Ultimate Strength of Plates and Stiffened Panels

by

Professor Jeom Paik

The LRET Research Collegium
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Ultimate Strength
of Plates and Stiffened Panels

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The LRET Research Centre of Excellence
at Pusan National University, Korea
Overview

- Aims and Scope
- Rationally-Based Structural Design
- Methods used for Benchmark Study
- Theory of the ALPS/ULSAP Method
  - Ultimate Strength of Plates
  - Ultimate Strength of Stiffened Panels
- Benchmark Study
  - Target Structure: Unstiffened panel
  - Ultimate Strength of Unstiffened Plates under Biaxial Compression
  - Target Structure: Stiffened panel
  - Nonlinear FEA Modeling
  - Ultimate Strength of Stiffened Panels
- Concluding Remarks
Aims and Scope

Stiffened panel structure

Stiffened panel subject to a combined in-plane and lateral pressure load
Rationally-Based Structural Design
(Optimum structural design procedure based on ultimate limit state)

Modeling of Structure & Loads

Structural Response Analysis
Calculate Load Effects, Q

Limit State Analysis
Calculate Limit Values of Load Effects, Q_L

Evaluation
(A) Formulate constraints
\[ \gamma_1 \gamma_2 \gamma_3 \ Q \leq Q_L \]
(B) Evaluate adequacy
Constraints satisfied? Objective achieved?

Optimization
Objective
Methods Used for Benchmark Study

- Candidate methods
  - Nonlinear finite element method (ANSYS, MSC/MARC, Abaqus)
  - ALPS/ULSAP method
  - DNV/PULS method
Theory of the ALPS/ULSAP Method
Theory of the ALPS/ULSAP Method

ALPS/ULSAP (Analysis of Large Plated Structures / Ultimate Limit State Assessment Program), developed by Prof. J.K. Paik, Pusan National University

Paik & Thayamballi

Ship Structural Analysis and Design (2010)
Hughes & Paik
Ultimate Strength of Plates: $\sigma_u = \min(\sigma_{u1}, \sigma_{u2}, \sigma_{u3})$

(a) Plasticity at the corners

$$\sigma_{eq1} = \sqrt{\sigma_{x_{\text{max}}}^2 - \sigma_{x_{\text{max}}} \sigma_{y_{\text{max}}} + \sigma_{y_{\text{max}}}^2 + 3\tau^2} = \sigma_y$$

(b) Plasticity at the longitudinal mid-edges

$$\sigma_{eq2} = \sqrt{\sigma_{x_{\text{max}}}^2 - \sigma_{x_{\text{max}}} \sigma_{y_{\text{min}}} + \sigma_{y_{\text{min}}}^2 + 3\tau^2} = \sigma_y$$

(c) Plasticity at the transverse mid-edges

$$\sigma_{eq3} = \sqrt{\sigma_{x_{\text{min}}}^2 - \sigma_{x_{\text{min}}} \sigma_{y_{\text{max}}} + \sigma_{y_{\text{max}}}^2 + 3\tau^2} = \sigma_y$$

* : Expected plasticity location
T: Tension
C: Compression
Ultimate Strength of Stiffened Panels: 6 Types of Collapse Modes

\[ \sigma_u = \min(\sigma_u^I, \sigma_u^{II}, \sigma_u^{III}, \sigma_u^{IV}, \sigma_u^{V}, \sigma_u^{VI}) \]
- Candidate methods
  - Nonlinear finite element method (ANSYS, MSC/MARC, Abaqus)
  - ALPS/ULSAP method
  - DNV/PULS method
Target Structure: Unstiffened Panel

• Material and Geometric Properties
  • Yield stress of plate, $\sigma_{yp} = 313.6 \text{ N/mm}^2$
  • Yield stress of stiffener, $\sigma_{ys} = 313.6 \text{ N/mm}^2$
  • Elastic modulus, $E = 205800 \text{ N/mm}^2$
  • Poisson’s ratio, $\nu = 0.3$
  • Plate length, $a = 2550 \text{ mm}$
  • Plate breath, $b = 850 \text{ mm}$
  • Plate thickness, $t_p = 11, 16, 22, 33 \text{ mm}$
  • Under biaxial compressive loads
  • All edges simply supported
  • No residual stress
  • No lateral pressure
Unstiffened Plates under Biaxial Compression (1/3)

- **Buckling half-wave number**

\[
\frac{(m^2 / a^2 + 1 / b^2)^2}{m^2 / a^2 + c / b^2} \leq \frac{[(m + 1)^2 / a^2 + 1 / b^2]^2}{(m + 1)^2 / a^2 + c / b^2}
\]

where, \( c = \sigma_y / \sigma_x \)

\( \sigma_x \) and \( \sigma_y \) are the component of the longitudinal and transverse axial buckling stress of the plate under combined biaxial loading.

\( \sigma_{x,1} \) and \( \sigma_{y,1} \) are the component of the longitudinal or transverse axial buckling stress of the plate under uniaxial loading.
Unstiffened Plates under Biaxial Compression (2/3)

$t_p=11\text{mm}$

$a \times b = 2550 \times 850 (\text{mm})$

$\sigma_{yu}/\sigma_Y$

$t_p=16\text{mm}$

$a \times b = 2550 \times 850 (\text{mm})$

$\sigma_{xu}/\sigma_Y$

- FEM
- ALPS/ULSAP
- DNV/PULS
Unstiffened Plates under Biaxial Compression (3/3)

**tp = 22mm**

$a \times b = 2550 \times 850$ (mm)

$\sigma_{yu}/\sigma_Y$

$\sigma_{xu}/\sigma_Y$

- FEM
- ALPS/ULSAP
- DNV/PULS

**tp = 33mm**

$a \times b = 2550 \times 850$ (mm)

$\sigma_{yu}/\sigma_Y$

$\sigma_{xu}/\sigma_Y$

- FEM
- ALPS/ULSAP
- DNV/PULS
Target Structure: Stiffened Panel (1/2)

- Material and Geometric Properties
  
  - Yield stress of plate, $\sigma_{Yp} = 313.6 \text{ N/mm}^2$
  
  - Yield stress of stiffener, $\sigma_{Ys} = 313.6 \text{ N/mm}^2$
  
  - Elastic modulus, $E = 205800 \text{ N/mm}^2$
  
  - Poisson’s ratio, $\nu = 0.3$
  
  - Plate length, $a = 4750 \text{ mm}$
  
  - Plate breath, $b = 950 \text{ mm}$
  
  - Plate thickness, $t_p = 11, 12.5, 15, 18.5, 25, 37 \text{ mm}$
  
  - Number of the stiffeners: 8 stiffeners in a panel
  
  - No residual stress
Target Structure: Stiffened Panel (2/2)

**Dimensions of the Stiffeners**

<table>
<thead>
<tr>
<th></th>
<th>Flat bar ($h_w \times t_w$)</th>
<th>Angle bar ($h_n \times b_f \times t_w/t_f$)</th>
<th>Tee bar ($h_n \times b_f \times t_w/t_f$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size 1</td>
<td>150x17</td>
<td>138x90x9/12</td>
<td>138x90x9/12</td>
</tr>
<tr>
<td>Size 2</td>
<td>250x25</td>
<td>235x90x10/15</td>
<td>235x90x10/15</td>
</tr>
<tr>
<td>Size 3</td>
<td>350x35</td>
<td>383x100x12/17</td>
<td>383x100x12/17</td>
</tr>
<tr>
<td>Size 4</td>
<td>550x35</td>
<td>580x150x15/20</td>
<td>580x150x15/20</td>
</tr>
</tbody>
</table>

**Target Structure:** Stiffened Panel (2/2)

**Dimensions of the Stiffeners**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Size 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*N.A. = neutral axis*
ANSYS Nonlinear FEA Modeling (1/2)

- **Extent of Analysis: Two bay/two span model**

- **Boundary Conditions**

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-A‴ and D-D‴</td>
<td>Symmetric condition with $R_x = R_z = 0$ and uniform displacement in the $y$ direction ($U_y =$ uniform), coupled the plate part</td>
</tr>
<tr>
<td>A-D and A‴-D‴</td>
<td>Symmetric condition with $R_y = R_z = 0$ and uniform displacement in the $x$ direction ($U_x =$ uniform), coupled with longitudinal stiffeners</td>
</tr>
<tr>
<td>A'-D', A‴-D‴,</td>
<td>$U_z = 0$</td>
</tr>
<tr>
<td>B-B' and C-C'</td>
<td>$U_z = 0$</td>
</tr>
</tbody>
</table>
## Mesh Sizes Applied

<table>
<thead>
<tr>
<th>Size</th>
<th>Flat type (h₀ × t₀)</th>
<th>Angle type (h₀ × b₀ × t₀/t₀)</th>
<th>Tee type (h₀ × b₀ × t₀/t₀)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size 1</td>
<td>150 × 17(mm)</td>
<td>138 × 90 × 9/12(mm)</td>
<td>138 × 90 × 9/12(mm)</td>
</tr>
<tr>
<td></td>
<td>27600 elements</td>
<td>30800 elements</td>
<td>30800 elements</td>
</tr>
<tr>
<td>Size 2</td>
<td>250 × 25(mm)</td>
<td>235 × 90 × 10/15(mm)</td>
<td>235 × 90 × 10/15(mm)</td>
</tr>
<tr>
<td></td>
<td>27600 elements</td>
<td>30800 elements</td>
<td>30800 elements</td>
</tr>
<tr>
<td>Size 3</td>
<td>350 × 35(mm)</td>
<td>383 × 100 × 12/17(mm)</td>
<td>383 × 100 × 12/17(mm)</td>
</tr>
<tr>
<td></td>
<td>40400 elements</td>
<td>43600 elements</td>
<td>43600 elements</td>
</tr>
<tr>
<td>Size 4</td>
<td>550 × 35(mm)</td>
<td>580 × 150 × 15/20(mm)</td>
<td>580 × 150 × 15/20(mm)</td>
</tr>
<tr>
<td></td>
<td>50000 elements</td>
<td>53200 elements</td>
<td>53200 elements</td>
</tr>
</tbody>
</table>
MSC/MARC Nonlinear FEA Modeling by Prof. M. Fujikubo, Osaka University

- Extent of Analysis: Two bay/two span model

- Mesh Sizes Applied (22753 elements)
Abaqus Nonlinear FEA Modeling by Dr. Amlashi Hadi, DNV

- Extent of Analysis: Three bay/one span model

- Mesh Sizes Applied (7740 elements)
Nonlinear FEA Modeling for ANSYS and MSC/MARC

- **Plate Initial Deflection**

\[ w_{opl} = A_o \sin \frac{m \pi x}{a} \sin \frac{\pi y}{b} \]

- **Column Type Initial Distortion of Stiffener**

\[ w_{oc} = B_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]

- **Sideways Initial Distortion of Stiffener**

\[ w_{os} = C_o \frac{z}{h_o} \sin \frac{\pi x}{a} \]

\[ A_o = 0.1 \beta^2 t_p; \quad B_o = C_o = 0.0015a; \quad \beta = \frac{b}{t_p} \sqrt{\frac{\sigma_{yp}}{E}} \]
Nonlinear FEA Modeling for ANSYS and MSC/MARC

- Plate Initial Deflection
- Sideways Initial Distortion of Stiffener
- Column Type Initial Distortion of Stiffener
- Local deflection with amplification factor of 20
- Overall deflection with amplification factor of 50
- Initial deflection with amplification factor of 20
Ultimate Strength of Stiffened Panels (1/26)

- Flat Bar Under Longitudinal Uniaxial Compression

**Size 1**

Panel C: \( h_w \times t_w = 150 \times 17 \) (mm) (F)

- FEA (ANSYS)
- FEA (ABAQUS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)

**Size 2**

Panel C: \( h_w \times t_w = 250 \times 25 \) (mm) (F)

- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)
Ultimate Strength of Stiffened Panels (2/26)

- Flat Bar Under Longitudinal Uniaxial Compression

Panel C: \( h_w \times t_w = 350 \times 35 \) (mm) (F)

Panel C: \( h_w \times t_w = 550 \times 35 \) (mm) (F)

\[ \frac{\sigma_{xU}}{\sigma_{Yeq}} \] vs \( \left( \frac{b}{t_p} \right) \sqrt{\frac{\sigma_{Yp}}{E}} \)

Legend:
- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)

Size 3

Size 4
Ultimate Strength of Stiffened Panels (3/26)

- Angle Bar Under Longitudinal Uniaxial Compression

### Size 1

Panel C: $h_w \times b_f \times t_w/t_f = 138 \times 90 \times 9/12$ (mm) (A)

- FEA (ANSYS)
- FEA (AB AQUS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)

### Size 2

Panel C: $h_w \times b_f \times t_w/t_f = 235 \times 90 \times 10/15$ (mm) (A)

- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)
Ultimate Strength of Stiffened Panels (4/26)

- Angle Bar Under Longitudinal Uniaxial Compression

Size 3

Panel C: \( h_w \times b_f \times t_w/t_f = 383 \times 100 \times 12/17 \text{ (mm)} \) (A)

Size 4

Panel C: \( h_w \times b_f \times t_w/t_f = 580 \times 150 \times 15/20 \text{ (mm)} \) (A)
Ultimate Strength of Stiffened Panels (5/26)

- Tee Bar Under Longitudinal Uniaxial Compression

Size 1

Panel C: \( h_w \times b_t \times t_w / t_f = 138 \times 90 \times 9/12 \) (mm) (T)

- FEA (ANSYS)
- FEA (ABAQUS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)

\[ \frac{\sigma_{\text{Xu}}}{\sigma_{\text{Yeq}}} \]

\[ (b / t_p ) \sqrt{\frac{\sigma_{\text{Yp}}}{E}} \]

Size 2

Panel C: \( h_w \times b_t \times t_w / t_f = 235 \times 90 \times 10/15 \) (mm) (T)

- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)
- Design Formula (DNV/PULS)

\[ \frac{\sigma_{\text{Xu}}}{\sigma_{\text{Yeq}}} \]

\[ (b / t_p ) \sqrt{\frac{\sigma_{\text{Yp}}}{E}} \]
Ultimate Strength of Stiffened Panels (6/26)

- Tee Bar Under Longitudinal Uniaxial Compression

Size 3

Panel C: \( h_w \times b_t \times t_w / t_f = 383 \times 100 \times 12/17 \text{(mm)} \) (T)

Size 4

Panel C: \( h_w \times b_t \times t_w / t_f = 580 \times 150 \times 15/20 \text{(mm)} \) (T)
Ultimate Strength of Stiffened Panels (7/26)

- Under Longitudinal Uniaxial Compression

Flat bar under longitudinal compressive loads
FEA (ANSYS)
Design formula (ALPS/ULSAP)

Angle bar under longitudinal compressive loads
FEA (ANSYS)
Design formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (8/26)

- Under Longitudinal Uniaxial Compression

Tee bar

All stiffener types

Flat    Angle   Tee

Tee bar under longitudinal compressive loads
- FEA (ANSYS)
- Design formula (ALPS/ULSAP)

\[ \frac{\sigma_{xu}}{\sigma_{ Yeq}} \]

\[ \left( \frac{a}{\pi r} \right) \sqrt{\frac{\sigma_{ Yeq}}{E}} \]
Ultimate Strength of Stiffened Panels (9/26)

- Flat Bar Under Transverse Uniaxial Compression

**Size 1**

Panel C: \( h_w \times t_w = 150 \times 17 \text{ (mm) (F)} \)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Mode I**

\[ \sigma_{yu} / \sigma_{Yeq} \]

\[ (b / t_p) \sqrt{\sigma_{Yp} / E} \]

**Size 2**

Panel C: \( h_w \times t_w = 250 \times 25 \text{ (mm) (F)} \)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Mode III**

\[ \sigma_{yu} / \sigma_{Yeq} \]

\[ (b / t_p) \sqrt{\sigma_{Yp} / E} \]
Ultimate Strength of Stiffened Panels (10/26)

- Flat Bar Under Transverse Uniaxial Compression

Size 3

Panel C: \( h_w \times t_w = 350 \times 35 \) (mm) (F)
- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

Size 4

Panel C: \( h_w \times t_w = 550 \times 35 \) (mm) (F)
- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (11/26)

- Angle Bar Under Transverse Uniaxial Compression

### Size 1

Panel C: $h_w \times b_f \times t_w/t_f = 138 \times 90 \times 9/12$ (mm) (A)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

### Size 2

Panel C: $h_w \times b_f \times t_w/t_f = 235 \times 90 \times 10/15$ (mm) (A)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (12/26)

- Angle Bar Under Transverse Uniaxial Compression

**Size 3**

Panel C: \( h_w \times b_f \times t_w / t_f = 383 \times 100 \times 12 / 17 \text{ (mm)} \) (A)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Size 4**

Panel C: \( h_w \times b_f \times t_w / t_f = 580 \times 150 \times 15 / 20 \text{ (mm)} \) (A)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (13/26)

- Tee Bar Under Transverse Uniaxial Compression

Size 1

Panel C: \( h_w \times b_t \times t_w / t_f = 138 \times 90 \times 9/12 \text{ (mm)} \) (T)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

Mode III

Size 2

Panel C: \( h_w \times b_t \times t_w / t_f = 235 \times 90 \times 10/15 \text{ (mm)} \) (T)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

Mode III
Ultimate Strength of Stiffened Panels (14/26)

- Tee Bar Under Transverse Uniaxial Compression

**Panel C**: $h_w \times b_t \times t_w / t_f = 383 \times 100 \times 12/17$ (mm) (T)

**Mode III**

**Panel C**: $h_w \times b_t \times t_w / t_f = 580 \times 150 \times 15/20$ (mm) (T)

**Mode V**
Ultimate Strength of Stiffened Panels (15/26)

- Under Transverse Uniaxial Compression

Flat bar

Angle bar

\[
\frac{\sigma_{yu}}{\sigma_{Yeq}} vs (a / \pi r) \sqrt{\sigma_{Yeq} / E}
\]

Flat bar under transverse compressive loads
- FEA (ANSYS)
- Design formula (ALPS/ULSAP)

Angle bar under transverse compressive loads
- FEA (ANSYS)
- Design formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (16/26)

- Under Transverse Uniaxial Compression

Tee bar under transverse compressive loads
- FEA (ANSYS)
- Design formula (ALPS/ULSAP)

Flat Angle Tee

Size 4 Size 3 Size 2 Size 1

σ_yu/σ_Yeq

(a/πr)√σ_Yeq/E

(σ_yu/σ_Yeq)

(a/πr)√σ_Yeq/E
Ultimate Strength of Stiffened Panels (17/26)

- Buckling half-wave number

\[
\frac{(m^2 / a^2 + 1 / b^2)^2}{m^2 / a^2 + c / b^2} \leq \frac{[(m + 1)^2 / a^2 + 1 / b^2]^2}{(m + 1)^2 / a^2 + c / b^2}
\]

where, \( c = \sigma_y / \sigma_x \)

\( \sigma_x \) and \( \sigma_y \) are the component of the longitudinal and transverse axial buckling stress of the plate under combined biaxial loading.

\( \sigma_{x,1} \) and \( \sigma_{y,1} \) are the component of the longitudinal or transverse axial buckling stress of the plate under uniaxial loading.
Ultimate Strength of Stiffened Panels (18/26)

- Under Biaxial Compression (Flat Bar, $t_p=18.5\text{mm}$)

**Size 1**

Panel C: $t_p=18.5\text{mm}$

$h_w \times t_w = 150 \times 17(\text{mm})$ (F)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Size 2**

Panel C: $t_p=18.5\text{mm}$

$h_w \times t_w = 250 \times 25(\text{mm})$ (F)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (19/26)

- Under Biaxial Compression (Flat Bar, $t_p=18.5\text{mm}$)

**Size 3**

Panel C: $t_p=18.5\text{mm}$

$$h_w \times t_w = 350 \times 35(\text{mm}) \ (F)$$

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Mode III**

**Mode II**

**Size 4**

Panel C: $t_p=18.5\text{mm}$

$$h_w \times t_w = 550 \times 35(\text{mm}) \ (F)$$

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

**Mode IV**

**Mode III**

**Mode II**

**Mode IV**
Ultimate Strength of Stiffened Panels (20/26)

- Under Biaxial Compression (Angle Bar, $t_p=18.5\text{mm}$)

![Graph of Mode III and IV for Size 1 and Size 2](image)

Panel C: $t_p=18.5\text{mm}$

Size 1:
- $h_w \times b_f \times t_w / t_f = 138 \times 90 \times 9 / 12 (\text{mm})$ (A)
- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)

Size 2:
- $h_w \times b_f \times t_w / t_f = 235 \times 90 \times 10 / 15 (\text{mm})$ (A)
- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (21/26)

- Under Biaxial Compression (Angle Bar, $t_p=18.5$mm)

Panel C: $t_p=18.5$mm

<table>
<thead>
<tr>
<th>Size 3</th>
<th>Size 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph Size 3" /></td>
<td><img src="image2" alt="Graph Size 4" /></td>
</tr>
</tbody>
</table>

Design Formula (ALPS/ULSAP)

- Panel C: $t_p=18.5$mm
  - Size 3: $h_w \times b_f \times \frac{t_w}{t_f} = 383 \times 100 \times 12/17$ (mm) (A)
  - Size 4: $h_w \times b_f \times \frac{t_w}{t_f} = 580 \times 150 \times 15/20$ (mm) (A)

- FEA (ANSYS)
- Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (22/26)

- Under Biaxial Compression (Tee Bar, $t_p=18.5$mm)

Panel C: $t_p=18.5$mm

Size 1

Panel C: $t_p=18.5$mm

- $h_w \times b_t \times t_w/t_f = 138 \times 90 \times 9/12$ (mm) (T)
- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)

Mode III

Size 2

Panel C: $t_p=18.5$mm

- $h_w \times b_t \times t_w/t_f = 235 \times 90 \times 10/15$ (mm) (T)
- FEA (ANSYS)
- FEA (MSC/MARC)
- Design Formula (ALPS/ULSAP)

Mode III
Ultimate Strength of Stiffened Panels (23/26)

- Under Biaxial Compression (Tee Bar, $t_p=18.5\text{mm}$)

![Graph showing the comparison of ultimate strength for different modes and sizes of panels.](graphic)

- Size 3
  - Panel C: $t_p=18.5\text{mm}$
  - $h_w \times b_f \times t_w / t_f = 383 \times 100 \times 12 / 17(\text{mm}) (T)$
  - FEA (ANSYS)
  - FEA (MSC/MARC)
  - Design Formula (ALPS/ULSAP)

- Size 4
  - Panel C: $t_p=18.5\text{mm}$
  - $h_w \times b_f \times t_w / t_f = 580 \times 150 \times 15 / 20(\text{mm}) (T)$
  - FEA (ANSYS)
  - FEA (MSC/MARC)
  - Design Formula (ALPS/ULSAP)
Ultimate Strength of Stiffened Panels (24/26)

- Under Combined Longitudinal Compression and Lateral Pressure Loads

Plate-sided Pressure

Stiffener-sided Pressure
Ultimate Strength of Stiffened Panels (25/26)

- Under Combined Longitudinal Compression and Lateral Pressure Loads

Plate thickness, $t_p = 15$ mm

$h_w \times b_f \times t_w/t_f = 383 \times 100 \times 12/17$ (mm) (T)

![Diagram of stiffened panel with dimensions and notation]

Graph showing $\sigma_{xu}/\sigma_{Yeq}$ vs $\sigma$ with different modes and pressure types (Plate-sided and Stiffener-sided).
Ultimate Strength of Stiffened Panels (26/26)

- Under Combined Longitudinal Compression and Lateral Pressure Loads

Plate thickness, $t_p = 15\ mm$

\[ h_w \times b_f \times t_v/t_t = 383 \times 100 \times 12/17(\text{mm}) \ (T) \]

Plate-sided Pressure

(With $p=0.25\ \text{MPa}$, amplification factor of 10)

Stiffener-sided Pressure

(With $p=-0.25\ \text{MPa}$, amplification factor of 10)
Concluding Remarks
Concluding Remarks

- The dimension and material properties of a real ship panel was selected as a standard panel and a wider range of plating and stiffener dimensions were considered by varying the panel’s properties.

- The objective of the benchmark study reported in this paper was to check the accuracy of the ALPS/ULSAP method’s use to calculate the ultimate strength of plate and stiffened panel, compared with nonlinear finite element method.

- The ALPS/ULSAP method was found in a good agreement with the nonlinear finite element method computations through a wide range of panel dimensions and different loading conditions.

- The ALPS/ULSAP method is based on design formulations, the computational time required is extremely short compared to the nonlinear finite element method. So, this will be of great advantage in the structures design and safety assessment of ship structures comprising a large number of plate and panels.
Ultimate Strength of Hull Girders

Prof. Jeom Kee Paik, Director
The LRET Research Centre of Excellence
at Pusan National University, Korea
Overview

1. Background
2. Rationally-based structural design
3. Presumed stress distribution-based method
4. Methods applied for the ultimate hull strength analysis
5. Analysis results
6. Statistical analysis
7. Concluding remarks
Background

- To develop the modified Paik-Mansour formula method for the ultimate strength calculations of ship hulls subject to vertical bending moments.
- To validate the accuracy and applicability of modified Paik-Mansour formula method by comparing with more refined other methods.
Rationally-based structural design

(Optimum structural design procedure based on ultimate limit state)

- Modeling of Structure & Loads
  - Structural Response Analysis
    - Calculate Load Effects, Q
  - Limit State Analysis
    - Calculate Limit Values of Load Effects, Q_L

- Optimization
  - Objective

- Evaluation
  - (A) Formulate constraints
    \[ \gamma_1 \gamma_2 \gamma_3 Q \leq Q_L \]
  - (B) Evaluate adequacy
    - Constraints satisfied?
    - Objective achieved?
Methods for the ultimate hull strength analysis

• Numerical
  - Nonlinear FEM
  - Intelligent supersize FEM
  - Idealized structural unit method

• Analytical
  - Design formula

• Experimental
Presumed stress distribution-based method (1/8)

Sagging Hogging

Caldwell’s original formula method (1965)

\[ \int 
\sigma_x \, dA = 0 \]

\[ g_u = \frac{\sum_{i=1}^{n} |\sigma_{xi}| a_i z_i}{\sum_{i=1}^{n} |\sigma_{xi}| a_i} \]

\[ M_u = \sum_{i=1}^{n} \sigma_{xi} a_i (z_i - g_u) \]
Presumed stress distribution-based method (2/8)

Longitudinal bending stress distribution of tanker hull at ULS obtained by Nonlinear FEA
Presumed stress distribution-based method (3/8)

Longitudinal bending stress distribution of ship hull at ULS obtained by Nonlinear FEA

Dow's frigate test hull in sagging

Container ship in hogging

Bulk carrier in hogging
Presumed stress distribution-based method (4/8)

Original Paik-Mansour formula method (1995)

\[
\int \sigma_x \, dA = 0
\]

\[
g_u = \frac{\sum_{i=1}^{n} |\sigma_{xi}| a_i z_i}{\sum_{i=1}^{n} |\sigma_{xi}| a_i}
\]

\[
M_u = \sum_{i=1}^{n} \sigma_{xi} a_i (z_i - g_u)
\]
Presumed stress distribution-based method (5/8)

Modified Paik-Mansour formula method

\[ \int \sigma_x dA = 0 \]

\[ g_u = \frac{\sum_{i=1}^{n} |\sigma_{xi}| a_i z_i}{\sum_{i=1}^{n} |\sigma_{xi}| a_i} \]

\[ M_u = \sum_{i=1}^{n} \sigma_{xi} a_i (z_i - g_u) \]
Presumed stress distribution-based method (6/8)

\[
\frac{\sigma_{xu}}{\sigma_{Yeq}} = \frac{1}{\sqrt{0.995 + 0.936\lambda^2 + 0.170\beta^2 + 0.188\lambda^2 \beta^2 - 0.067\lambda^4}}
\]

\[
\lambda = \frac{a}{\pi r} \sqrt{\frac{\sigma_{Yeq}}{E}} \quad \beta = \frac{b}{t} \sqrt{\frac{\sigma_{yp}}{E}} \quad r = \sqrt{\frac{I}{A_s}}
\]

(Paik and Thayamballi, 1997)
Presumed stress distribution-based method (7/8)

Plate element: $a/b \geq 1$ (Paik at al. 2004)

$$\frac{\sigma_{xu}}{\sigma_{yp}} = \begin{cases} 
-0.032 \beta^4 + 0.002 \beta^2 + 1.0 & \text{for } \beta \leq 1.5 \\
1.274 / \beta & \text{for } 1.5 < \beta \leq 3.0 \\
1.248 / \beta^2 + 0.283 & \text{for } \beta > 3.0
\end{cases}$$

$$\beta = \frac{b}{t \sqrt{\frac{\sigma_{yp}}{E}}}$$

Plate element: $a/b < 1$ (Paik at al. 2004)

$$\frac{\sigma_{xu}}{\sigma_{yp}} = \frac{a}{b} \frac{\sigma^*_{xu}}{\sigma_{yp}} + \frac{0.475}{b} \left(1 - \frac{a}{b}\right)$$

$$\sigma^*_{xu} = \begin{cases} 
-0.032 \alpha^4 + 0.002 \alpha^2 + 1.0 & \text{for } \alpha \leq 1.5 \\
1.274 / \alpha & \text{for } 1.5 < \alpha \leq 3.0 \\
1.248 / \alpha^2 + 0.283 & \text{for } \alpha > 3.0
\end{cases}$$

$$\alpha = \frac{a}{t \sqrt{\frac{\sigma_{yp}}{E}}}$$
Presumed stress distribution-based method (8/8)

Modeling: Modified Paik-Mansour formula method
Methods applied for the ultimate hull strength analysis (1/5)

- Nonlinear FEM: ANSYS
- Intelligent supersize FEM: ALPS/HULL
- Idealized structural unit method (Smith method): IACS CSR by Dr. C.H. Huang
  (China Corporation Register of Shipping)
- Modified Paik-Mansour formula method

NLFEM model
ISFEM model
ISUM model
Modified P-M method
## Methods applied for the ultimate hull strength analysis (2/5)

<table>
<thead>
<tr>
<th>Geometric modeling</th>
<th>NLFEM (ANSYS)</th>
<th>ISUM/Smith Method (IACS CSR)</th>
<th>ISFEM (ALPS/HULL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Finite element model</td>
<td>Plate-stiffener combination model</td>
<td>Plate-stiffener separation model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formulation technique</th>
<th>NLFEM (ANSYS)</th>
<th>ISUM/Smith Method (IACS CSR)</th>
<th>ISFEM (ALPS/HULL)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma = \int [B]^T[D][B]d\text{vol})</td>
<td>(\sigma = \Phi \sigma_i)</td>
<td>(\sigma = \int [B]^T[D][B]d\text{vol})</td>
</tr>
<tr>
<td></td>
<td>: Numerical formulation</td>
<td>:Closed-form solution</td>
<td>: Numerical formulation</td>
</tr>
<tr>
<td></td>
<td>([D]): Numerical formulation</td>
<td>(\Phi = \text{edge function})</td>
<td>([D]): Closed-form solution</td>
</tr>
<tr>
<td></td>
<td>(\begin{aligned} &amp;-1 \quad \text{for} \quad \varepsilon &lt; -1 \ &amp;= \varepsilon \quad \text{for} \quad -1 &lt; \varepsilon &lt; 1 \ &amp;= 1 \quad \text{for} \quad \varepsilon &gt; 1 \end{aligned})</td>
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<table>
<thead>
<tr>
<th>Computational cost</th>
<th>Expensive</th>
<th>Cheap</th>
<th>Cheap</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Feature (1)</th>
<th>2 and 3-dimensional</th>
<th>2-dimensional</th>
<th>2 and 3-dimensional</th>
</tr>
</thead>
</table>

| Feature (2) | Can deal with interaction between local and global failures | Can not deal with interaction between local and global failures | Can deal with interaction between local and global failures |
Methods applied for the ultimate hull strength analysis (3/5)

Extent of the analysis

(a) The entire hull model

(b) The three cargo hold model

(c) The two cargo hold model

(d) The one cargo hold model

(e) The two-bay sliced hull cross-section model

(f) The one-bay sliced hull cross-section model
Methods applied
for the ultimate hull strength analysis (4/5)

Initial imperfections

\[ w_{\text{opl}} = A_o \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \]

\[ w_{\text{oc}} = B_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{B} \]

where, \( A_o = 0.1\beta^2 t_p \); \( B_o = C_o = 0.0015a \); \( \beta = \frac{b}{t_p} \sqrt{\frac{\sigma_{yp}}{E}} \)

\[ w_{\text{os}} = C_o \frac{z}{h_w} \sin \frac{\pi x}{a} \]
Application of vertical bending moments keeping the hull cross-section plane

Methods applied for the ultimate hull strength analysis (5/5)

Displacement control with neutral axis changes

\[
g = \frac{\sum_{i=1}^{n} \sigma_{xi} a_i z_i}{\sum_{i=1}^{n} |\sigma_{xi}| a_i}
\]

- \( g \) = **neutral axis position** from the baseline
- \( z_i \) = **distance from the ship’s baseline** (reference position) to the horizontal neutral axis of the \( i \)th structural component
- \( \sigma_{xi} \) = **longitudinal stress** of the \( i \)th structural component following the presumed stress distribution
- \( a_i \) = **cross-sectional area** of the \( i \)th structural component
- \( n \) = total number of structural components
NLFEM (ANSYS) Modeling

Dow’s Test Hull
- Total number of elements: 36,432
- Elements distribution:
  - Plate: 10
  - Web: 4
  - Flange: 2

Container Ship
- Total number of elements: 76,992
- Elements distribution:
  - Plate: 8
  - Web: 4
  - Flange: 1

Bulk Carrier
- Total number of elements: 271,680
- Elements distribution:
  - Plate: 10
  - Web: 8
  - Flange: 2

D/H Suezmax
- Total number of elements: 262,630
- Elements distribution:
  - Plate: 10
  - Web: 6
  - Flange: 2

S/H VLCC
- Total number of elements: 222,858
- Elements distribution:
  - Plate: 10
  - Web: 6
  - Flange: 1

D/H VLCC
- Total number of elements: 297,888
- Elements distribution:
  - Plate: 8
  - Web: 6
  - Flange: 2
ISFEM (ALPS/HULL) Modeling

Dow’s Test Hull
- Total number of elements: 196
  - Plate: 106 elements
  - Beam-column: 90 elements

Container Ship
- Total number of elements: 389
  - Plate: 231 elements
  - Beam-column: 158 elements

Bulk Carrier
- Total number of elements: 431
  - Plate: 243 elements
  - Beam-column: 188 elements

D/H Suezmax
- Total number of elements: 605
  - Plate: 367 elements
  - Beam-column: 238 elements

S/H VLCC
- Total number of elements: 453
  - Plate: 341 elements
  - Beam-column: 112 elements

D/H VLCC
- Total number of elements: 834
  - Plate: 442 elements
  - Beam-column: 392 elements
ISUM (CSR) Modeling by Dr. C. H. Huang
(China Corporation Register of Shipping, Taiwan)
Result - Case I: Dow’s Test Hull (1/6)

Curvature versus vertical bending moments

<table>
<thead>
<tr>
<th>Dow’s test hull</th>
<th>Design formula method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_u$ (MNm)</td>
<td>Original P-M</td>
<td>Modified P-M</td>
<td>$M_u$ ANSYS (MNm)</td>
<td>$M_u$ ALPS/HULL (MNm)</td>
</tr>
<tr>
<td></td>
<td>$h_C$ (mm) $h_Y$ (mm)</td>
<td>$h_C$ (mm) $h_Y$ (mm)</td>
<td>$h_C$ (mm) $h_Y$ (mm)</td>
<td></td>
<td></td>
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<tr>
<td>Hogging</td>
<td>10.338</td>
<td>210.000 0.000</td>
<td>210.000 0.000</td>
<td>11.235</td>
<td>10.698</td>
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<tr>
<td>Sagging</td>
<td>9.329</td>
<td>760.200 0.000</td>
<td>760.200 0.000</td>
<td>10.618</td>
<td>9.940</td>
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</table>

Note: $M_u$ = ultimate moment, $h_C$ = height of collapsed hull part, $h_Y$ = height of yielded hull part.
Result - Case II: Container Ship (2/6)

Curvature versus vertical bending moments

<table>
<thead>
<tr>
<th>Container ship</th>
<th>Design formula method</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M_u ) (GNm)</td>
<td>( h_C ) (mm)</td>
<td>( h_Y ) (mm)</td>
<td>ANSYS</td>
</tr>
<tr>
<td>Hogging</td>
<td>6.400</td>
<td>698.800</td>
<td>0.000</td>
<td>6.969</td>
</tr>
<tr>
<td>Sagging</td>
<td>7.077</td>
<td>10330.800</td>
<td>0.000</td>
<td>6.951</td>
</tr>
</tbody>
</table>

Note: \( M_u \) = ultimate moment, \( h_C \) = height of collapsed hull part, \( h_Y \) = height of yielded hull part.
**Result - Case III: Bulk Carrier (3/6)**

Curvature versus vertical bending moments

![Graph showing curvature versus bending moments for bulk carrier](image)

<table>
<thead>
<tr>
<th>Bulk carrier</th>
<th>Design formula method</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_u$ (GNm)</td>
<td>Original P-M</td>
<td>Modified P-M</td>
<td>$M_u$ ANSYS (GNm)</td>
<td>$M_u$ ALPS/HULL (GNm)</td>
<td>$M_u$ CSR (GNm)</td>
</tr>
<tr>
<td>Sagging</td>
<td>14.798</td>
<td>17935.000</td>
<td>17935.000</td>
<td>0.000</td>
<td>15.800</td>
<td>15.380</td>
</tr>
</tbody>
</table>

Note: $M_u = \text{ultimate moment}$, $h_C = \text{height of collapsed hull part}$, $h_Y = \text{height of yielded hull part}$. 
**Result - Case IV: Double Hull Suezmax Class Tanker (4/6)**

Curvature versus vertical bending moments

![Curvature versus vertical bending moments graph](graph)

<table>
<thead>
<tr>
<th></th>
<th>Double hull Suezmax tanker</th>
<th>Design formula method</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mu (GNm)</td>
<td>Original P-M</td>
<td>Mu (GNm)</td>
<td>Mu (GNm)</td>
<td>Mu (GNm)</td>
<td>Mu (GNm)</td>
</tr>
<tr>
<td></td>
<td>h_c (mm) h_y (mm)</td>
<td></td>
<td>h_c (mm) h_y (mm)</td>
<td>h_c (mm) h_y (mm)</td>
<td>h_c (mm) h_y (mm)</td>
<td>h_c (mm) h_y (mm)</td>
</tr>
<tr>
<td>Hogging</td>
<td>13.965</td>
<td>-</td>
<td>12.100</td>
<td>2210.600</td>
<td>14.066</td>
<td>13.308</td>
</tr>
<tr>
<td>Sagging</td>
<td>12.213</td>
<td>16078.500 0.000</td>
<td>16078.500 0.000</td>
<td>11.151</td>
<td>11.097</td>
<td>12.420</td>
</tr>
</tbody>
</table>

Note: \( M_u \) = ultimate moment, \( h_c \) = height of collapsed hull part, \( h_y \) = height of yielded hull part.
**Result - Case V: Single Hull VLCC Class Tanker (5/6)**

Curvature versus vertical bending moments

![Graphs showing curvature versus bending moments](image)

<table>
<thead>
<tr>
<th>Single hull VLCC tanker</th>
<th>Design formula method</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_u$ (GNm)</td>
<td>Original P-M</td>
<td>Modified P-M</td>
<td>$M_u$ ANSYS (GNm)</td>
<td>$M_u$ ALPS/HULL (GNm)</td>
<td>$M_u$ CSR (GNm)</td>
</tr>
<tr>
<td></td>
<td>$h_C$(mm)</td>
<td>$h_Y$(mm)</td>
<td>$h_C$(mm)</td>
<td>$h_Y$(mm)</td>
<td>$h_C$(mm)</td>
<td>$h_Y$(mm)</td>
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<tr>
<td>H hogging</td>
<td>18.701</td>
<td>7035.200</td>
<td>0.000</td>
<td>7035.200</td>
<td>0.000</td>
<td>17.355</td>
</tr>
<tr>
<td>S agging</td>
<td>17.825</td>
<td>15225.500</td>
<td>0.000</td>
<td>15225.500</td>
<td>0.000</td>
<td>16.179</td>
</tr>
</tbody>
</table>

Note: $M_u$ = ultimate moment, $h_C$ = height of collapsed hull part, $h_Y$ = height of yielded hull part.
Result - Case VI: Double Hull VLCC Class Tanker (6/6)

Curvature versus vertical bending moments

| Double hull VLCC tanker | Design formula method |  |  |  |  |
|-------------------------|-----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                         | $M_u$ (GNm)           | $h_c$ (mm)      | $h_y$ (mm)      | $M_u$ ANSYS (GNm) | $M_u$ ALPS/HULL (GNm) | $M_u$ CSR (GNm) |
| Hoggings                | 25.667                | -               | -               | 15.900           | 3816.000         | 27.335          |
| Sagging                | 22.390                | 20240.700       | 0.000           | 20240.700        | 0.000            | 22.495          | 22.000          | 24.798          |

Note: $M_u$ = ultimate moment, $h_c$ = height of collapsed hull part, $h_y$ = height of yielded hull part.
Result: Analysis Video
## Modified P-M Formula Method versus ANSYS Nonlinear FEA (1/2)

<table>
<thead>
<tr>
<th>Ship</th>
<th>$M_p$ (GNm)</th>
<th>Hogging Formula</th>
<th>Hogging ANSYS</th>
<th>Hogging Formula/ ANSYS</th>
<th>Sagging Formula</th>
<th>Sagging ANSYS</th>
<th>Sagging Formula/ ANSYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.010</td>
<td>0.772</td>
<td>0.011</td>
<td>0.840</td>
<td>0.920</td>
<td>0.009</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.400</td>
<td>0.694</td>
<td>6.969</td>
<td>0.756</td>
<td>0.918</td>
<td>7.077</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>16.576</td>
<td>0.813</td>
<td>17.500</td>
<td>0.858</td>
<td>0.947</td>
<td>14.798</td>
</tr>
<tr>
<td>D/H Suezmax</td>
<td>17.677</td>
<td>13.965</td>
<td>0.790</td>
<td>14.066</td>
<td>0.796</td>
<td>0.993</td>
<td>12.213</td>
</tr>
<tr>
<td>S/H VLCC</td>
<td>22.578</td>
<td>18.701</td>
<td>0.828</td>
<td>17.355</td>
<td>0.769</td>
<td>1.078</td>
<td>17.825</td>
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<tr>
<td>D/H VLCC</td>
<td>32.667</td>
<td>25.667</td>
<td>0.786</td>
<td>27.335</td>
<td>0.837</td>
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<td>0.828</td>
<td>17.355</td>
<td>0.769</td>
<td>1.078</td>
<td>17.825</td>
</tr>
<tr>
<td>D/H VLCC</td>
<td>32.667</td>
<td>25.667</td>
<td>0.786</td>
<td>27.335</td>
<td>0.837</td>
<td>0.939</td>
<td>22.390</td>
</tr>
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<td>Mean</td>
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<td></td>
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</tr>
<tr>
<td>S-D</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>COV</td>
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<td></td>
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</tbody>
</table>

### Note:
- $M_p$ = fully plastic bending capacity
- $M_{uh}$ = ultimate hogging moment
- $M_{us}$ = ultimate sagging moment
- S-D = standard deviation
- COV = coefficient of variation
Modified P-M Formula Method versus ANSYS Nonlinear FEA (2/2)

Formula/ANSYS:

Hog / Sag
Mean : 0.966 / 1.004
Standard deviation : 0.061 / 0.088
COV : 0.063 / 0.087

Statistical analysis Statistical analysis -- Mean and COV (2/12)
## Modified P-M Formula Method versus ALPS/HULL ISFEM (1/2)

<table>
<thead>
<tr>
<th>Ship</th>
<th>$M_p$ (GNm)</th>
<th>Hogging</th>
<th>Sagging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Formula</td>
<td>ALPS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_{uh}$ (GNm)</td>
<td>$M_{uh}/M_p$</td>
</tr>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.010</td>
<td>0.772</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.400</td>
<td>0.694</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>16.576</td>
<td>0.813</td>
</tr>
<tr>
<td>D/H Suezmax</td>
<td>17.677</td>
<td>13.965</td>
<td>0.790</td>
</tr>
<tr>
<td>S/H VLCC</td>
<td>22.578</td>
<td>18.701</td>
<td>0.828</td>
</tr>
<tr>
<td>D/H VLCC</td>
<td>32.667</td>
<td>25.667</td>
<td>0.786</td>
</tr>
<tr>
<td>Mean</td>
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<tr>
<td>S-D</td>
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<td>0.055</td>
</tr>
<tr>
<td>COV</td>
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<td></td>
<td>0.055</td>
</tr>
</tbody>
</table>

Note: $M_p$ = fully plastic bending capacity, $M_{uh}$ = ultimate hogging moment, $M_{us}$ = ultimate sagging moment, $S-D$ = standard deviation, COV = coefficient of variation
**Statistical analysis - Mean and COV (4/12)**

**Modified P-M Formula Method versus ALPS/HULL ISFEM (2/2)**

**Formula/ALPS:**

- **Hog / Sag**
- **Mean**: 1.003 / 1.020
- **Standard deviation**: 0.055 / 0.061
- **COV**: 0.055 / 0.060

**Graphs:**

- **Graph 1:**
  - Y-axis: $\frac{(M_u/M_p)}{\text{Formula}}$
  - X-axis: $\frac{(M_u/M_p)}{\text{ALPS/HULL}}$
  - Data points for different ship types.

- **Graph 2:**
  - Y-axis: $(M_u)_{\text{Formula}}$ (GNm)
  - X-axis: $(M_u)_{\text{ALPS/HULL}}$ (GNm)
  - Data points for different ship types.

**Legend:**

- **Hog / Sag**: Dow’s frigate test hull, Container ship, Bulk carrier, D/H Suezmax, S/H VLCC, D/H VLCC.
## Modified P-M Formula Method versus CSR ISUM (1/2)

### Statistical analysis - Mean and COV (5/12)

<table>
<thead>
<tr>
<th>Ship</th>
<th>( M_p ) (GNm)</th>
<th>( M_{uh} ) (GNm)</th>
<th>( M_{uh}/M_p )</th>
<th>( M_{uh}/CSR )</th>
<th>( M_{us} ) (GNm)</th>
<th>( M_{us}/M_p )</th>
<th>( M_{us}/CSR )</th>
<th>Formula/CSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.010</td>
<td>0.772</td>
<td>0.012</td>
<td>0.888</td>
<td>0.870</td>
<td>0.009</td>
<td>0.697</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.400</td>
<td>0.694</td>
<td>8.040</td>
<td>0.872</td>
<td>0.796</td>
<td>7.077</td>
<td>0.768</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>16.576</td>
<td>0.813</td>
<td>17.941</td>
<td>0.880</td>
<td>0.924</td>
<td>14.798</td>
<td>0.726</td>
</tr>
<tr>
<td>D/H Suezmax</td>
<td>17.677</td>
<td>13.965</td>
<td>0.790</td>
<td>15.714</td>
<td>0.889</td>
<td>0.889</td>
<td>12.213</td>
<td>0.691</td>
</tr>
<tr>
<td>S/H VLCC</td>
<td>22.578</td>
<td>18.701</td>
<td>0.828</td>
<td>19.889</td>
<td>0.881</td>
<td>0.940</td>
<td>17.825</td>
<td>0.789</td>
</tr>
<tr>
<td>D/H VLCC</td>
<td>32.667</td>
<td>25.667</td>
<td>0.786</td>
<td>28.352</td>
<td>0.868</td>
<td>0.905</td>
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<td>Mean</td>
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<td>0.887</td>
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<td>0.953</td>
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<tr>
<td>S-D</td>
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<td>0.051</td>
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<td>0.054</td>
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</tr>
<tr>
<td>COV</td>
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<td></td>
<td>0.058</td>
<td></td>
<td>0.056</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( M_p \) = fully plastic bending capacity, \( M_{uh} \) = ultimate hogging moment, \( M_{us} \) = ultimate sagging moment, S-D = standard deviation, COV = coefficient of variation
**Statistical analysis - Mean and COV (6/12)**

**Modified P-M Formula Method versus CSR ISUM (2/2)**

**Formula/CSR:**
- **Hog / Sag**
  - **Mean**: 0.887 / 0.953
  - **Standard deviation**: 0.051 / 0.054
  - **COV**: 0.058 / 0.056

**Graphs:**
- **(M_u/M_p)_{Formula}** vs. **(M_u/M_p)_{CSR}**
- **(M_u)_{Formula} (GNm)** vs. **(M_u)_{CSR} (GNm)**
# ANSYS Nonlinear FEA versus ALPS/HULL ISFEM (1/2)

<table>
<thead>
<tr>
<th>Ship</th>
<th>$M_p$ (GNm)</th>
<th><strong>Hogging</strong></th>
<th></th>
<th></th>
<th><strong>Sagging</strong></th>
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<th></th>
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<tbody>
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<td></td>
<td></td>
<td>ANSYS</td>
<td>ALPS</td>
<td>ANSYS / ANSYS</td>
<td>ANSYS</td>
<td>ALPS</td>
<td>ALPS / ANSYS</td>
</tr>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.011</td>
<td>0.840</td>
<td>0.011</td>
<td>0.799</td>
<td>0.952</td>
<td>0.011</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.969</td>
<td>0.756</td>
<td>6.916</td>
<td>0.750</td>
<td>0.992</td>
<td>6.951</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>17.500</td>
<td>0.858</td>
<td>16.602</td>
<td>0.814</td>
<td>0.949</td>
<td>15.800</td>
</tr>
<tr>
<td>D/H Suezmax</td>
<td>17.677</td>
<td>14.066</td>
<td>0.796</td>
<td>13.308</td>
<td>0.753</td>
<td>0.946</td>
<td>11.151</td>
</tr>
<tr>
<td>S/H VLCC</td>
<td>22.578</td>
<td>17.355</td>
<td>0.769</td>
<td>17.335</td>
<td>0.768</td>
<td>0.999</td>
<td>16.179</td>
</tr>
<tr>
<td>D/H VLCC</td>
<td>32.667</td>
<td>27.335</td>
<td>0.837</td>
<td>25.600</td>
<td>0.784</td>
<td>0.937</td>
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</tr>
<tr>
<td><strong>Mean</strong></td>
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<td></td>
<td>0.962</td>
<td></td>
<td></td>
<td></td>
<td>0.984</td>
</tr>
<tr>
<td><strong>S-D</strong></td>
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<td>0.026</td>
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<td>0.045</td>
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<tr>
<td><strong>COV</strong></td>
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<td></td>
<td>0.027</td>
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<td>0.046</td>
</tr>
</tbody>
</table>

*Note: $M_p$ = fully plastic bending capacity, $M_{uh}$ = ultimate hogging moment, $M_{us}$ = ultimate sagging moment, S-D = standard deviation, COV = coefficient of variation*
Statistical analysis - Mean and COV (8/12)

ANSYS Nonlinear FEA versus ALPS/HULL ISFEM (2/2)

ALPS/ANSYS:

Hog / Sag
Mean : 0.962 / 0.984
Standard deviation : 0.026 / 0.045
COV : 0.027 / 0.046

Hog / Sag
: Dow’s frigate test hull
: Container ship
: Bulk carrier
: D/H Suezmax
: S/H VLCC
: D/H VLCC

(M_u/M_p)_{ALPS/HULL} vs (M_u/M_p)_{ANSYS}

(M_u)_{ALPS/HULL} (GNm) vs (M_u)_{ANSYS} (GNm)
### ANSYS Nonlinear FEA versus CSR ISUM (1/2)

<table>
<thead>
<tr>
<th>Ship</th>
<th>( M_p ) (GNm)</th>
<th>( M_{uh} ) (GNm)</th>
<th>( M_{uh}/M_p )</th>
<th>( M_{uh}/M_{ANSYS} )</th>
<th>( M_{us} ) (GNm)</th>
<th>( M_{us}/M_p )</th>
<th>( M_{us}/M_{ANSYS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.011</td>
<td>0.840</td>
<td>0.012</td>
<td>0.888</td>
<td>0.011</td>
<td>0.793</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.969</td>
<td>0.756</td>
<td>8.040</td>
<td>0.872</td>
<td>6.951</td>
<td>0.754</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>17.500</td>
<td>0.858</td>
<td>17.941</td>
<td>0.880</td>
<td>15.800</td>
<td>0.775</td>
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<tr>
<td>D/H Suezmax</td>
<td>17.677</td>
<td>14.066</td>
<td>0.796</td>
<td>15.714</td>
<td>0.889</td>
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<td>S/H VLCC</td>
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<td>D/H VLCC</td>
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<td>0.868</td>
<td>22.495</td>
<td>0.689</td>
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<tr>
<td><strong>Mean</strong></td>
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<td><strong>S-D</strong></td>
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<td>0.056</td>
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<td></td>
</tr>
<tr>
<td><strong>COV</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.052</td>
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</tr>
</tbody>
</table>

**Note:** \( M_p \) = fully plastic bending capacity, \( M_{uh} \) = ultimate hogging moment, \( M_{us} \) = ultimate sagging moment, S-D = standard deviation, COV = coefficient of variation
ANSYS Nonlinear FEA versus CSR ISUM (2/2)

Statistical analysis - Mean and COV (10/12)

CSR/ANSYS:

Hog / Sag
Mean : 1.090 / 1.055
Standard deviation : 0.056 / 0.091
COV : 0.052 / 0.086

Dow’s frigate test hull
Container ship
Bulk carrier
D/H Suezmax
S/H VLCC
D/H VLCC

(M_u/M_p)CSR
(M_u/M_p)ANSYS

(M_u)CSR(GNm)
(M_u)ANSYS(GNm)
### Statistical analysis - Mean and COV (11/12)

#### ALPS/HULL ISFEM versus CSR ISUM (1/2)

<table>
<thead>
<tr>
<th>Ship</th>
<th>(M_p) (GNm)</th>
<th>Hogging</th>
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<th>Sagging</th>
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<th>CSR/ALPS</th>
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</thead>
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<tr>
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<td>ALPS</td>
<td>CSR</td>
<td>ALPS</td>
<td>CSR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(M_{uh}) (GNm)</td>
<td>(M_{uh}/M_p)</td>
<td>(M_{uh}) (GNm)</td>
<td>(M_{uh}/M_p)</td>
<td>(M_{us}) (GNm)</td>
<td>(M_{us}/M_p)</td>
</tr>
<tr>
<td>Dow's test hull</td>
<td>0.013</td>
<td>0.011</td>
<td>0.799</td>
<td>0.012</td>
<td>0.888</td>
<td>1.111</td>
</tr>
<tr>
<td>Container ship</td>
<td>9.220</td>
<td>6.916</td>
<td>0.750</td>
<td>8.040</td>
<td>0.872</td>
<td>1.163</td>
</tr>
<tr>
<td>Bulk carrier</td>
<td>20.394</td>
<td>16.602</td>
<td>0.814</td>
<td>17.941</td>
<td>0.880</td>
<td>1.081</td>
</tr>
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<td>D/H Suezmax</td>
<td>17.677</td>
<td>13.308</td>
<td>0.753</td>
<td>15.714</td>
<td>0.889</td>
<td>1.181</td>
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<td>S/H VLCC</td>
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<td>17.335</td>
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<tr>
<td>S-D</td>
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<tr>
<td>COV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** \(M_p\) = fully plastic bending capacity, \(M_{uh}\) = ultimate hogging moment, \(M_{us}\) = ultimate sagging moment, S-D = standard deviation, COV = coefficient of variation.
Statistical analysis - Mean and COV (12/12)

ALPS/HULL ISFEM versus CSR ISUM (2/2)

CSR/ALPS:

Hog / Sag
Mean : 1.132 / 1.072
Standard deviation: 0.038 / 0.087
COV : 0.034 / 0.081

(Csr/Mp)CSR
(Csr/Mp)ALPS/HULL

Hog / Sag
: Dow’s frigate test hull
: Container ship
: Bulk carrier
: D/H Suezmax
: S/H VLCC
: D/H VLCC

(Mu)CSR (GNm)
(Mu)ALPS/HULL (GNm)
Concluding Remarks (1/2)

- Four methods, namely NLFEM (ANSYS), ISFEM (ALPS/HULL), ISUM (CSR method), and Modified P-M formula method have been considered.

- Modified P-M formula method calculations are in good agreement with ANSYS nonlinear FEA and ALPS/HULL progressive collapse simulations.
Concluding Remarks (2/2)

- Statistical analysis of the hull girder ultimate strength based on comparisons among the various computation is carried out in terms of their mean values and coefficient of variation.

<table>
<thead>
<tr>
<th>Ship</th>
<th>Formula/ ANSYS</th>
<th>Formula/ ALPS</th>
<th>Formula/ CSR</th>
<th>ALPS/ ANSYS</th>
<th>CSR/ ANSYS</th>
<th>CSR/ ALPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow's test hull</td>
<td>0.920</td>
<td>0.879</td>
<td>0.966</td>
<td>0.939</td>
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</tr>
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<td>Container ship</td>
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<td>1.067</td>
<td>0.796</td>
<td>0.902</td>
</tr>
<tr>
<td>Bulk carrier</td>
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<td>0.937</td>
<td>0.998</td>
<td>0.962</td>
<td>0.924</td>
<td>1.022</td>
</tr>
<tr>
<td>D/H Suezmax</td>
<td>0.993</td>
<td>1.095</td>
<td>1.049</td>
<td>1.101</td>
<td>0.889</td>
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<td>S/H VLCC</td>
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<td>1.079</td>
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<td>1.003</td>
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<td>S-D</td>
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<td>0.061</td>
<td>0.051</td>
<td>0.054</td>
</tr>
<tr>
<td>COV</td>
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<td>0.087</td>
<td>0.055</td>
<td>0.060</td>
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<td>0.056</td>
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