Extended-body effects in general relativity
What is possible?

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(with Michael Gaffney)

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Southampton Gravity Seminar
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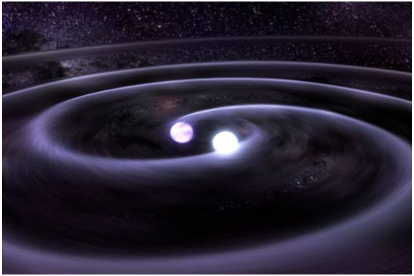

How do things fall?

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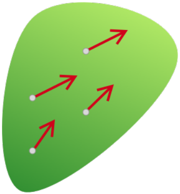
Equivalence principle: Free-fall is universal

Is that *really* how things fall?

In Newtonian gravity,

$$\ddot{\boldsymbol{z}} = - \int (\rho/m) \boldsymbol{\nabla} \phi dV.$$



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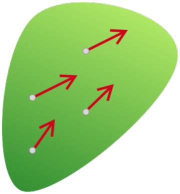

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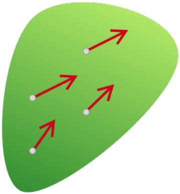

When is this *exactly* equal to $-\boldsymbol{\nabla}\phi$?

. . . at least when bodies are spherical.

Corrections to spherical-body motion

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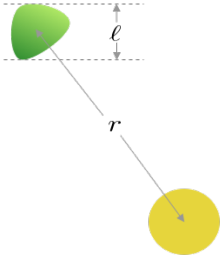
$$\ddot{z}_i = \underbrace{-\nabla_i \phi_{\rm ext}}_{\rm monopole} - \underbrace{\tfrac{1}{2}(\overbrace{Q^{jk}}^{\rm (quadrupole\ moment)}/m)\nabla_i \nabla_j \nabla_k \phi_{\rm ext}}_{\rm quadrupole} + \dots$$

Two types of corrections:

1. Self-interaction requires that ϕ be replaced by $\phi_{\rm ext}$.
2. Extended-body effects bring in aspects of the internal structure: Free-fall is no longer universal.

Newtonian extended-body effects

$$\ddot{z}_i = -\nabla_i \phi_{\mathrm{ext}} - \underbrace{\tfrac{1}{2}(Q^{jk}/m)\nabla_i \nabla_j \nabla_k \phi_{\mathrm{ext}}}_{\mathscr{O}[(\ell/r)^2 \,\times\, (\text{monopole})]} + \dots$$



At leading nontrivial order, effects of internal structure are determined by the **quadrupole moment** $Q^{ij} \sim m\ell^2$.

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. . . and maneuvers are forbidden mainly by symmetries.

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You need *gradients* to "grab onto..."

Motion in GR?

Extended-body effects derived in full by Dixon (1974), at least without self-interaction. Self-interaction incorporated later AIH (2012, 2015).

$$\dot{p}_a = -\frac{1}{2} R_{abcd} \dot{z}^b S^{cd} + (\text{int struct}),$$
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There can also be a nontrivial momentum-velocity relation:

$$p^a = m \dot{z}^a + \text{(hidden momentum)}$$

Universality of free-fall in GR

There are spin effects, but these aren't controllable.

First non-universal contributions to motion are from the quadrupole moment. This has 10 components in vacuum spacetimes.

Irrelevant quadrupole components

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1. Type D spacetimes:
   - At least 4 components are irrelevant.
   - In *Schwarzschild*, 5 components are irrelevant.
2. Type N:
   - At least 6 components are irrelevant.
   - For *linearly-polarized plane waves*, 9 are irrelevant.
3. . . .

Suppose that:

Rocket-free maneuvers in Schwarzschild [AIH, 2020]

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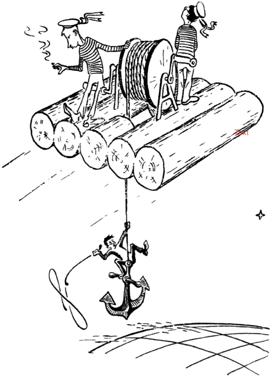
5 quadrupole components are irrelevant. 4 must be controlled to control the torque. There is 1 left over... What can this do?

Everything is in the mass

Extended-body effects enter *only* via changes in mass:

$$d m = - \mathscr{J} \, d(M/r^3).$$

E and $\vec{J}$ are conserved. E/m and $\vec{J}/m$ are not.



[Beletsky, 2001]

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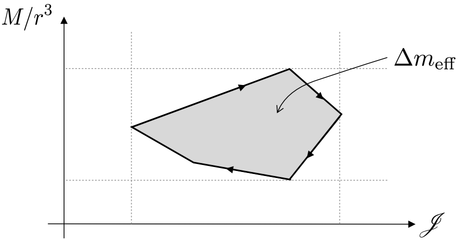

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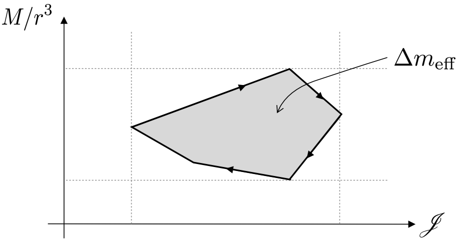
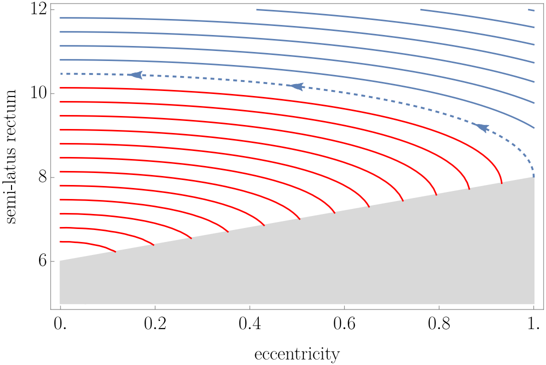


Repeating cycles with **nonzero area** raises or lowers the mass.

What do mass changes accomplish?



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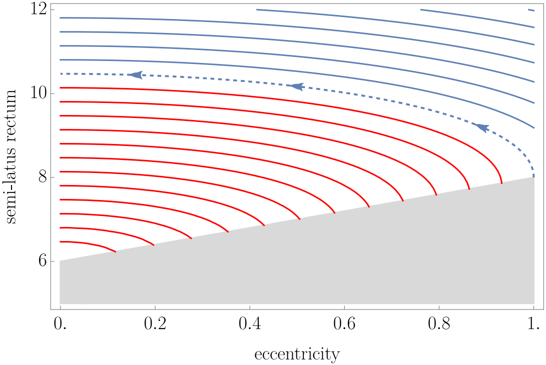

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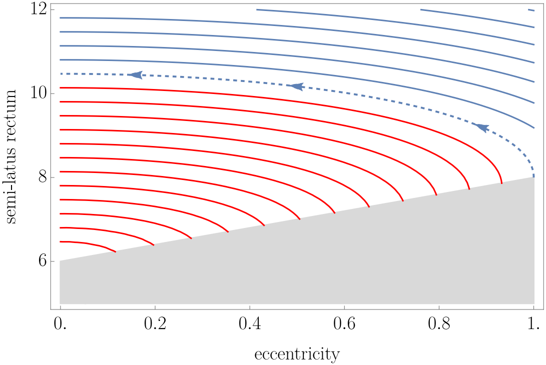
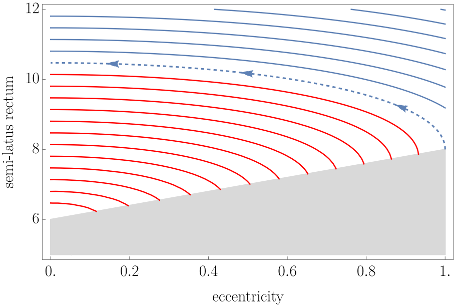


- Increasing mass $\Rightarrow$ circularization
- Decreasing mass is more complicated. . .

Effect of decreasing mass



1. If $r_i > 2(3+\sqrt{5})M \approx 10.5M$, you can escape!

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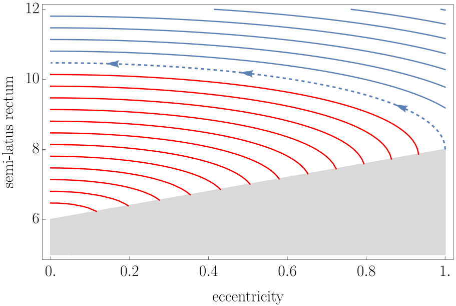
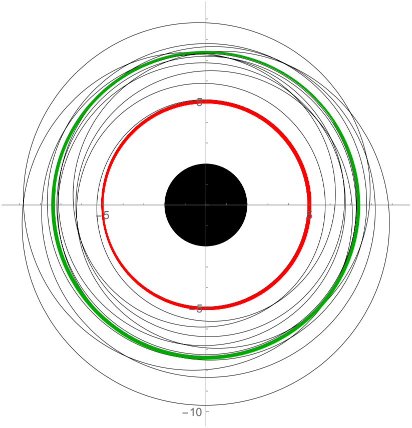
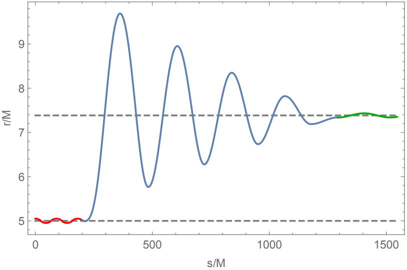


1. If $r_i > 2(3+\sqrt{5})M \approx 10.5M$, you can escape!
2. Otherwise, decreasing mass results in an approach to an unstable circular orbit, or a plunge.

Circular transfers

Move between pairs of circular geodesics with $4 < r/M < 10.5$.





Relativistic vs. Newtonian effects

[AIH (2020) vs. AIH & Gaffney (2020)]

In the spin-free, torque-free context, they're almost identical:

Concept	Relativistic	Newtonian
Torque-free force	$\nabla(\mathscr{I} M/r^3)$	$\nabla(\mathscr{I} M/r^3)$
Always constant	E, $\vec{L}$	m_N, $\vec{L}$
Const when $\mathscr{I}$ jumps	m	$E_{\rm pt}$
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$$E - m \sim E_{\mathrm{pt}}, \qquad E - m_{\mathrm{eff}} \sim E_{\mathrm{pt}} + U_{\mathrm{ext}}$$

Conclusions

1. Extended-body effects allow objects to control their motion.
2. They can change mass, allowing for bound orbits to be unbound, for unstable orbits to be stabilized, and more.

Even more is possible with spin and torque... There are many options for those who wait!