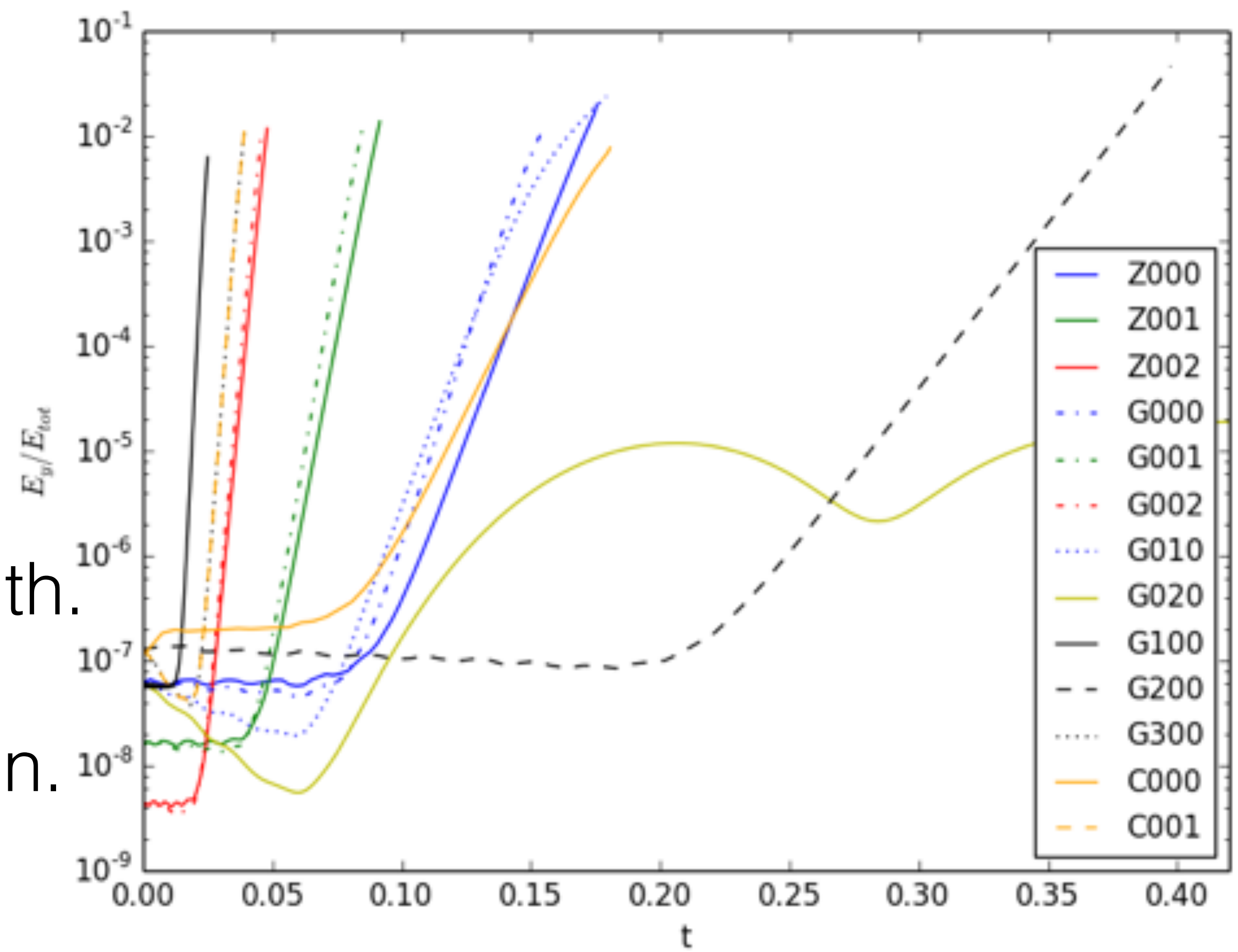
growth rate proportional to B

01

Gourgouliatos et al. 2015

Summary of Evolution

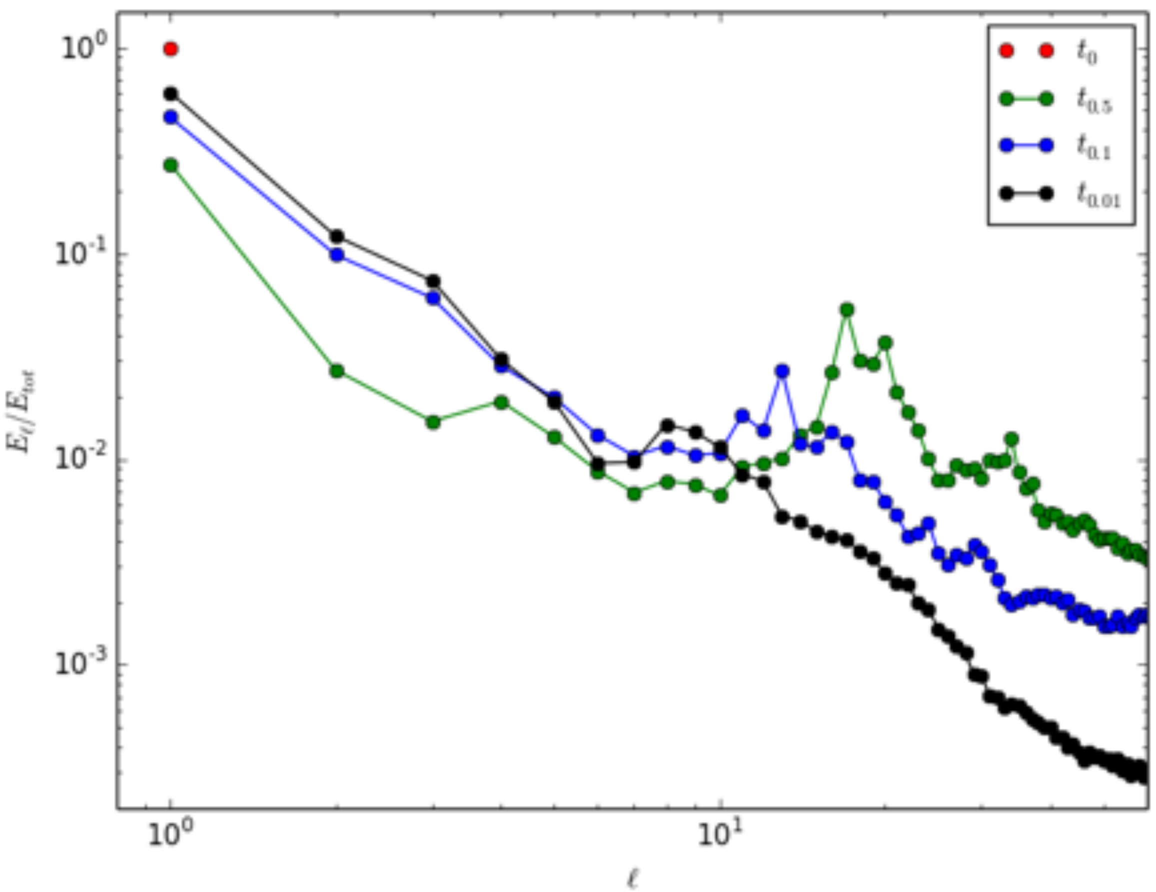
- Adjustment of the perturbing field.
- Exponential growth of perturbation.
- Deformation of the background field.

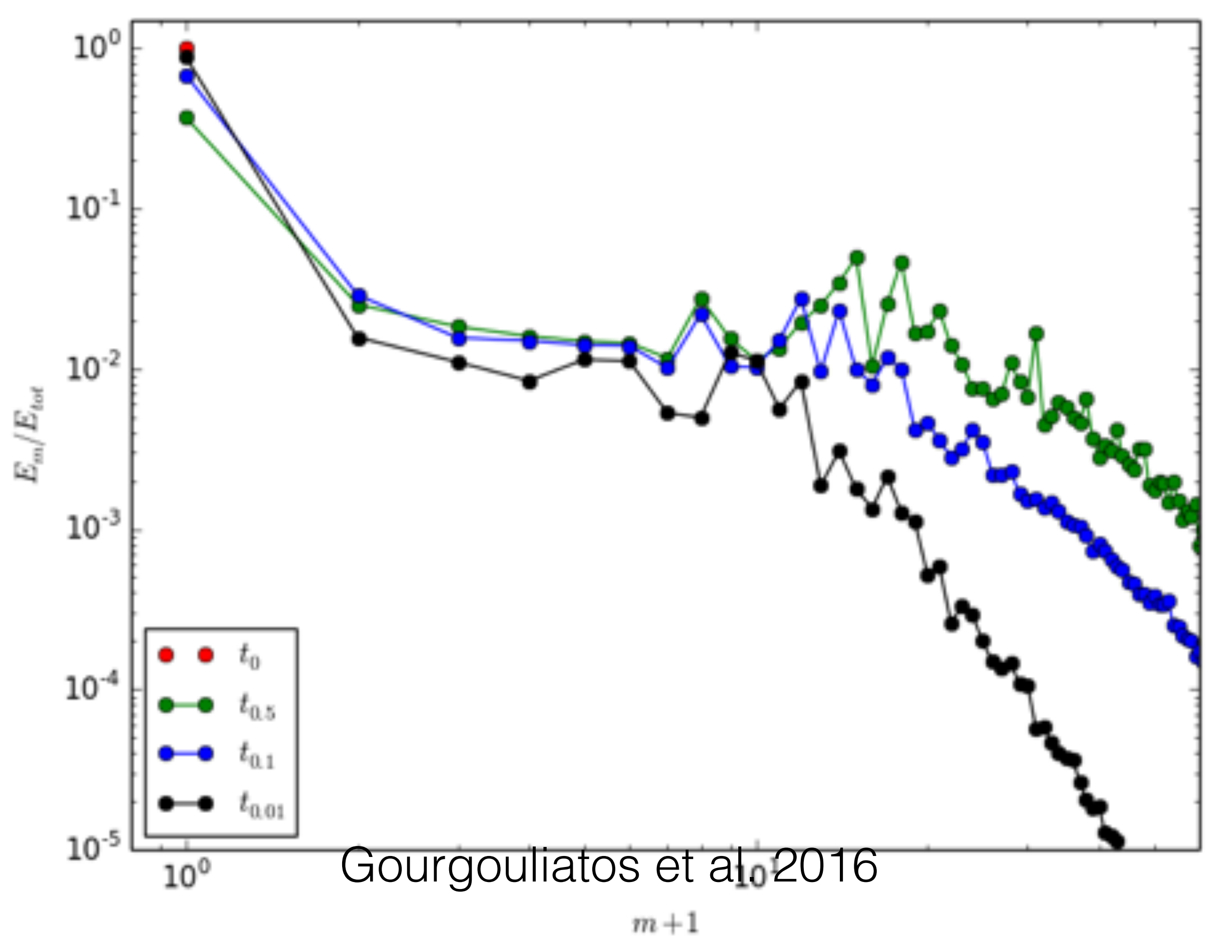


- Insensitive to small resistivity.
- Dependence on magnetic field strength.
- Dependence on the scale-height.
- Strong resistivity may lead to saturation.

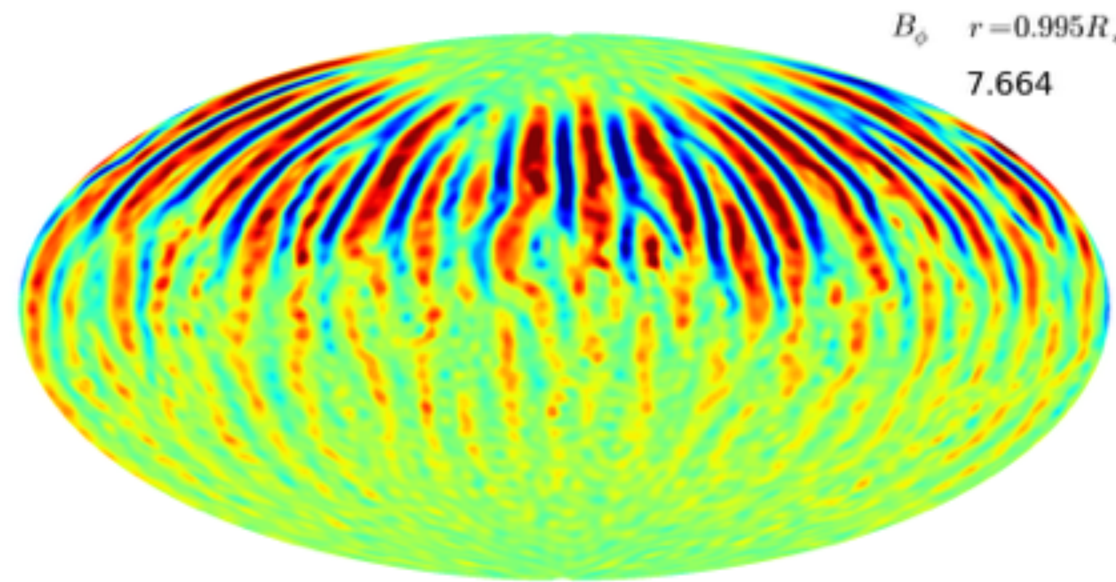
- Wavenumber - growth rate in accordance with analytical expectations within a few percent.

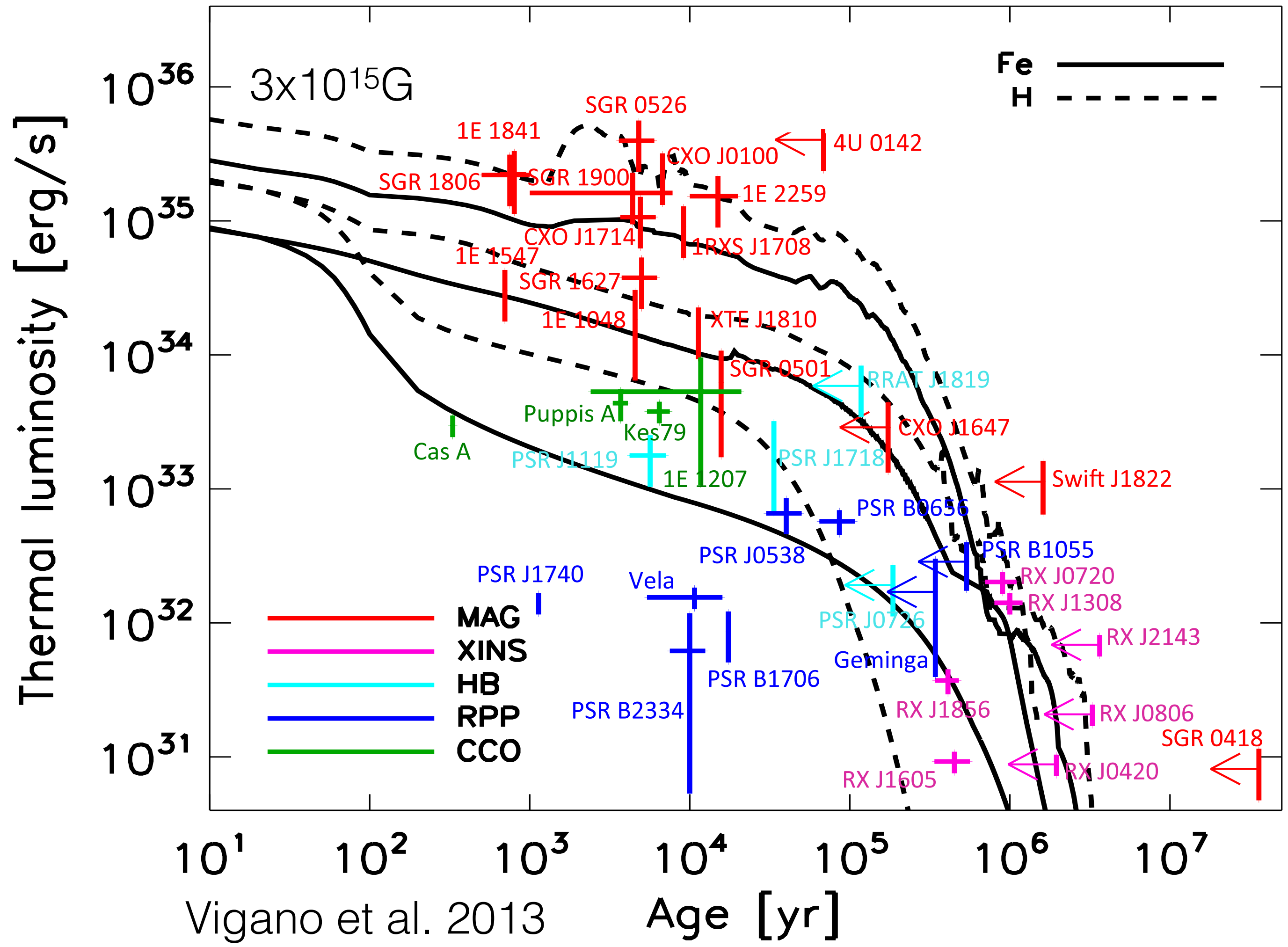
toroidal l=1, no poloidal

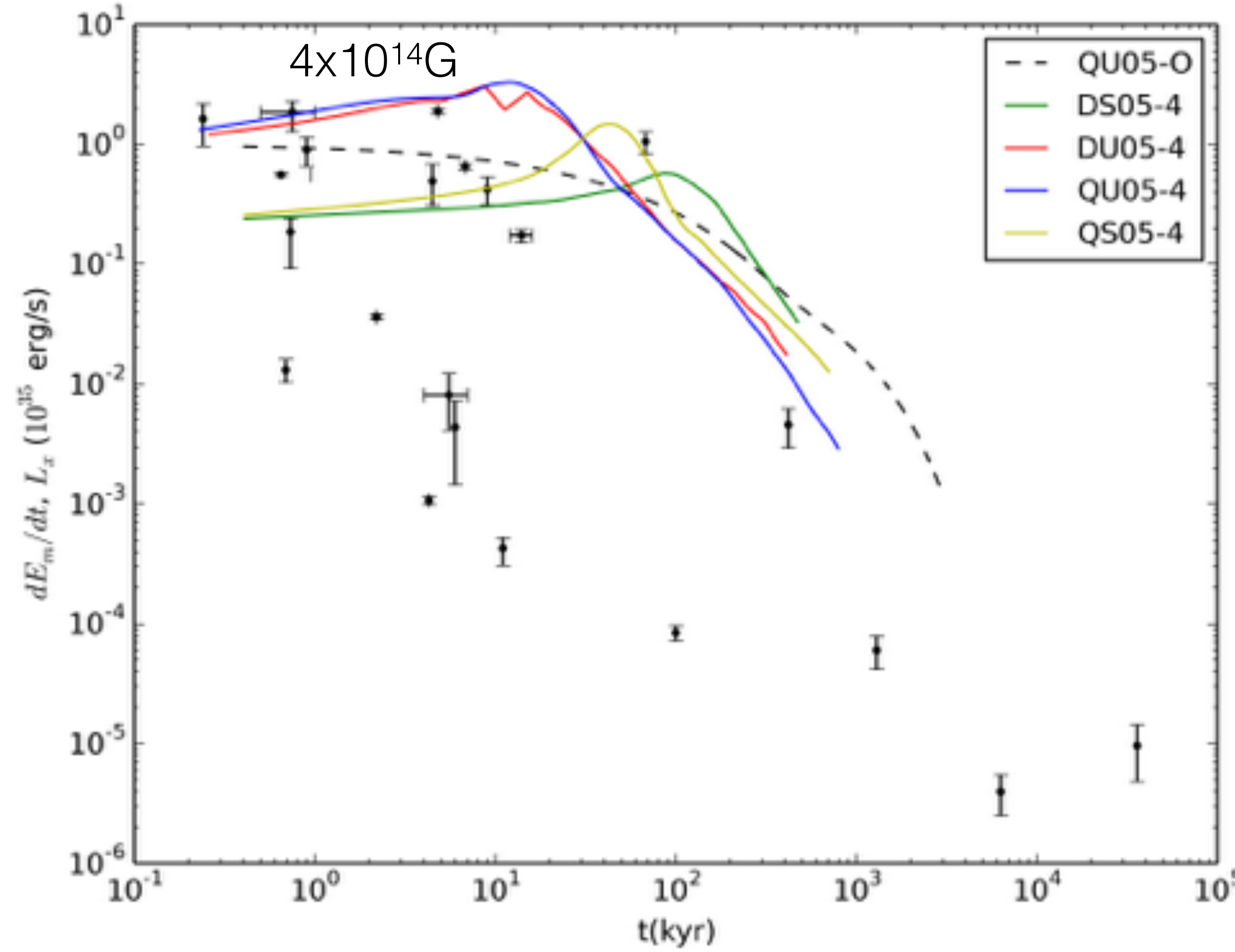




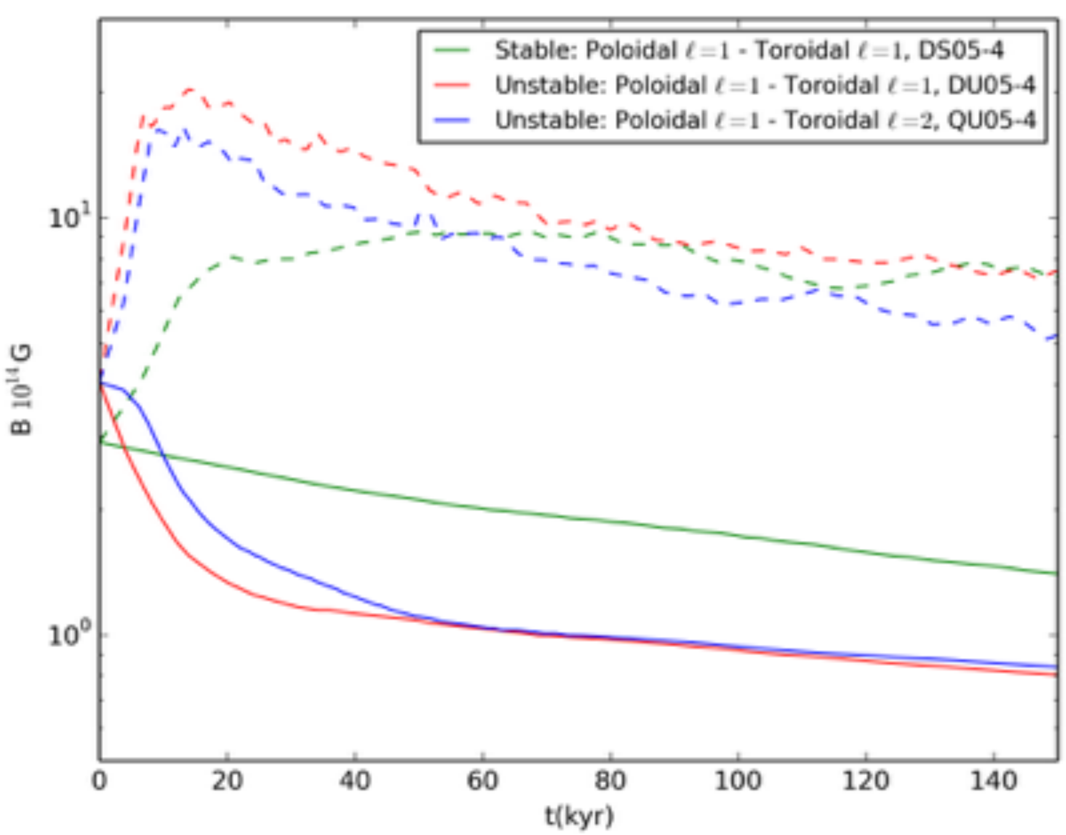
Gourgouliatos et al. 2016







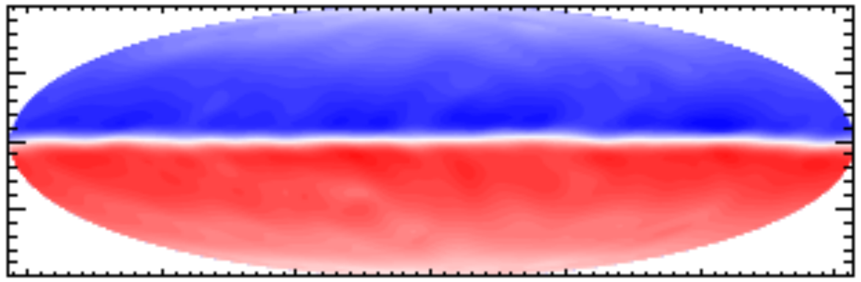
survival of localised magnetic field ~10 times stronger than the dipole.



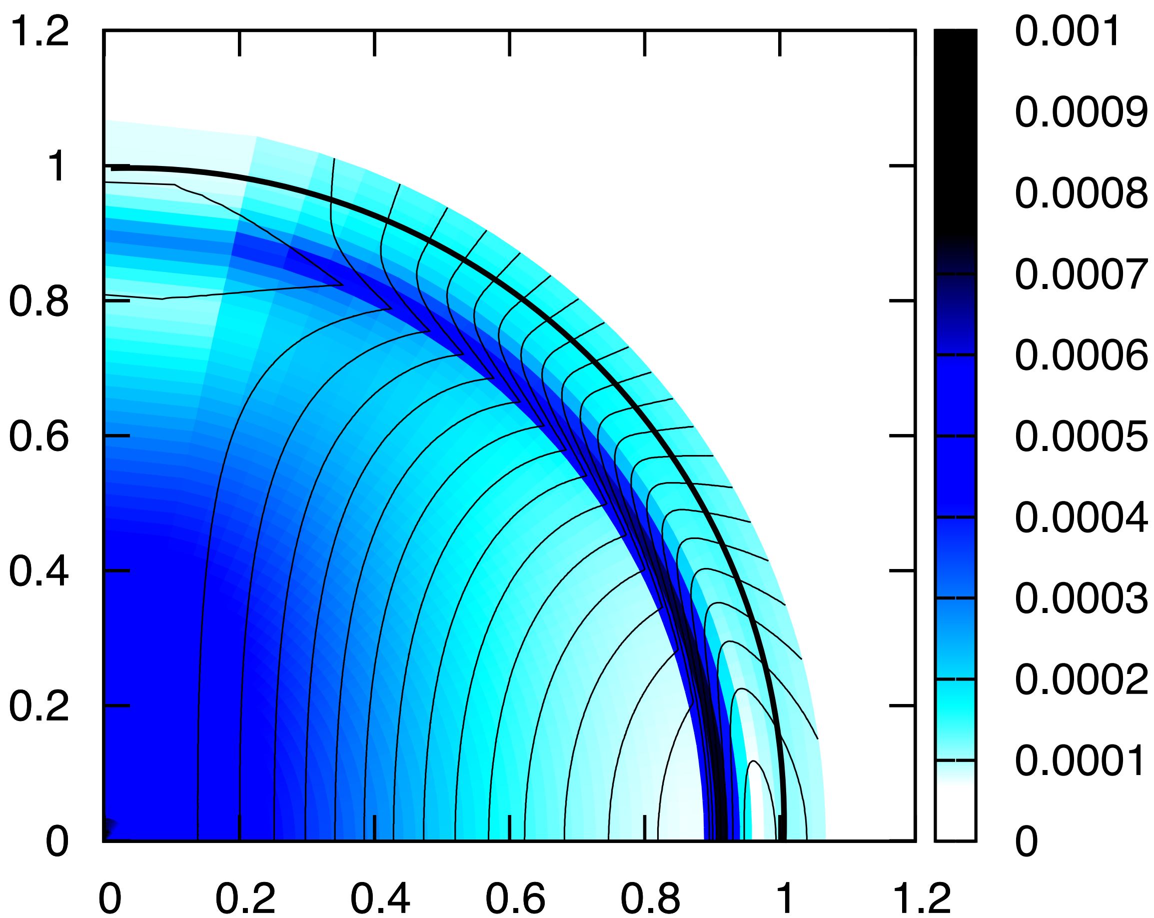
Faster magnetic field decay. The required magnetic energy to power magnetars ~30 times smaller compared to the axisymmetric model.

Resistive Tearing Instability

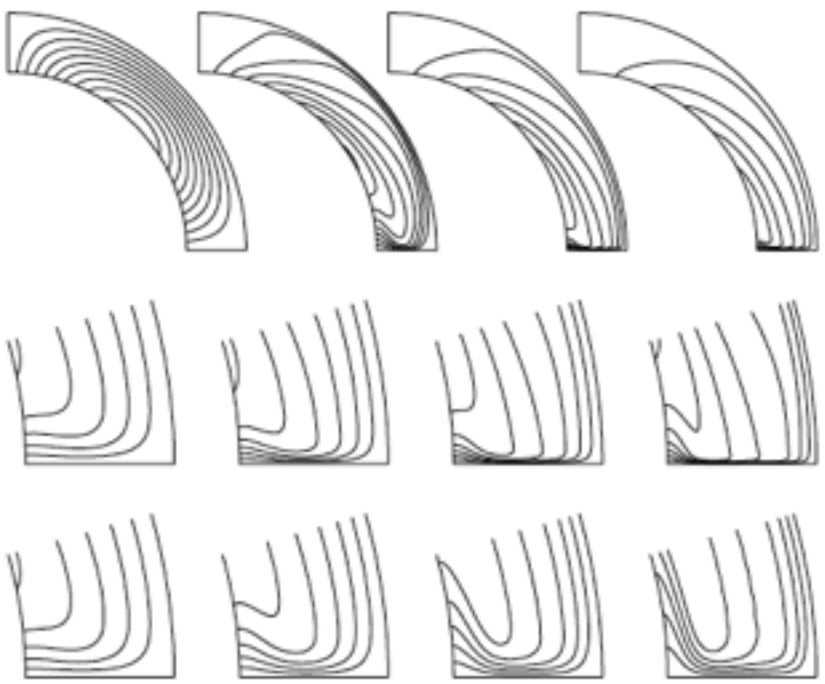
Motivation:



Wood and Hollerbach 2015

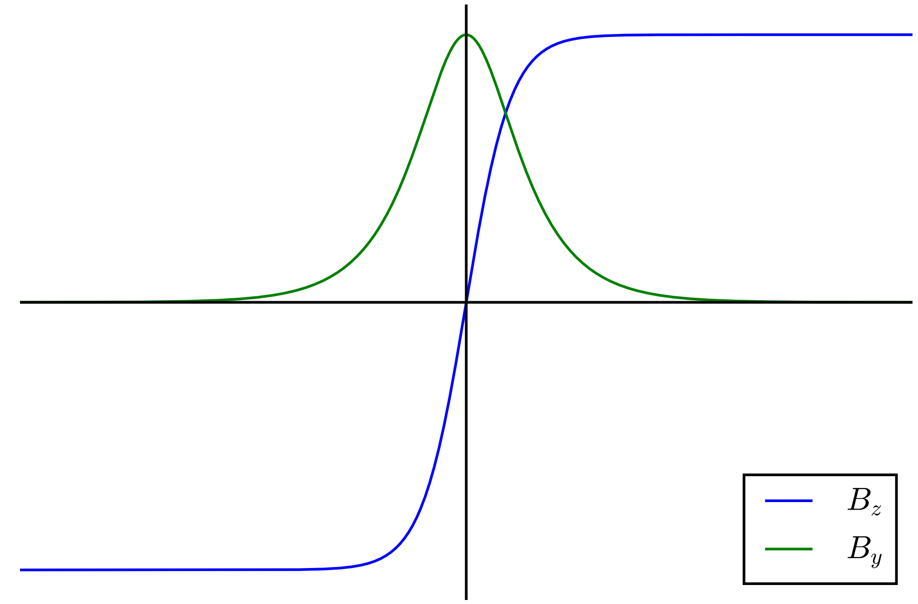


Lander 2014



Hollerbach and Rudiger 2004

Setup



$$\mathbf{B} = B_{y,0}\,\mathrm{sech}\left(\frac{x}{x_0}\right)\,\hat{\mathbf{y}} + B_{z,0}\tanh\left(\frac{x}{x_0}\right)\,\hat{\mathbf{z}}$$

(Hall equilibrium)

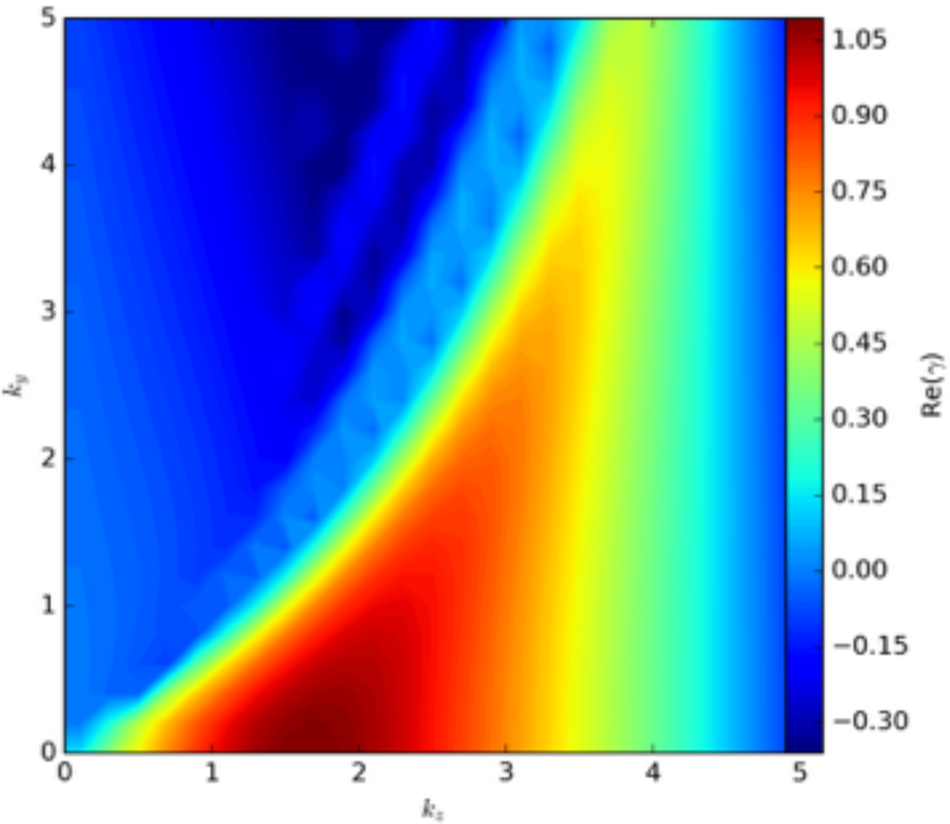
Perturbation:

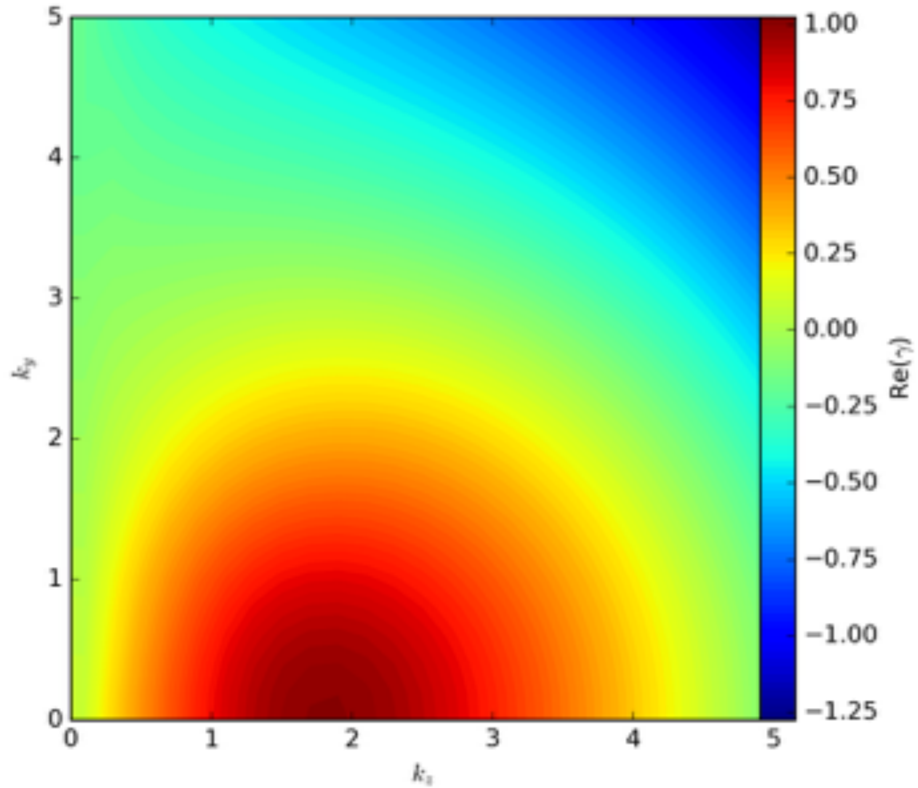
$$\boldsymbol{b} = \exp\left(\gamma t + i k_y y + i k_z z\right)\left[b_x(x)\hat{\boldsymbol{x}} + b_y(x)\hat{\boldsymbol{y}} + \left(i k_z^{-1} b_x' - k_y k_z^{-1} b_y\right)\hat{\boldsymbol{z}}\right]$$

$$\gamma b_x + \frac{c}{4\pi e n_e}\Big[k_z^2 B_z b_y - i k_z B_y' b_x + k_y\Big(i\left\{B_z' b_x - B_z b_x' - k_y k_z^{-1} B_y b_x'\right\} + \left\{k_z B_y + k_y^2 k_z^{-1} B_y + k_y B_z\right\} b_y\Big)\Big] + \frac{c^2}{4\pi\sigma}\left[\left(k_y^2 + k_z^2\right) b_x - b_x''\right] = 0\,,$$

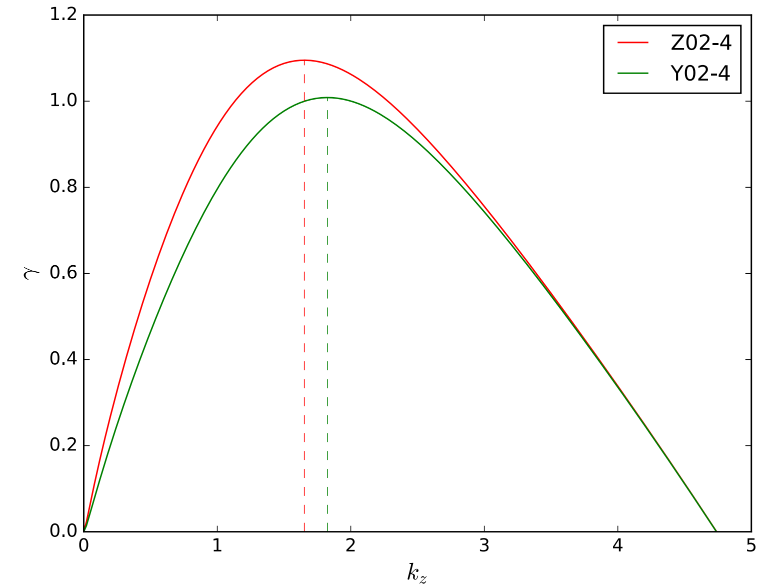
$$\gamma b_y + \frac{c}{4\pi e n_e}\Big[-k_z^2 B_z b_x - B_z'' b_x + B_z b_x'' + k_y\Big(i\left\{k_y k_z^{-1}\left(B_y b_y\right)' + \left(B_z b_y\right)'\right\} - k_z B_y b_x + k_z^{-1}\left(B_y b_x'\right)'\Big)\Big] + \frac{c^2}{4\pi\sigma}\left[\left(k_y^2 + k_z^2\right) b_y - b_y''\right] = 0$$

Evaluation of γ vs k_y, k_z through discretisation.





Real part of γ



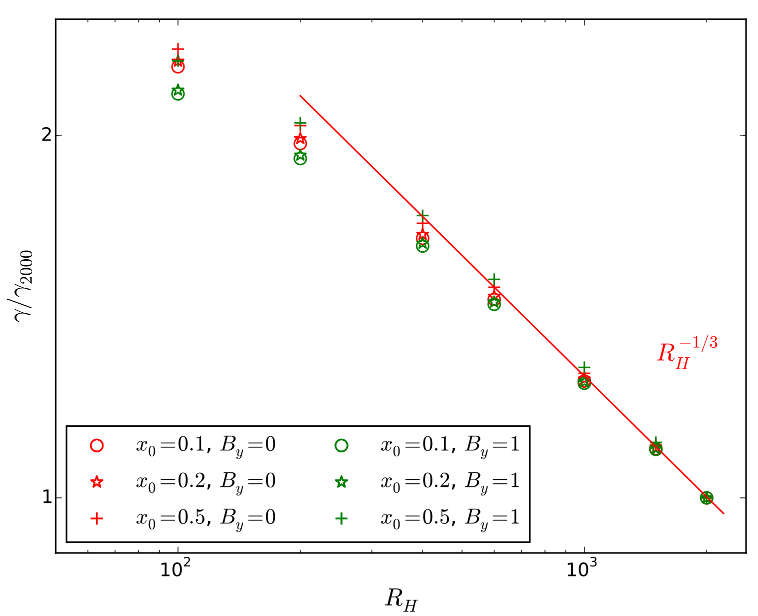
$B_{y,0}$=0, R_H=400, x_0=0.2

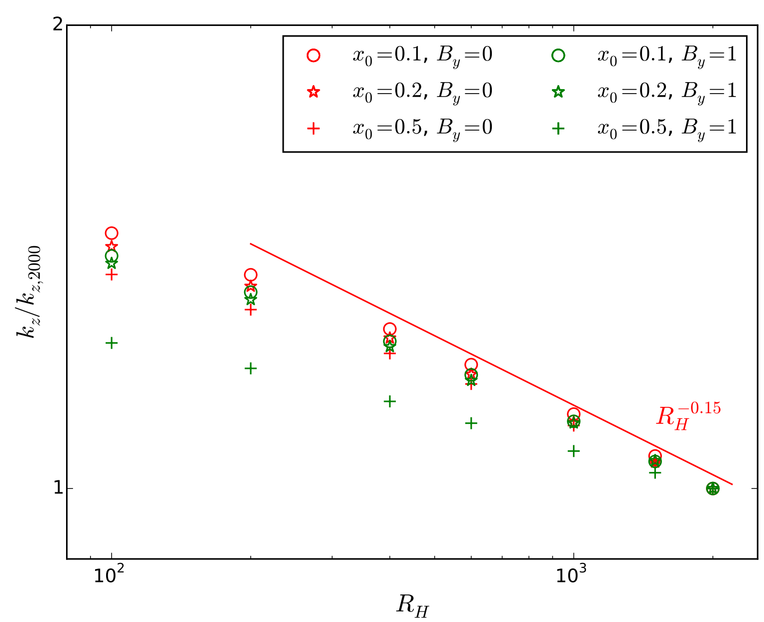
$B_{y,0}$=$B_{z,0}$, R_H=400, x_0=0.2

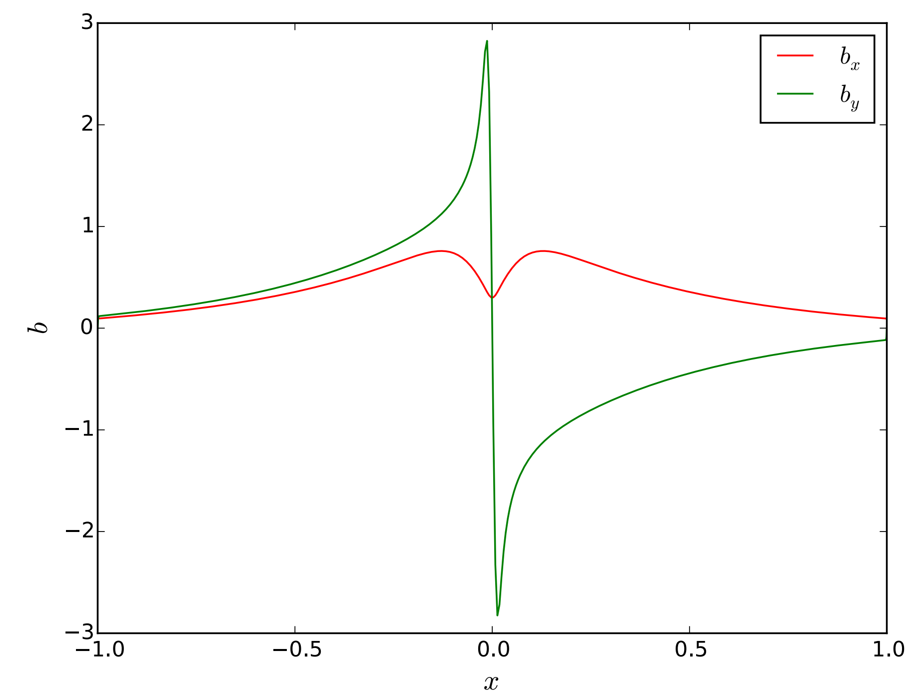
k_y=0

Perturbations in the y direction have a stabilising effect.

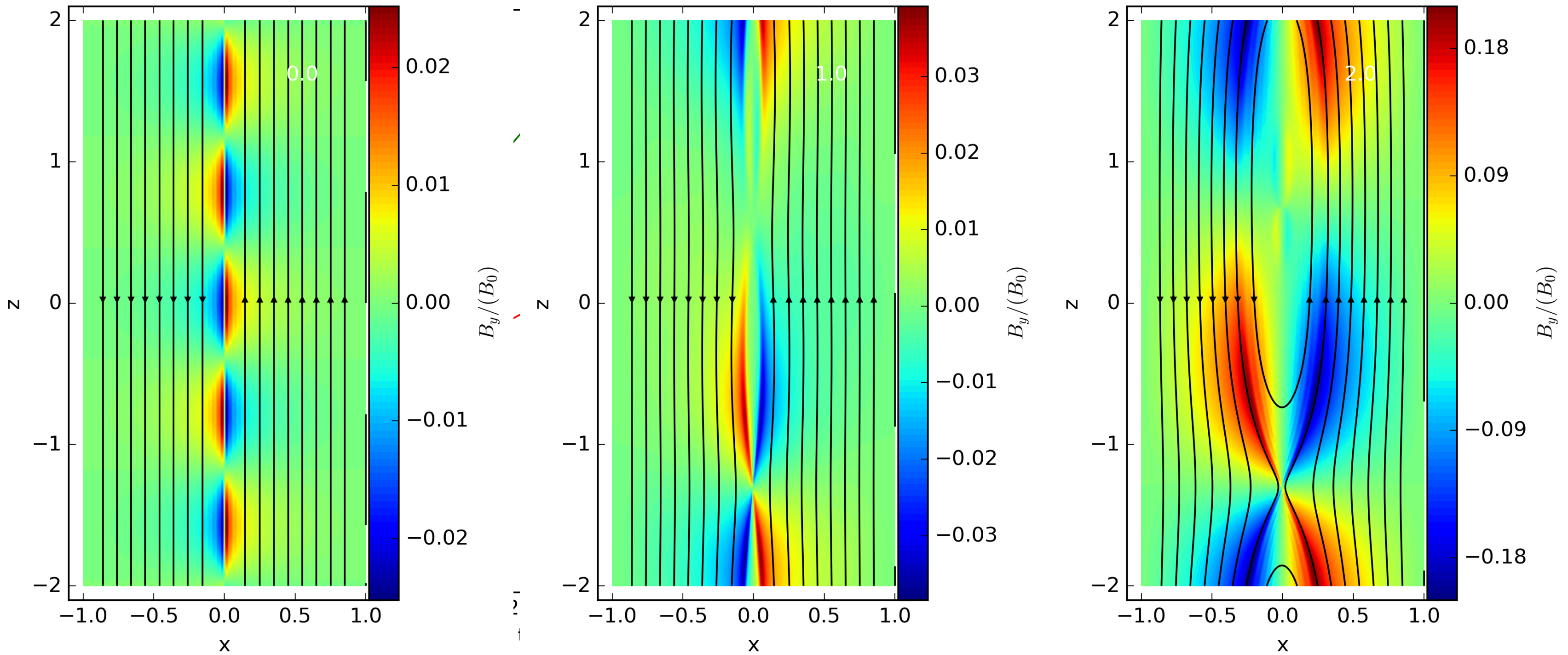
$$\boxed{\gamma \propto \sigma^{-1/3} B^{2/3}}$$





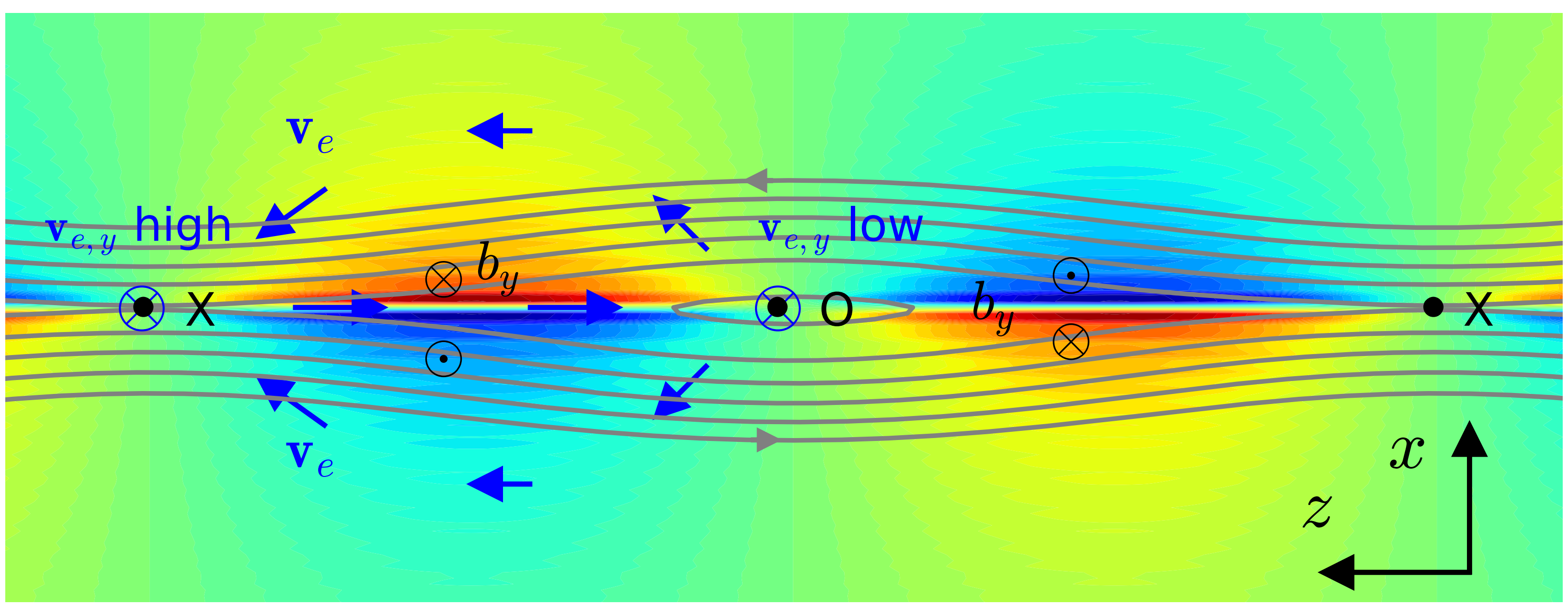


eigenfunctions



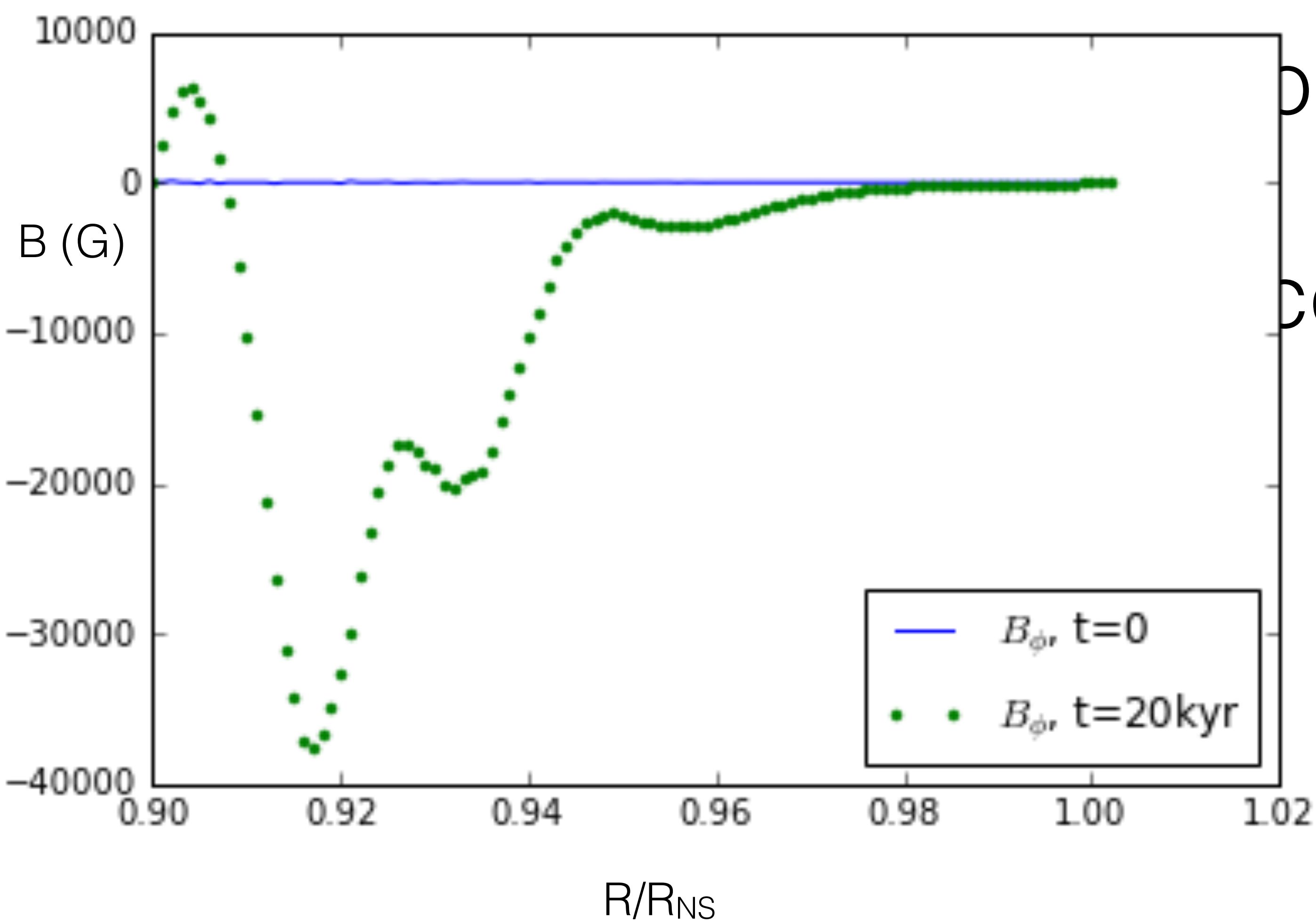
Non-linear evolution:

Early growth rate reproduces the results of linear theory.



Effect of tearing instability

- Speed up decay in current sheets.
- Generate smaller structure.
- onger magnetic fields.
- ced in simulations (but…)



Azimuthal field on the equator from an axially and equatorally symmetric simulation.

Summary

- Both ideal and non-ideal instabilities have been confirmed in Hall-MHD.
- Analytical and numerical exploration, agree very well with each other.
- Effect on neutron stars:
    - Ideal: generation of non-axisymmetric strong localised fields, hotspots, faster decay.
    - Resistive: speeds up magnetic dissipation in current sheets.

Thanks!